

Design Optimization of High-Frequency Power Transformer by Genetic Algorithm and Simulated Annealing

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Abstract

This paper highlights the transformer design optimization using genetic algorithm (GA) and simulated annealing (SA). Any optimization problem, a given objective function is to be minimized keeping in view the constraints. Similarly, transformer design optimization problem involves minimizing the total mass (or cost) of the core and wire material by satisfying constraints imposed by international standards and transformer user specification. The constraints include appropriate limits on efficiency, voltage regulation, temperature rise, no-load current and winding fill factor. The design optimizations seek a constrained minimum mass (or cost) solution by optimally setting the transformer geometry parameters and require magnetic properties. This paper solves the said design problem by using genetic algorithm (GA) and simulated annealing (SA) techniques. The results of geometric programming (GP) technique have been compared with the results obtained by applying GA and SA techniques. It is quite evident from the results that the dimensions as well as mass of copper and core have been reduced in comparison to GP using same set of constraints. Therefore, the paper presents improved design of power transformer by these two techniques. The results of GA and SA have been obtained using optimization tool box MATLAB Release 9.1 which have not been applied for power transformer design so far. First it provides efficient and reliable solution for the design optimization problem with several variables. Second, it guaranteed that the obtained solution is global optimum. Hence paper demonstrates a better and efficient solution to high frequency power transformer design.

Keywords: optimization, power transformer, genetic algorithm, simulated annealing technique, geometric programming

1. Introduction

In today era of competition optimal design plays an important role. Engineering optimization helps in achieving optimal performance, cost competitiveness and optimal efficiency of any equipment/apparatus. In the problem of power transformer optimization mass of core and copper are the main materials for any transformer manufacturing have been optimized using GA and SA. Since dimensions of transformer core and winding are complex system and in conventional optimization technique such a problem formulation involves large number of variables making transformer design optimization complex. The objective of transformer design optimization (TDO) is to optimize mass of core and mass of copper subject to constraints imposed by international standards and transformer user specification.

The transformer design is worked out using various methods based on accumulated experience. Transformer design methods vary among transformer manufacturers. While designing a transformer, much emphasis should be placed on lowering its cost by saving materials. The design should be satisfactory with respect to dielectric strength and mechanical endurance, and windings must withstand dynamic and thermal stresses in the event of short-circuit. In order to meet the above requirements, the transformer designer should be familiar with the prices of basic materials used in the transformer

This paper presents a transformer design methodology based on an artificial intelligence technique. In case of a power transformer design, a design engineer has to consider several aspects of the transformer design such as core shape, size, properties, copper wire size etc and arrives at an optimum design. Improper design results in under utilization of materials. The study of designing power transformers using a computer was pioneered by [1] [2] and [3]. Later, [4] suggested a method for obtaining an optimized design of power transformers. However, procedure was not general and cannot be used conveniently for all power transformer design problems. Several other techniques were also used for design of power transformer [5]-[11]. In [12], attention was focused on a class of single-phase transformers that are employed when a high voltage withstand test is required for verifying the quality of the dielectric insulation of a given component.

2. Minimum mass Design of A High-Frequency Transformer

2.1 Nomenclature

E_p rated primary voltage (V, rms sine wave); E_s rated secondary voltage (V, rms sine wave); S rated apparent power rating (VA); pf rated power factor T_a ambient temperature ($^{\circ}\text{C}$); η_m maximum allowed efficiency;

VR_m maximum allowed voltage regulation; k_ϕ maximum allowed ratio of no-load to full load current. A_c physical cross-sectional area of the center leg $= 2ct(m^2)$; A_m effective cross-sectional area of the center leg $= k_f A_c(m^2)$. ($k_f \cong 0.95$ is the core stacking factor) MLT mean length of a turn $= 4c + 2t + \pi b_w(m)$; W_a window area $= b_w h_w(m^2)$;

V_c volume of core $= A_c(2h_w + 2b_w + 4c)(m^3)$; V_w volume of winding $= MLT.W_a(m^3)$; ρ_c mass density of core material; k_f core stacking factor; K_{Cu} material parameter; J, d current density, thickness of foil or layer; P_o, P_c, P_{Cu} no load loss, core loss, copper loss; B, I_m, ρ_w max flux density, magnetizing current, electrical Resistivity of winding.

2.3. Mathematical model

1) *Objective Function*: The objective of the design is to minimize the total mass of copper and core material. The objective function is

$$m_c + m_{cu} = \rho_c k_f V_c + \rho_{Cu} k_{Cu} V_w \quad (1)$$

Where ρ_{Cu} is the mass density of copper in kg/m^3 . Note that (1) can be easily changed to a cost function by weighting m_c and m_{cu} using appropriate monetary values per kilogram [5].

2) *Induced Voltage Constraint*: For a sinusoidal waveform, the rms value of the induced voltage in the primary winding is given by [14]

$$E_p = \sqrt{2\pi} f N_p B A_m = \sqrt{2\pi} k_f N_p f B A_c \quad (2)$$

Where N_p is the number of turns on the primary. The number of turns on the secondary is given by

$$N_s = (E_s / E_p) N_p \quad (3)$$

3) *Copper Fill Factor Constraints*: The optimum transformer design criterion dictates that the power loss on the primary side is equal to that on the secondary side [13], [14]. Mathematically, the optimum criterion can be expressed as

$$A_{Cu,s} / A_{Cu,p} = N_p / N_s \quad (4)$$

Where $A_{Cu,s}$ and $A_{Cu,p}$ are the secondary and primary conductor areas, respectively. There are two implications from (4). First, the primary and secondary conductors equally share the window area. Second, both the primary and secondary conductors carry the same current density J (A/m rms sine wave) [17]. The copper fill factor constraint is therefore

$$k_{Cu} = \frac{N_p A_{Cu,p} + N_s A_{Cu,s}}{W_a} = \frac{2N_p A_{Cu,p}}{W_a} = \frac{2N_p I_p}{W_a J} \quad (5)$$

Where $I_p = \frac{S}{E_p}$ is the rated primary current in amperes (rms sine wave). Moreover, additional constraints should account for the layer thickness corresponding to the number of layers per secondary section. By assuming that the same layer thickness is used for the primary and secondary windings, it follows from (4) that

$$N_{lp} / N_{ls} = N_p / N_s \quad (6)$$

Where N_{lp} and N_{ls} are, respectively, the number of turns per layer in the primary and secondary windings. To ensure that the transformer windings fit into the transformer window, the following additional requirement has to be met [14]:

$$d = \frac{A_{Cu,s}}{F_l h_w / N_{ls}} = \frac{(N_p I_p) / (N_s J)}{F_l h_w / N_{ls}} \quad (7)$$

4) *Temperature Rise Constraints*: The thermal resistance formula is necessary in optimal transformer design since it links the thermal performance to core and winding loss [15]. In a dry-type transformer with natural air circulation, the dominant heat-transfer mechanism is by convection [15], [18]. The equivalent thermal resistance in C/W is [6]

$$R_\theta = \frac{1}{1.42 A_t} \sqrt[4]{\frac{h_w + 2_c}{\Delta T}} \quad (8)$$

Where $A_t = k_a \sqrt{2ct h_w b_w}$ (m^2) is the transformer surface area. Hurley *et al.* [18] suggest a choice of $ka = 40$. Both the core loss P_c and copper loss P_{Cu} contribute to raising the transformer temperature. The core loss is given by (5) whereas the copper loss, under the optimum design criterion embodied in (5), can be computed from

$$\begin{aligned} P_{cu} &= 2F_r \rho_w \frac{N_p MLT}{A_{Cu,p}} I_p^2 = F_r \rho_w (2N_p A_{cu,p}) MLT . J^2 \\ &= F_r \rho_w (k_{Cu} W_a) MLT . J^2 = F_r \rho_w k_{Cu} V_w J^2 \end{aligned} \quad (9)$$

The temperature rise constraint is therefore

$$R_\theta (P_c + P_{Cu}) = \frac{1}{1.42 A_t} \sqrt[4]{\frac{h_w + 2_c}{\Delta T}} (P_c + P_{Cu}) \leq \Delta T \quad (10)$$

5) *Efficiency Constraint*: The transformer efficiency is required to be greater than a prespecified value η_m

$$\frac{P_o}{P_o + P_c + P_{Cu}} \geq \eta_m \quad (11)$$

Where $P_o = S.p_f$ is the output power in watts. Equation (11) can be rearranged into

$$\frac{\eta_m}{1 - \eta_m} P_c + \frac{\eta_m}{1 - \eta_m} P_{Cu} \leq P_o \quad (12)$$

6) *No-Load Current Constraint*: Proper design requires limiting the transformer no-load current to a small fraction of the full load current. The no-load exciting current (A, rms equivalent sine wave) can be found from the core loss component I_c , and the magnetizing component I_m

$$I_\phi = \sqrt{(I_c^2 + I_m^2)} \quad (13)$$

The core loss component is [2]

$$I_c = \frac{P_c}{E_p} \quad (14)$$

The magnetizing component is given by Ampere's law

$$I_m = \frac{B}{\mu} \frac{2h_w + 2b_w + 4c}{\sqrt{2N_p}} \quad (15)$$

Where $\mu = \mu_r \mu_o$ the permeability, and B is subject to the restriction $B \leq B_{sat}$. The limit on the no-load current can be therefore expressed as

$$I_c^2 + I_m^2 \leq k_\phi^2 I_p^2 \quad (16)$$

7) *Voltage regulation Constraint*: Both the transformer resistance and leakage reactance contribute to the transformer voltage drop. The resistive voltage drop referred to the primary side (under the optimum design criterion) can be computed from

$$V_R = 2F_r \rho_w \frac{N_p MLT}{A_{Cu,p}} I_p pf = 2F_r \rho_w N_p MLT \cdot J \cdot pf \quad (17)$$

The reactive voltage drop referred to the primary side is given by

$$V_X = 2\pi f L_l I_p rf \quad (18)$$

Where $rf = \sqrt{1 - pf^2}$ is the reactive factor and L_l is the leakage inductance referred to the primary side. A general expression for the leakage inductance of a split winding arrangement is [14]

$$L_l \cong \frac{\mu_o N_p^2 MLT \cdot b_w}{3h_w p^2} \quad (19)$$

In (19), p is the number of interfaces between winding sections. For a split winding with more than one interface, p should be an even integer [17], [14]. It is given by twice the number of sections of the secondary winding

$$p = \frac{2}{m} \frac{N_s}{N_{ls}} \quad (20)$$

The voltage regulation constraint, for lagging power factor, can be approximated as [16]

$$\frac{V_R + V_x}{E_p} \leq VR_m \quad (21)$$

2.4. Genetic Algorithm Format

The primary problem variables are chosen as $c, t, h_w, b_w, N_p, N_s, N_{ls}, B, J$, in addition to the upper variable limits

$P_c, P_{Cu}, I_m, V_R, V_X$, and $\bar{h}$. All these variables are by definition positive. Note that this is not the minimum variable set to completely define the problem. An equivalent formulation in terms of c, t, h_w, b_w, B , and $\bar{h}$ could have been used, though the constraint equations would have appeared in a much more complex form
1) *GA Objective Function*: The objective of the design is to minimize the total mass of copper and core material. The objective function is

$$\begin{aligned} mc + mcu &= \rho_c K_f V_c + \rho_{cu} K_{cu} V_{cu} \\ &= 4k_f \rho_c c t h_w + 8k_f \rho_c c^2 t + 4k_f \rho_c c t b_w \\ &\quad + 4k_{Cu} \rho_{Cu} c b_w h_w + 2k_{Cu} \rho_{Cu} t b_w h_w + \pi k_{Cu} \rho_{Cu} c b_w^2 h_w \end{aligned} \quad (22)$$

where m_c , m_{cu} are mass of core and mass of copper respectively.

subject to the following constraint :-

$$\text{Induced Voltage GA constraint} \quad (23)$$

$$\text{Copper Fill Factor GA Constraint} \quad (24)$$

$$\text{Temperature Rise GA Constraint} \quad (25)$$

$$\text{Efficiency GA Constraint} \quad (26)$$

$$\text{No-Load Current GA Constraint} \quad (27)$$

$$\text{Voltage Regulation GA Constraint} \quad (28)$$

3. Implementation of Genetic Algorithm Technique for OPTD

The input to the program includes detail like distance between core centre(c), thickness of core (t), width of window (bw), height of window (hw), connection, frequency etc besides the specification values. A set of random values are assigned to the four independent values and the initial total mass of core and copper is calculated. The objective function i.e. mass of core and mass of copper has been written in MATLAB and it is optimized by Genetic Algorithm Tool (gatool) and Simulated Annealing algorithm (simulannealbnd), by formulating the constraints in terms of design variable i.e. c,t,bw,hw and using standards limits on its dimensions.

4. Results and Discussion

High frequency design example [19]. The GA technique was used to design a transformer operating at 100 kHz. The design inputs are as follows:-

$S = 1200 \text{ VA}$

$E_p = 300 \text{ V}$; $E_s = 75 \text{ V}$, (rms sine wave voltages)

Frequency $f = 100 \text{ kHz}$

Maximum temperature: $T_a = 40^\circ \text{C}$, $\Delta T = 60^\circ \text{C}$

In addition, the following design constraints were imposed:

Rated power factor $\text{pf} = 0.80$ (lagging)

Maximum efficiency $\eta_m = 0.97$

Maximum voltage regulation $\text{VR}_m = 0.03$

Maximum no-Load / full load current $K\phi = 0.02$

Table 1. Transformer Design at 100kHz

Design Variable	Min Mass design by GP	Min Mass design by GA (1 st pass)	Min Mass design by GA (2 nd pass)	Min Mass design by SA
c(cm)	0.49	0.49	0.49	0.49
t(cm)	2.21	2.20	2.11	2.10
bw(cm)	0.68	0.68	0.68	0.68
hw(cm)	2.44	2.44	2.44	2.44
N_p	40	40	40	40
N_s	10	10	10	10
N_{Ip}	4	4	4	4
N_{Is}	1	1	1	1
P	10	10	10	10
$A_{cup}(\text{mm}^2)$	1.30	1.30	1.30	1.30
$A_{cus}(\text{mm}^2)$	5.21	5.21	5.21	5.21
$m_c + m_{cu}(\text{g})$	157.74	157.394	153.4325	151.8620
$\eta\%$	99.56	99.56	99.56	99.56

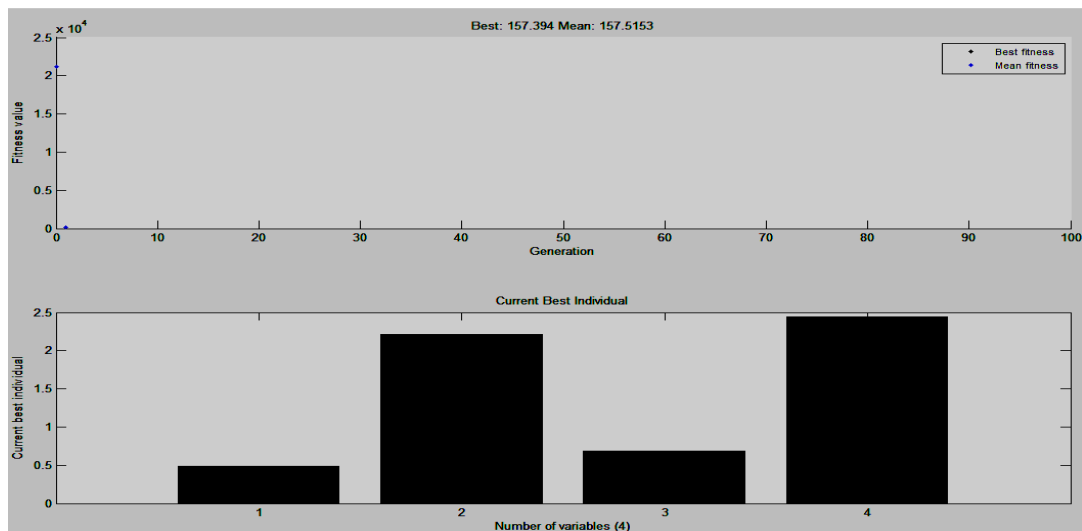


Figure 1. Variation of mass of copper and core with iteration by GA phase 1

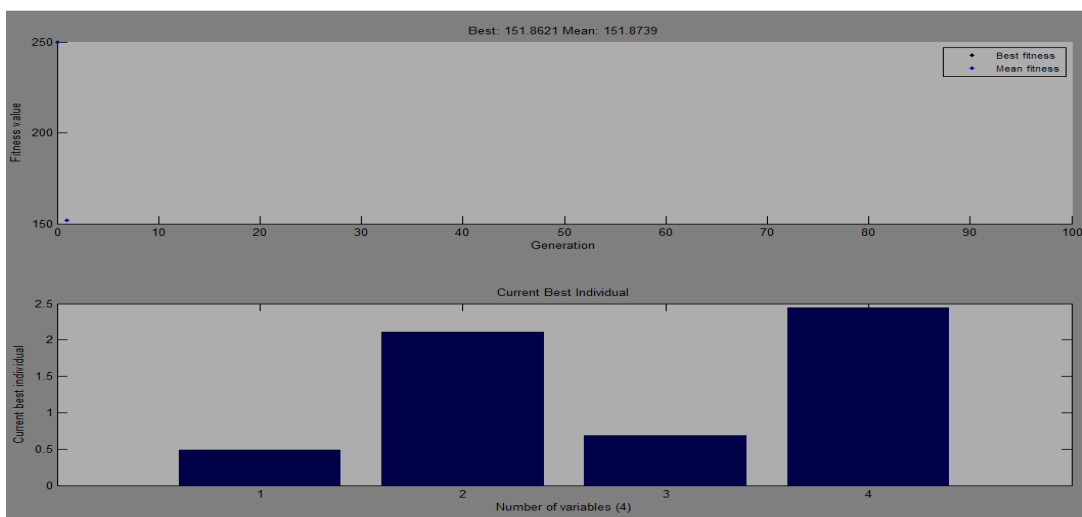


Figure 2. Variation of mass of copper and core with iteration by GA phase 2

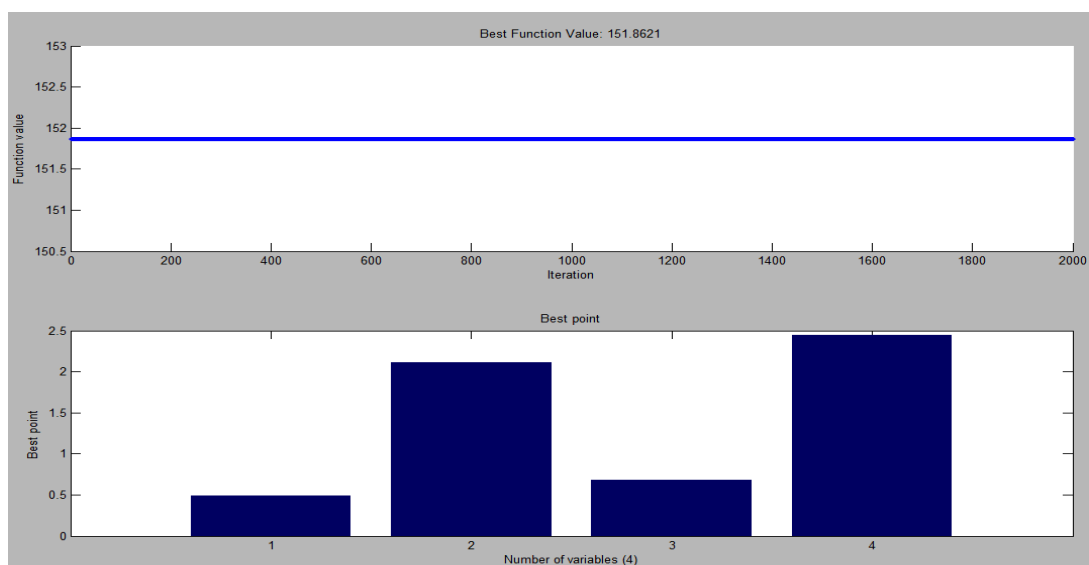


Figure 3. Variation of mass of copper and core with iteration by SA

5. Conclusions

Geometric Programming involves monomial and posynomial functions to solve the objective function. It is complex and time taking to solve the transformer design problem by GP so artificial intelligence technique i.e. GA and SA are used to solve transformer design optimization problem. SA and GA are the most flexible techniques available for solving hard combinatorial transformer design problems. The main advantage of GA and SA are that it can be applied to large problems regardless of the conditions of differentiability, continuity, and convexity that are normally required in conventional optimization methods. In this work Genetic Algorithm (GA) and Simulated Annealing (SA) algorithm has been used for the optimum design of transformer. The result obtained by GP has been compared with GA and SA. It has been shown graphically and analytically that GA, SA are capable of finding a design which is superior to GP. The GA, SA has been used to find minimum mass of core and copper by changing the core's dimensions and its constraints. The optimum mass of core has been given by SA. Thus GA, SA based design optimization is simple, robust and reliable for design optimization of transformer. Thus GA, SA has been used a viable tool for obtaining optimal design of transformer.

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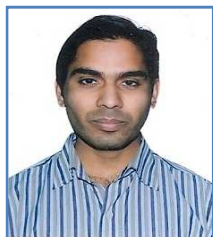
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