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# New fast Walsh–Hadamard–Hartley transform algorithm

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**ABSTRACT**

This paper presents an efficient fast Walsh–Hadamard–Hartley transform (FWHT) algorithm that incorporates the computation of the Walsh-Hadamard transform (WHT) with the discrete Hartley transform (DHT) into an orthogonal, unitary single fast transform possesses the block diagonal structure. The proposed algorithm is implemented in an integrated butterfly structure utilizing the sparse matrices factorization approach and the Kronecker (tensor) product technique, which proved a valuable and fast tool for developing and analyzing the proposed algorithm. The proposed approach was distinguished by ease of implementation and reduced computational complexity compared to previous algorithms, which were based on the concatenation of WHT and FHT by saving up to 3N-4 of real multiplication and 7.5N-10 of real addition.



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## 1. INTRODUCTION

The orthogonal transforms and their fast algorithms have a vital role in a variety of fields such as signal processing [1], image encryption [2], [3] digital watermarking [4], wireless communication systems [5]–[7] and many other fields [8]–[10]. The importance of these transforms has prompted numerous researchers to use various techniques and methodologies, which have developed a wide variety of fast algorithms and novel transforms to solve current problems and challenges or satisfy the criteria of modern applications. One of the most effective approaches to creating new transforms is combining two orthogonal transforms to provide a realistic and cost-effective solution while retaining quality by benefiting from their unique advantages, besides exchanging resources [11]–[15].

Discrete Hartley transform (DHT) is a significant member of the orthogonal transformations family, representing an efficient tool for a wide range of applications [2], [4], [15]–[18]. The importance of DHT is due to its use as an alternative to the discrete Fourier transform (DFT) for real input; therefore, utilizing DHT achieves a significant increase in computing efficiency by saving arithmetic operations and memory storage. Additionally, the DHT has the self-inverse characteristic, which means that except for the scale factor, the same algorithm is used for forward and inverse [16], [17], [19]. Another essential member of orthogonal transforms is the Walsh-Hadamard transform (WHT), a common tool in a wide range of applications [3], [5], [8], [20]–[23]. WHT's key distinguishing feature is that its computation does not include any multiplication or division.

Therefore, in this paper, a radix-2 fast Walsh–Hadamard–Hartley transform (FWHT) algorithm integrates the computation of both WHT and DHT into a single fast algorithm for the length power-of-two sequences. The primary advantage of the radix-2 FWHT algorithm is that it concurrently computes both transforms (WHT and FHT) utilizing a single butterfly. Furthermore, the FWHT has more efficient performance and lower arithmetic complexity than the traditional technique based on the concatenation of fast WHT and DHT algorithms. Accordingly, the proposed algorithm can be applied to a wide range of applications, such as in orthogonal frequency division multiplexing (OFDM) systems.

The remainder of the paper is structured in the following manner. The development of the proposed algorithm is completely derived in section 2. Section 3 discusses the applications of the developed algorithm. Section 4 discusses computational complexity and comparisons. The conclusion is presented in section 5.

## 2. DERIVATION OF THE ALGORITHM

Radix-2 FWHT starts by constructing the $H_N$ matrix, which is equal to the product of the WHT and DHT matrices as (1):

$$H_N = \frac{1}{N} WH_N \hat{\mathrm{DH}}_N \tag{1}$$

where $H_N$ is used to denote the H-transform matrix, $WH_N$ is the Walsh–Hadamard matrix, and $\hat{\mathrm{DH}}_N$ denotes the discrete Hartley by rearranging the rows in bit reversed order. The DHT matrix in (1) has order $N$ and may be expressed in terms of the lower order $N$/2. [24], [25] as shown in (2).

$$\hat{\mathrm{DH}}_N = \begin{bmatrix} \hat{\mathrm{DH}}_{\frac{N}{2}} & \hat{\mathrm{DH}}_{\frac{N}{2}} \\ \hat{\mathrm{DH}}_{\frac{N}{2}} D_{\frac{N}{2}} & -\hat{\mathrm{DH}}_{\frac{N}{2}} D_{\frac{N}{2}} \end{bmatrix} \tag{2}$$

$$\mathrm{D}_{N/2} = \mathrm{diag}(\cos(\tfrac{2\pi k}{N})) + \mathrm{diag}(\sin(\tfrac{2\pi k}{N})) \times (1 \oplus \mathrm{J}_{\frac{N}{2}-1}) \tag{3}$$

where $0 \le k \le \frac{N}{2} - 1$ and $J_N$ is the exchange matrix of order $N$ (i.e., reverse diagonal unity matrix) specified by:

$$\mathrm{J}_N = \begin{bmatrix} & & & 1 \\ & & 1 & \\ & \iddots & & \\ 1 & & & \end{bmatrix}$$

as shown in (2) may be factorized into the (4).

$$\begin{aligned} \hat{\mathrm{DH}}_N &= \begin{bmatrix} \hat{\mathrm{DH}}_{N/2} & 0 \\ 0 & \hat{\mathrm{DH}}_{N/2} \end{bmatrix} \begin{bmatrix} \mathrm{I}_{N/2} & \mathrm{I}_{N/2} \\ \mathrm{D}_{N/2} & -\mathrm{D}_{N/2} \end{bmatrix} \\ &= \begin{bmatrix} \hat{\mathrm{DH}}_{N/2} & 0 \\ 0 & \hat{\mathrm{DH}}_{N/2} \end{bmatrix} \begin{bmatrix} \mathrm{I}_{N/2} & 0 \\ 0 & \mathrm{D}_{N/2} \end{bmatrix} \begin{bmatrix} \mathrm{I}_{N/2} & \mathrm{I}_{N/2} \\ \mathrm{I}_{N/2} & -\mathrm{I}_{N/2} \end{bmatrix} \end{aligned} \tag{4}$$

The factorization shown in (4) can be stated in terms of tensor (Kronecker) product as (5).

$$\hat{\mathrm{DH}}_N = \left(\mathrm{I}_2 \otimes \hat{\mathrm{DH}}_{N/2}\right) \Delta_N \;\; \mathrm{DH}_2 \otimes \mathrm{I}_{N/2} \tag{5}$$

Using similar approach, $\mathrm{WH}_N$ can be written as (6):

$$\mathrm{WH}_N = \mathrm{I}_2 \otimes \mathrm{WH}_{N/2} \quad \mathrm{WH}_2 \otimes \mathrm{I}_{N/2} \tag{6}$$

Substituting (5) and (6) into (1), the general radix-2 H-matrix of order $N$=2$^{m}$ can be expressed as (7).

$$H_N = \frac{1}{N}\ I_2 \otimes WH_{2^{m-1}}\quad WH_2 \otimes I_{2^{m-1}}\ \left(I_2 \otimes \hat{DH}_{2^{m-1}}\right)\Delta_{2^m}\ \ DH_2 \otimes I_{2^{m-1}} \tag{7}$$

The terms product $WH_2 \otimes I_{2^{m-1}}\ \left(I_2 \otimes \hat{DH}_{2^{m-1}}\right)$ can be exchanged with each other, with the aid of a result of the multiplication rule for tensor product [26].

Therefore (7) can be written as (8):

$$\begin{aligned} H_N &= \frac{1}{N}\ I_2 \otimes WH_{2^{m-1}}\ \left(I_2 \otimes \hat{DH}_{2^{m-1}}\right)\ WH_2 \otimes I_{2^{m-1}}\quad \Delta_{2^m}\ \ DH_2 \otimes I_{2^{m-1}} \\ &= \frac{1}{N}\left(I_2 \otimes WH_{2^{m-1}}\ \hat{DH}_{2^{m-1}}\right)\ WH_2 \otimes I_{2^{m-1}}\quad \Delta_{2^m}\ \ DH_2 \otimes I_{2^{m-1}} \\ &= \frac{1}{N}\left(I_2 \otimes WH_{2^{m-1}}\ \hat{DH}_{2^{m-1}}\right) H_N^{I} \end{aligned} \tag{8}$$

$$H_N^{I} = \ WH_2 \otimes I_{2^{m-1}}\quad \Delta_{2^m}\ \ DH_2 \otimes I_{2^{m-1}} \tag{9}$$

with the aid of (6) and (5), respectively, the product $WH_{2^{m-1}}\ \hat{DH}_{2^{m-1}}$ can be factorized to,

$$\hat{DH}_{2^{m-1}} = \left(I_2 \otimes \hat{DH}_{2^{m-2}}\right)\Delta_{2^{m-1}}\ \ DH_2 \otimes I_{2^{m-2}} \tag{10}$$

$$WH_{2^{m-1}} = \ I_2 \otimes WH_{2^{m-2}}\quad WH_2 \otimes I_{2^{m-2}} \tag{11}$$

Substituting $WH_{2^{m-1}}$ and $\hat{DH}_{2^{m-1}}$ by their values in (10) and (11) into (8), we obtain,

$$H_N = \frac{1}{N}\left(I_2 \otimes\ I_2 \otimes WH_{2^{m-2}}\quad WH_2 \otimes I_{2^{m-2}}\ \left(I_2 \otimes \hat{DH}_{2^{m-2}}\right)\Delta_{2^{m-1}}\ \ DH_2 \otimes I_{2^{m-2}}\ \right) H_N^{I} \tag{12}$$

Employing the same strategy as in (7); therefore, (12) can be expressed as (13):

$$H_N = \frac{1}{N}\left(I_4 \otimes WH_{2^{m-2}}\ \hat{DH}_{2^{m-2}}\right) H_N^{II}\ H_N^{I} \tag{13}$$

where

$$H_N^{II} = \ I_2 \otimes\ WH_2 \otimes I_{2^{m-2}}\quad I_2 \otimes \Delta_{2^{m-1}}\quad I_2 \otimes DH_2 \otimes I_{2^{m-2}} \tag{14}$$

This factorization will be repeated, and after $\log_2 N$ stages, the final stage will be denoted as (15):

$$\begin{aligned} \left(I_{2^{m-2}} \otimes WH_4\ \hat{DH}_4\right) &= \ I_{2^{m-2}} \otimes\ \ I_2 \otimes WH_2\quad WH_2 \otimes I_2\quad I_2 \otimes DH_2\ \ \Delta_4\ \ DH_2 \otimes I_2 \\ &= \ I_{2^{m-2}} \otimes\ \ I_2 \otimes WH_2\quad I_2 \otimes DH_2\quad WH_2 \otimes I_2\ \ \Delta_4\ \ DH_2 \otimes I_2 \\ &= \ I_{2^{m-1}} \otimes WH_2\quad I_{2^{m-1}} \otimes DH_2\quad I_{2^{m-2}} \otimes WH_2 \otimes I_2\quad I_{2^{m-2}} \otimes \Delta_4\quad I_{2^{m-2}} \otimes DH_2 \otimes I_2 \end{aligned} \tag{15}$$

By combining (9)–(15) and utilizing the fact $\Delta_2 = I_2$ ,The H-matrix can be decomposed into,

$$H_N = \frac{1}{N}\; I_{2^{m-1}} \otimes WH_2 \quad I_{2^{m-1}} \otimes \Delta_2 \quad I_{2^{m-1}} \otimes DH_2 \quad I_{2^{m-2}} \otimes WH_2 \otimes I_2 \quad I_{2^{m-2}} \otimes \Delta_4$$
$$\times\; I_{2^{m-2}} \otimes DH_2 \otimes I_2 \;\cdots\cdots\; I_2 \otimes WH_2 \otimes I_{2^{m-2}} \quad I_2 \otimes \Delta_{2^{m-1}} \quad I_2 \otimes DH_2 \otimes I_{2^{m-2}} \tag{16}$$
$$\times\; WH_2 \otimes I_{2^{m-1}} \quad \Delta_{2^m} \quad DH_2 \otimes I_{2^{m-1}}$$

Since $N$ is a power of two, after that (16) can be written in compact form as (17):

$$H_N = \prod_{i=0}^{m-1} I_{2^{m-i-1}} \otimes WH_2 \otimes I_{2^i} \quad I_{2^{m-i-1}} \otimes \frac{1}{2}\Delta_{2^{i+1}} \quad I_{2^{m-i-1}} \otimes DH_2 \otimes I_{2^i} \tag{17}$$

## 3. APPLICATIONS OF THE DEVELOPED ALGORITHM

As an example, and without losing the generality, let us assume the transform length N=16; the $H_N$ matrix can be represented as (18).

$$H_{16} = \prod_{i=0}^{3} I_{2^{3-i}} \otimes WH_2 \otimes I_{2^i} \quad I_{2^{3-i}} \otimes \frac{1}{2}\Delta_{2^{i+1}} \quad I_{2^{3-i}} \otimes DH_2 \otimes I_{2^i} \tag{18}$$

To fully clarify the proposed algorithm, we apply (18) separately for each stage as follows:
For stage one ($i$=0)

$$\begin{aligned} H_{16} &= I_8 \otimes WH_2 \quad I_8 \otimes \frac{1}{2}\Delta_2 \quad I_8 \otimes DH_2 \\ &= I_8 \otimes \frac{1}{2} WH_2\; \Delta_2\; DH_2 \end{aligned} \tag{19}$$

Since $\Delta_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and $WH_2 = DH_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$. Hence $H_{16} = I_8 \otimes I_2 = I_{16}$. For stage two ($i$=1)

$$\begin{aligned} H_{16} &= I_4 \otimes WH_2 \otimes I_2 \quad I_4 \otimes \frac{1}{2}\Delta_4 \quad I_4 \otimes DH_2 \otimes I_2 \\ &= \left( I_4 \otimes \begin{bmatrix} I_2 & I_2 \\ I_2 & -I_2 \end{bmatrix} \right) \left( I_4 \otimes \frac{1}{2} \begin{bmatrix} I_2 & 0 \\ 0 & I_2 \end{bmatrix} \right) \left( I_4 \otimes \begin{bmatrix} I_2 & I_2 \\ I_2 & -I_2 \end{bmatrix} \right) \\ &= I_{16} \end{aligned} \tag{20}$$

for stages three ($i$=2)

$$H_{16} = I_2 \otimes WH_2 \otimes I_4 \left( I_2 \otimes \frac{1}{2}\Delta_8 \right) I_2 \otimes DH_2 \otimes I_4$$
$$= I_2 \otimes \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.85355 & 0 & 0.35355 & 0 & 0.14645 & 0 & -0.35355 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.35355 & 0 & 0.14645 & 0 & -0.35355 & 0 & 0.85355 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0.14645 & 0 & -0.35355 & 0 & 0.85355 & 0 & 0.35355 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -0.35355 & 0 & 0.85355 & 0 & 0.35355 & 0 & 0.14645 \end{bmatrix} \tag{21}$$

Following the same procedure for stage four ($i$=3).

$$H_{16} = WH_2 \otimes I_8 \quad \frac{1}{2}\Delta_{16} \quad DH_2 \otimes I_8$$
$$H_{16} =$$

Therefore, it is feasible to calculate transformations using the matrices provided in (18) using the butterfly structure, as illustrated in Figure 1.

$$\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.96194 & 0 & 0 & 0 & 0 & 0 & 0.19134 & 0 & 0.03806 & 0 & 0 & 0 & 0 & 0 & -0.19134 \\
0 & 0 & 0.85355 & 0 & 0 & 0 & 0.35355 & 0 & 0 & 0 & 0.14645 & 0 & 0 & 0 & -0.35355 & 0 \\
0 & 0 & 0 & 0.69134 & 0 & 0.46194 & 0 & 0 & 0 & 0 & 0 & 0.30866 & 0 & -0.46194 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.46194 & 0 & 0.30866 & 0 & 0 & 0 & 0 & 0 & -0.46194 & 0 & 0.69134 & 0 & 0 \\
0 & 0 & 0.35355 & 0 & 0 & 0 & 0.14645 & 0 & 0 & 0 & -0.35355 & 0 & 0 & 0 & 0.85355 & 0 \\
0 & 0.19134 & 0 & 0 & 0 & 0 & 0 & 0.03806 & 0 & -0.19134 & 0 & 0 & 0 & 0 & 0 & 0.96194 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.03806 & 0 & 0 & 0 & 0 & 0 & -0.19134 & 0 & 0.96194 & 0 & 0 & 0 & 0 & 0 & 0.19134 \\
0 & 0 & 0.14645 & 0 & 0 & 0 & -0.35355 & 0 & 0 & 0 & 0.85355 & 0 & 0 & 0 & 0.35355 & 0 \\
0 & 0 & 0 & 0.30866 & 0 & -0.46194 & 0 & 0 & 0 & 0 & 0 & 0.69134 & 0 & 0.46194 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.46194 & 0 & 0.69134 & 0 & 0 & 0 & 0 & 0 & 0.46194 & 0 & 0.30866 & 0 & 0 \\
0 & 0 & -0.35355 & 0 & 0 & 0 & 0.85355 & 0 & 0 & 0 & 0.35355 & 0 & 0 & 0 & 0.14645 & 0 \\
0 & -0.19134 & 0 & 0 & 0 & 0 & 0 & 0.96194 & 0 & 0.19134 & 0 & 0 & 0 & 0 & 0 & 0.03806
\end{bmatrix} \tag{22}$$

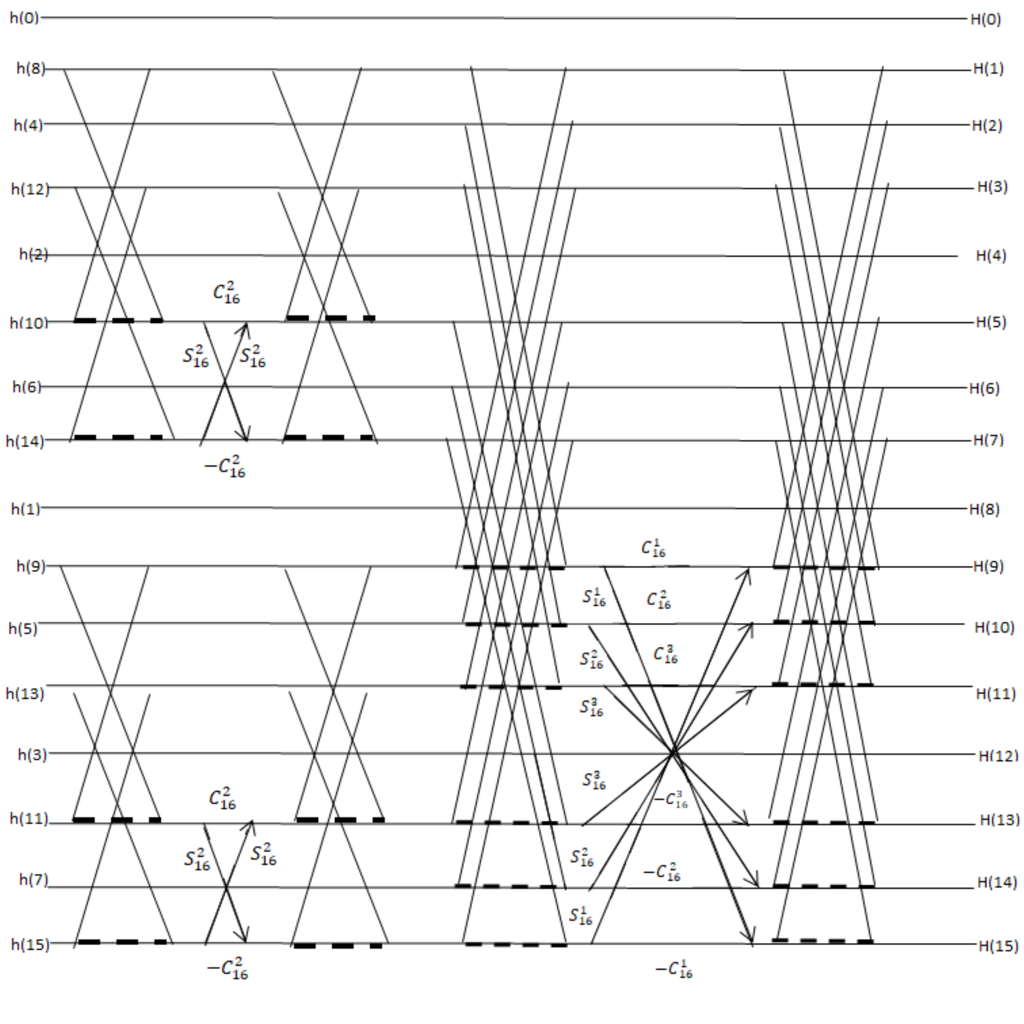



Figure 1. Radix-2 FWHT signal flow diagram when N=16, with (20) multiplications and (50) additions where the solid and dotted lines denote additions and subtractions, respectively

## 4. COMPUTATIONAL COMPLEXITY

According to Figure 1, the proposed algorithm reduces the total number of stages to $(\log_2 N - 2)$ as demonstrated in (19) and (20). Additionally, (21) and (22) show removing the butterflies at the points where $\cos_N^k + \sin_N^k = 1$ , at k=0, $\frac{N}{4}$ . Therefore, we can construct an in-place butterfly for the developed algorithm, as shown in Figure 2.

Hence, the whole transformation satisfies the following:

$$M_N = N(\log_2 N-2)-(N-4) \quad (23)$$

$$A_N = \frac{5}{2}\ N(\log_2 N-2)-(N-4) \quad (24)$$

$A_N$ and $M_N$ denote the overall number of real additions and multiplications. The comparison in the total number of operations number between the FWHT and radix-2 WHT followed by the radix-2 FHT is shown in Table 1. The comparison shows that the FWHT algorithm requires (3N-4) real multiplications and (7.5N-10) real additions less than the existing algorithm. Furthermore, the proposed algorithm was also applied on MATLAB (R2021b), loaded on a laptop computer processor (Intel Core i7), and Windows-10 system to validate and confirm the results of the mathematical operations, as shown in Figures 3(a) and 3(b).

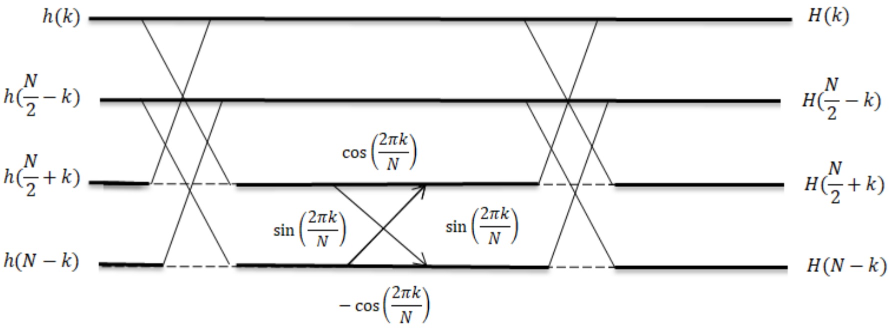
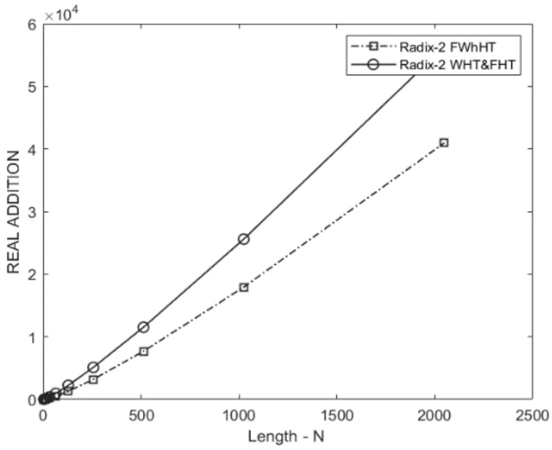


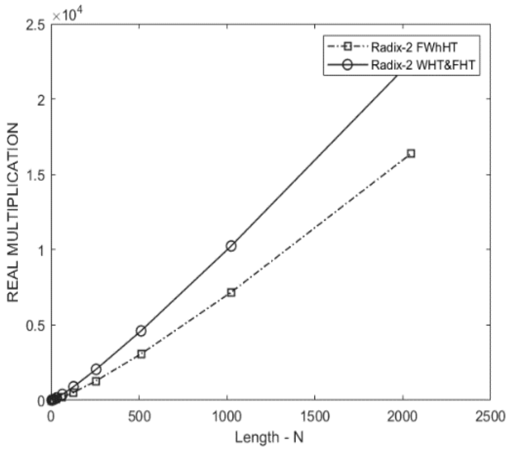

Figure 2. An in-place butterfly of radix-2 FWHT algorithm

Table 1. Comparison of real arithmetic operations

| Transform Length(N) | Radix-2 WHT&FHT | | Radix-2 FWHT | |
|---|---|---|---|---|
| | Multiplications | Additions | Multiplications | Additions |
| 4 | 8 | 20 | 0 | 0 |
| 8 | 24 | 60 | 4 | 10 |
| 16 | 64 | 160 | 20 | 50 |
| 32 | 160 | 400 | 68 | 170 |
| 64 | 384 | 960 | 196 | 490 |
| 128 | 896 | 2240 | 516 | 1290 |
| 256 | 2048 | 5120 | 1284 | 3210 |
| 512 | 4608 | 11520 | 3076 | 7690 |
| 1024 | 10240 | 25600 | 7172 | 17930 |
| 2048 | 22528 | 56320 | 16388 | 40970 |




(a)




(b)

Figure 3. Shows the overall number of real operations (additions and multiplications) for the proposed radix-2 FWHT and WHT+FHT (a) real additions and (b) real multiplications

## 5. CONCLUSION

The paper has been presented an efficient FWHT algorithm as a combination of the fast version of the WHT and the DHT. The developed algorithm is based on sparse matrices factorization using Kronecker product technique. The in-place butterfly structure has been used to implement the newly developed radix-2 FWHT algorithm, and the arithmetic complexity of the proposed algorithm has been computed and investigated in detail. The number of arithmetic operations has been compared with the conventional WHT-FHT method. The result of this comparisons reveals that the proposed algorithm significantly reduced the number of arithmetic operations (multiplications and additions) performed in addition to the simplicity of implementation. The unique characteristics of the transform developed in this paper imply a variety of exciting applications. Although this topic is beyond the scope of this paper, it will be discussed in more depth in a forthcoming publication.

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