The cross-association relation based on intervals ratio in fuzzy time series

Etna Vianita[1], Muhammad Sam'an[2], A. Nafis Haikal[1], Ina Salamatul Mufaricha[3], Redemtus Heru Tjahjana[4], Titi Udjiani[4]

[1]Postgraduate Program of Mathematics, Department of Mathematics, Faculty of Sains and Mathematics, Universitas Diponegoro, Semarang, Indonesia
[2]Departement of Informatics, Universitas Muhammadiyah Semarang, Semarang, Indonesia
[3]Postgraduate Program of Mathematics, Department of Mathematics, Faculty of Mathematics and Natural Science, Universitas Gadjah Mada, Yogyakarta, Indonesia
[4]Department of Mathematics, Faculty of Sains and Mathematics, Universitas Diponegoro, Semarang, Indonesia

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ABSTRACT

The fuzzy time series (FTS) is a forecasting model based on linguistic values. This forecasting method was developed in recent years after the existing ones were insufficiently accurate. Furthermore, this research modified the accuracy of existing methods for determining and the partitioning universe of discourse, fuzzy logic relationship (FLR), and variation historical data using intervals ratio, cross association relationship, and rubber production Indonesia data, respectively. The modified steps start with the intervals ratio to partition the determined universe discourse. Then the triangular fuzzy sets were built, allowing fuzzification. After this, the FLR are built based on the cross-association relationship, leading to defuzzification. The average forecasting error rate (AFER) was used to compare the modified results and the existing methods. Additionally, the simulations were conducted using rubber production Indonesia data from 2000-2020. With an AFER result of 4.77%<10%, the modification accuracy has a smaller error than previous methods, indicating very good forecasting criteria. In addition, the coefficient values of D_1 and D_2 were automatically obtained from the intervals ratio algorithm. The future works modified the partitioning of the universe of discourse using frequency density to eliminate unused partition intervals.

Corresponding Author:

Etna Vianita
Postgraduate Program of Mathematics, Department of Mathematics, Faculty of Science and Mathematics, Universitas Diponegoro
St. Prof. Sudarto No.13, Tembalang, Semarang, 50275, Indonesia
Email: etnavianita@student.undip.ac.id

1. INTRODUCTION

Zadeh [1] introduced the basic concept of fuzzy set theory in 1965 for continuous feasible sets. According to Zadeh, a fuzzy set is expressed by a membership function, where each domain corresponds to one number in range from zero to one [1]. The fuzzy set theory, the concept of linguistic variables, and the fuzzy applications to approximating reasoning were developed by Zadeh [1], [2] which have successfully penetrated into the realm of forecasting using time series data. Time series is the observation of historical data, while forecasting is the future prediction in daily life, such as economics, employment sector, tourism, agriculture, climatology, stock, and others, hence losses can be avoided using a well-timed forecast. The fuzzy time series (FTS) was introduced by Song and Chissom [3], [4] in 1993 to forecast the enrollment data at Alabama University. Furthermore, the observation of historical data is formed in linguistic value to predict the future in the FTS, which is simpler than genetic algorithms and neural networks. This forecasting models calculates in first [5]–[23] and high-order [17], [18], [22], [24]–[43].

The FTS introduced by Song and Chissom [3], [4] in 1993 to forecast the enrollment data at Alabama University. However this had a complicated calculation using max-min composition, hence Chen [44], [45] used arithmetic operation to simplify the calculation. Several kinds of research showed that partitioning of universe of discourse is determined by equal interval lengths. According to Huarng and Yu [46], different interval lengths using ratio is used to increase the accuracy and implement enrollment, stock index, and inventory demand data. Huarng and Yu [46] also demonstrated the sensitivity analyses of various ratio sample percentiles, concluding that the 50^{th} percentile is the appropriate choice. Chen [44] created a fuzzy logic relationship (FLR) involving historical data called the first-order. However, this was ambiguous and solved by involving two or more historical data called the high-order [45], which improved the accuracy of Chen [44]. Many research used the short association relationship, whereas Li and Yu [47] proposed a relationship called the long-association relationship, which was applied to the synthetics time series data and is presented by the long distance historical data. Furthermore, Lee *et al.* [48] modified Chen [45] to achieve two-or multi-factors to handle Taiwan futures exchange (TAIFEX) and temperature forecasts. Lee *et al.* [48] added Taiwan Stock exchange capitalization weighted stock index (TAIEX) as a second or influence factor in TAIFEX, while cloudy density was added in the factor of temperature. The limitation of the multi-factors FTS forecasting method [48] is using FLR, which only indicates the association between the main and second factors. However, FTS overcomes this shortcoming with cross-association which is proposed to deal with fuzzy time series forecasting problems. Recently, Li and Yu [49] describe the cross-associations relationship when building the FLR in multi-factors for one-order. The cross-associations relationship was integrated with multi-factors for high order [50]. The FLR was divided to be four, including the short- and long-association FLR, which describes the main factor, and the short and long cross-associations, indicating influence factor to the main factor.

According to the literature, determining the lengths of the intervals and building FLR are important processes. This study modified the accuracy of existing methods for determining the interval lengths using ratio and building FLR using a cross-association relationship. The modified results were compared with Li *et al.* [50] method and the average forecasting error rate (AFER) was used to test the results and errors of the method. According to [48], two-factors high-order has the lowest error in third-order, hence the simulation applied in third-order to compare.

Rubber is an agricultural commodity with an essential role, as a source of income, employment opportunities, and foreign exchange [51]. Although Indonesia has the largest rubber plantation area in the world, around 3.60 million ha in 2019, it is in second place after Thailand in terms of production, at 3.30 million tons [52]. An increase in rubber plant productivity is necessary, considering the prospects and development of rubber agribusiness. This is possible by making an accurate forecast of rubber production. The production estimation process is based on the data in previous years [53], hence rubber production Indonesia data from 2000-2020 in *Badan Pusat Statistika* (BPS-Statistics Indonesia) [54] was used in this study for simulation data.

The following section summarizes the theoretical background of previous methods and testing in section 2. The research method with modification to the previous ones to forecast the time series data is elaborated in section 3. The simulation of model performance for rubber production Indonesia data is explained in section 4, and conclusion and future works is displayed in section 5.

2. THEORETICAL BACKGROUND

2.1. Huarng's method

Huarng's method described the first-order of FTS forecasting model. This method modified Chen's [44] method in partitioning the universe of discourse using intervals ratio. This method algorithm includes: i) determine the universe of discourse with D_1 and D_2 from the intervals ratio algorithm, ii) partitioning the universe of discourse into several intervals with different lengths using intervals ratio, iii) fuzzification, iv) build FLR and fuzzy logic relations group (FLRG) based on Chen's [44] method, and v) defuzzification.

2.2. Lee's method

Lee's method demonstrated high-order of FTS forecasting model. This method modified Chen's [45] method by adding another factor. The following is Lee's method algorithm: i) determine the universe of discourse with D_1 and D_2 an arbitrary positive number elected by the researcher, ii) partitioning the universe of discourse into nine intervals with equal length for main factor and seven intervals with equal length of the influence factor, iii) fuzzification, iv) build FLR and FLRG based on Chen's [45] method, and v) defuzzification.

2.3. Li's method

Li's method described multi-factors high-order of FTS forecasting model and used the triangular fuzzy sets to modify Lee *et al.* [48] method in grades of membership 1, 0.5, and 0. This method modified the FLR using a cross-association relationship. The FLR is divided into four, hence Li's method algorithm is: i) determine the universe of discourse with D_1 and D_2, which is an arbitrary positive number that elected by the researcher; ii) partitioning the universe of discourse into several intervals for main factor and influence factors with equal length; iii) define a group of fuzzy sets; iv) fuzzification; v) build FLR using cross-association relationship; and vi) defuzzification.

2.4. Method testing

This research used AFER obtained from [47] to determine the error value between the actual data and the forecasting. The formula of AFER is:

$$AFER = \frac{\Sigma_{\theta=1}^{n}\left|\frac{F_j - A_j}{A_j}\right|}{n} \times 100\% \tag{1}$$

where A_j is the actual data result and F_j is the forecasting result.

3. RESEARCH METHOD

This research modified Li *et al.* method [50] of determining D_1 and D_2 for the build of the universe of discourse. Moreover, it is also partitioning the universe of discourse using intervals ratio [46] with 50th percentile. The following is the proposed method algorithm:

a) Determine the universe of discourse with D_1 and D_2, which is automatically obtained from the intervals ratio algorithm.
b) Partitioning the universe of discourse into several intervals with different length using intervals ratio.
c) Define a group of triangular fuzzy sets.
   Build fuzzy sets of main factor $\{A_k | k = 1,2, \dots, l+1\}$ on $\{u_{j_1} | j = 1,2, \dots, l\}$:

$$A_1 = [Q(1), Q(1), Q(1) + q_k\ ];$$
$$A_k = [Q(k) - q_{k-1}, Q(k), Q(k) + q_k\ ], k = 2, 3, \dots, l;$$
$$A_{l+1} = [Q(l), Q(l) + q_l\ , Q(l) + q_l\ ].$$

   where $Q(k)$ is the left endpoint, q_k is the length of $u_{k_1}(k\ \epsilon\ \{1, 2, \dots, l\})$. Fuzzy sets of influence factors $\{B_k^i | k = 1,2, \dots, l_i + 1, i = 1,2, \dots, n\}$ are built on $\{u_{j_i} | j = 1,2, \dots, l_i, i = 2, \dots, n\}$ analog to constructing $\{A_k | k = 1,2, \dots, l+1\}$.
   Definition 3.1. A fuzzy number with the main factor $A = (a, b, c)$ is said to be triangular if its membership function is given:

$$\mu_A(x) \begin{cases} \frac{(x-a)}{(b-a)} & if\ a \le x \le b, \\ 1 & if\ x = b, \\ \frac{(c-x)}{(c-b)} & if\ b \le x \le c, \\ 0 & otherwise. \end{cases}$$

   Also, the influence factor $B_k^i = (a, b, c)$ is said to be triangular fuzzy number if its membership function is given:

$$\mu_{B_k^i}(x) \begin{cases} \frac{(x-a)}{(b-a)} & if\ a \le x \le b, \\ 1 & if\ x = b, \\ \frac{(c-x)}{(c-b)} & if\ b \le x \le c, \\ 0 & otherwise. \end{cases}$$

d) Fuzzification.
   Each data point in the main factor *X* is fuzzified to a set, where data point data x_l is fuzzified to $A_{(k+1)}$, if $\mu_{A_k}(x_l) \le \mu_{A_{(k+1)}}(x_l)$, otherwise to $A_k(k \in \{1,2, \dots, l\}$. Similarly, data point y_s^i in influence

factors Y^i is fuzzified to B^i_j, where y^i_s is the s[th] investigation of the (i+1)[th] influence factors ($j \in \{1,2,\dots,l_i+1\}, s \in \{1,2,\dots,m\}, i=1,2,\dots,n$). Then, some fuzzy time series is obtained from that given. Suppose $F(t), G_i(t)$ is to be obtained from fuzzy time series of X and Y^i ($i=1,2,\dots,n$) respectively.

e) Build FLR using a cross-association relationship.
f) Defuzzification.

The prediction of x_t at moment t. Four kinds of FLRs can be used in forecasing, four predictions $\left\{x^*_{t_S}, x^*_{t_L}, x^*_{t_{SC_i}}, x^*_{t_{LC_i}} | i=1,2,\dots,n\right\}$ with final prediction x^*_t at moment t.

$$x^*_t = \frac{v_1\times\bar{x}(l_1)+v_2\times\bar{x}(l_2)+\cdots+v_N\times\bar{x}(l_N)}{v_1+v_2+\cdots+v_N}$$

where $\bar{x}(l_r)$ is given by $A_{f_h}\dots A_{f_2}A_{f_1}\to A_{l_r}(v_r)(f_u, l_r \in \{1,2,\dots,k+1\}, u=1,2,\dots,h, h>1, r=1,2,\dots,N)$ for high-order short-association fuzzy logical relationship (HSAFLR), $A_{f_{ph}}\dots A_{f_{p2}}A_{f_{p1}}\to A_{l_r}(v_r)\big(p_u, p_h \in Z^+, p_{(u+1)} > p_u, p_h > h, f_{pu}, f_{ph}, l_r \in \{1,2,\dots,k+1\}, u=1,2,\dots,(h-1), h>2, r=1,2,\dots,N\big)$ for high-order long-association fuzzy logical relationship (HLAFLR), $B^i_{j_h}\dots.B^i_{j_2}B^i_{j_1}\to A_{l_r}(v_r)(j_u \in \{1.2\dots., k_i+1\},\ l_r \in \{1,2,\dots,k+1\}, u=1,2,\dots,h, h>1, i=1,2,\dots,n, r=1,2,\dots,N)$ for high-order short-cross association fuzzy logical relationship (HSCAFLR), $B^i_{j_{Ph}}\dots.B^i_{j_{P2}}B^i_{j_{P1}}\to A_{l_r}(v_r)\big(p_u, p_h \in Z^+, p_{(u+1)} > p_u, p_h > h, f_{pu}, f_{ph}, l_r \in \{1,2,\dots,k+1\},\ u=1,2,\dots,(h-1), h>2, r=1,2,\dots,N\big)$ and calculated by using the following equation:

$$\bar{x}(a) = \begin{cases} \frac{0.5m(1)+Q(1)}{1.5}, a=1 \\ \frac{0.5m(a-1)+Q(a)+0.5m(a+1)}{2}, 2\le a\le k \\ \frac{0.5m(a-1)+Q(a-1)+q_k}{1.5}, a=k+1 \end{cases}$$

where $m(a)$ is midpoint and $Q(a)$ is left endpoint of interval $\{u_j | j=1,2,\dots,l\}$. The final prediction x^*_t:

$$x^*_t = \begin{cases} \dfrac{\sum_{i=1,i\notin W_1}^{n} x^*_{t_{SC_i}} + \sum_{i=1,i\notin W_2}^{n} x^*_{t_{LC_i}} + x^*_{t_S}}{2n+1-|W_1|-|W_2|}, x^*_{t_L}\ does\ not\ exist \\ \dfrac{\sum_{i=1,i\notin W_1}^{n} x^*_{t_{SC_i}} + \sum_{i=1,i\notin W_2}^{n} x^*_{t_{LC_i}} + x^*_{t_S} + x^*_{t_L}}{2(n+1)-|W_1|-|W_2|}, x^*_{t_L}\ exists \end{cases}$$

where $W_1 = \{i|$ no available HSCAFLR exists in SC_i, $1\le i\le n\}$, $W_2 = \{i|$ no available high-order long-cross association fuzzy logical relationship (HLCAFLR) exists in LC_i, $1\le i\le n\}$.

4. RESULTS AND DISCUSSION

In this research, the simulation data is retrieved from the *Badan Pusat Statistika* (BPS) website [54]. The data of Indonesia's rubber production and the land area from 2000-2020, which are the main and influence factor respectively, were used. Figure 1 shows that there were 21 data which were used for the forecasting simulation with the help of Microsoft Excel.

4.1. Proposed method

The first step is to determine the maximum and minimum values from the data. According to the data, the main factor has a minimum value D_{min_1} of 1125.2 and a maximum value D_{max_1} of 3111.3, while the influence factor has minimum and maximum value of 2747.9 and 3305.4, respectively. Following this, the universe of discourse $U_i = [D_{min_i} - D_{1_i}; D_{max_i} + D_{2_i}]$, with D_{1_i} and D_{2_i} (i=1,2) is determined using intervals ratio algorithm. The intervals ratio algorithm is:

a) The absolute difference between two successive data was taken. For example, the rubber production in 2000 and 2001 were 1125.2 and 1723.3, respectively, hence the absolute difference is 598.1. Similarly, the differences for the next year and the data observations for the influence factor can be calculated.
b) From each absolute difference, the relative difference is calculated. For example, the relative difference between 2001 and 2000 is $r_{X1} = \frac{|1723.3-1125.2|}{1125.2} = 53.2\%$. Table 1 shows the relative differences for year $t(r_t)$.

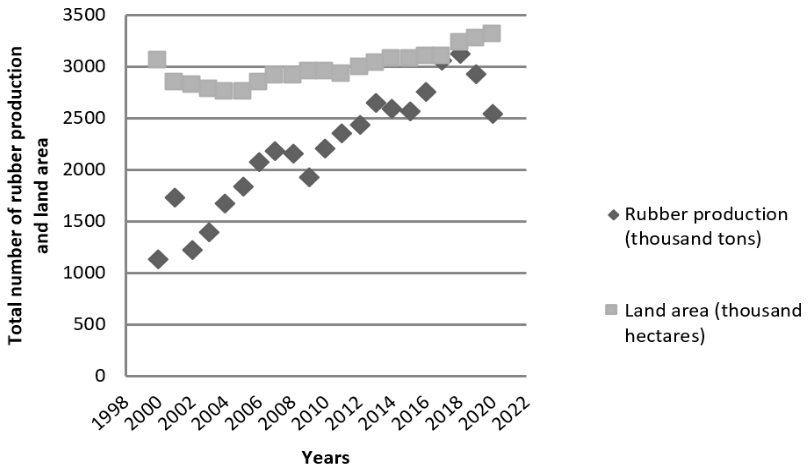

Figure 1. Data on Indonesia's rubber production and land area [54]

Table 1. Observations data and the relative differences

Years	Actual rubber production data	r_{Xt} of rubber production data	Actual land area data	r_{Yt} of land area data
2000	1125.2	-	3046	-
2001	1723.3	53.2%	2838.4	6.8%
2002	1226.6	28.8%	2825.5	0.5%
2003	1396.2	13.8%	2772.5	1.9%
2004	1662.0	19.0%	2747.9	0.9%
⋮	⋮	⋮	⋮	⋮
2020	2533.5	13.4%	3305.4	1.1%

c) Mapping to the base table for each factors by mapping $MIN(r_{Xt_{1,\dots}}, r_{Xt_{n-1}})$=0.57% for main factor and $MIN(r_{Y^1t_{1,\dots}}, r_{Y^1t_{n-1}})$=0.02% for influence factor. Mapping the main factor 0.57% to the base table shown in Table, basis=1% and mapping the influence factor 0.02% to the base table shown in Table 2, basis=0.01%.

Table 2. Base table

$MIN(r_{Xt_{1,\dots}}, r_{Xt_{n-1}})$	Base	$MIN(r_{Yt_{1,\dots}}, r_{Yt_{n-1}})$	Base
$MIN(r_{Xt_{1,\dots}}, r_{Xt_{n-1}}) \leq 0.05\%$	0.01%	$MIN(r_{Yt_{1,\dots}}, r_{Yt_{n-1}}) \leq 0.05\%$	0.01%
$0.05\% < MIN(r_{Xt_{1,\dots}}, r_{Xt_{n-1}}) \leq 0.5\%$	0.1%	$0.05\% < MIN(r_{Yt_{1,\dots}}, r_{Yt_{n-1}}) \leq 0.5\%$	0.1%
$0.5\% < MIN(r_{Xt_{1,\dots}}, r_{Xt_{n-1}}) \leq 5.0\%$	1%	$0.5\% < MIN(r_{Yt_{1,\dots}}, r_{Yt_{n-1}}) \leq 5.0\%$	1%
$5\% < MIN(r_{Xt_{1,\dots}}, r_{Xt_{n-1}}) \leq 50\%$	10%	$5\% < MIN(r_{Yt_{1,\dots}}, r_{Yt_{n-1}}) \leq 50\%$	10%
…	…	…	…

d) Plot the cumulative distribution of r_{Xt_i} on the basis table of 1% and $r_{Y^1t_i}$ on the basis table of 0.01%. First, $MIN(r_{Xt_{1,\dots}}, r_{Xt_{n-1}})$=0.57% and $MIN(r_{Y^1t_{1,\dots}}, r_{Y^1t_{n-1}})$=0.02%; gives a cumulative sum of 1. Furthermore, an increase from the horizontal axis on the principal factor to 1.57% and then 2.57% on a 1% basis, results in the same cumulative sum. Figure 2 illustrated the distribution of cumulative ratios for main factors, where the horizontal axis was plotted from 7.57% to 11.57% for simplicity. Conversely, when the horizontal axis of the influence factor increases to 0.03% and then 0.04% with a base of 0.01%, the cumulative number is 1. Subsequently, the cumulative distributives should be plotted. Figure 3 plotted the horizontal axis from 1.10% to 1.14% for simplicity.

e) Setting α as the 50th percentile. The total number from 2000 to 2020 is 21. Therefore, it is calculated as 50% of the total amount, which is 10. From Figure 2, 9.57% and 10.57% are both greater than 10 and the smaller is chosen as the ratio of the main factor, namely the ratio of the main factors of 9.57%. While from Figure 3, 1.12%, 1.13%, 1.14% all three are greater than 10 and the smaller one is chosen as the ratio of the influence factor, namely the ratio of the influence factor is 1.12%. So, results in a ratio of 9.57% and 1.12% for the main and influence factor, respectively.

f) The intervals are calculated as. First,

$$truncate\left(MIN(x_t)\right) = m.n \times 10^z$$

$$truncate(1125.2) = 1.1 \times 10^3$$

Second, n is reduced by 1 to obtain n'=1-1=0. Then, the initial value is given as $initial = m.n' \times 10^z$=$1.0 \times 10^3$. $upper_0 = initial = 1.0 \times 10^3$. For j≥1, $lower_j = upper_{j-1}$ and $upper_j = (1 + ratio)^{\,j} \times upper_0$, hence intervals $interval_j = [lower_j, upper_j]$. Furthermore, $upper_1 = (1 + 9.57\%)^{\,1} \times 1000 = 1095.7$, results in an interval [1000;1095.7]. Tables 3 and 4 shows the result of the intervals.

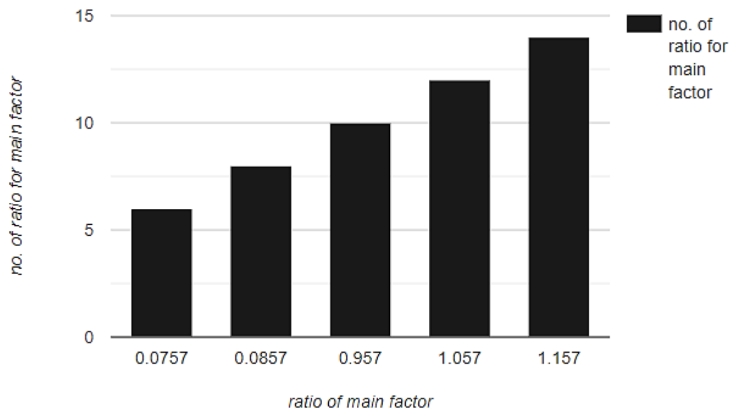

Figure 2. Distribution of cumulative ratios for main factor

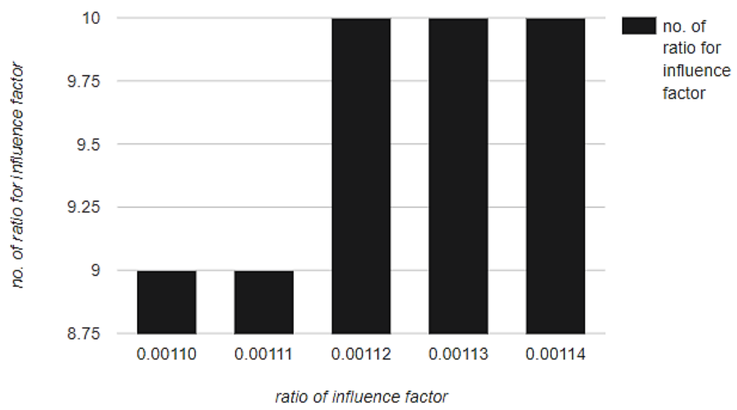

Figure 3. Distribution of cumulative ratios for influencing factor

From the interval ratio algorithm D_{1_i}, D_{2_i} and partition of U_i is obtained, where the main factor $D_{1_1} = 125.2$ and $D_{2_1} = 169.589$, while for influence factor $D_{1_2} = 147.9$ and $D_{2_2} = 16.516$. Therefore, the universe of discourse for the main factor is:

$$U_1 = \left[D_{min_1} - D_{1_1};\ D_{max_1} + D_{2_1}\right]$$
$$U_1 = [1000; 3280.889]$$

Furthermore, the universe of discourse for the influence factor is:

$$U_2 = \left[D_{min_2} - D_{1_2};\ D_{max_2} + D_{2_2}\right]$$
$$U_2 = [2600; 3321.916]$$

Partition of U_1 from the intervals ratio algorithm results in different lengths of intervals for the main factor as shown in Table 3, and partition of U_2 from intervals ratio algorithm gets different lengths of intervals for the influence factor as shown in Table 4.

Table 3. Partition of the universe of discourse U_1

Index	Interval
u_{1_1}	[1000; 1095.7]
u_{2_1}	[1095.7; 1200.558]
u_{3_1}	[1200.558; 1315.452]
$\vdots$	$\vdots$
u_{13_1}	[2994.332; 3280.889]

Table 4. Partition of the universe of discourse U_2

Index	Interval
u_{1_2}	[2600; 2629.12]
u_{2_2}	[2629.12; 2658.566]
u_{3_2}	[2658.566; 2688.342]
$\vdots$	$\vdots$
u_{22_2}	[3285.123; 3321.916]

After the partitioning of the universe of discourse, the triangular fuzzy sets $A_k(k = 1,2,3,\dots,14)$ and $B_k(k = 1,2,3,\dots,23)$ is built for the main and influence factors, respctively, corresponding to the linguistic intervals of Tables 3 and 4:

$$\begin{aligned}
A_1 &= (1000,1000,1095.7)\\
A_2 &= (990.842,1095.7,1200.558)\\
A_3 &= (1085.664,1200.558,1315.452)\\
&\vdots\\
A_{14} &= (2994.332,3280.889,3280.889)
\end{aligned}$$

and

$$\begin{aligned}
B_1 &= (2600,2600,2629.12)\\
B_2 &= (2599.674,2629.12,2658.566)\\
B_3 &= (2628.79,2658.566,2688.342)\\
&\vdots\\
B_{23} &= (3285.123,3321.916,3321.916)
\end{aligned}$$

Definition 3.1 was used to describe the grades of membership for each datum corresponding to the triangular fuzzy set [49], [55] $A_k(k = 1,2,3,\dots,14)$ and $B_k(k = 1,2,3,\dots,23)$ as:

$$A_1 = \emptyset,\ A_2 = \frac{0.72}{1125.2},\ A_3 = \frac{0.28}{1125.2} + \frac{0.77}{1226.6},\ A_4 = \frac{0.23}{1226.6} + \frac{0.36}{1396.2},\ A_5 = \frac{0.64}{1396.2},\ A_6 = \frac{0.45}{1662} + \frac{0.05}{1723.3},\ A_7 = \frac{0.55}{1662} + \frac{0.95}{1723.3} + \frac{0.35}{1838.7},\ A_8 = \frac{0.65}{1838.7} + \frac{0.88}{1918},\ A_9 = \frac{0.97}{2082.6} + \frac{0.5009}{2176.7} + \frac{0.64}{2148.7} + \frac{0.12}{1918} + \frac{0.42}{2193.4},\ A_{10} = \frac{0.03}{2082.6} + \frac{0.4991}{2176.7} + \frac{0.36}{2148.7} + \frac{0.58}{2193.4} + \frac{0.62}{2359.8},\ A_{11} = \frac{0.38}{2359.8} + \frac{0.30}{2429.5} + \frac{0.32}{2655.94} + \frac{0.63}{2583.4} + \frac{0.69}{2568.6} + \frac{0.83}{2533.5},\ A_{12} = \frac{0{,}70}{2429.5} + \frac{0.68}{2655.94} + \frac{0.37}{2583.4} + \frac{0.31}{2568.6} + \frac{0.92}{2754.7} + \frac{0.31}{2926.6} + \frac{0.26}{2533.5},\ A_{13} = \frac{0.08}{2754.7} + \frac{0.81}{3050.2} + \frac{0.59}{3111.3} + \frac{0.26}{2926.6},\ A_{14} = \frac{0.19}{3050.2} + \frac{0.41}{3111.3}.$$

$$B_i = \emptyset\ (i\text{=1, 2, 3, 4, 18, 19}),\ B_5 = \frac{0.03}{2747.9},\ B_6 = \frac{0.23}{2772.5} + \frac{0.97}{2747.9} + \frac{0.41}{2767},\ B_7 = \frac{0.77}{2772.5} + \frac{0.59}{2767},\ B_8 = \frac{0.12}{2838.4} + \frac{0.53}{2825.5} + \frac{0.30}{2833},\ B_9 = \frac{0.88}{2838.4} + \frac{0.47}{2825.5} + \frac{0.70}{2833},\ B_{10} = \frac{0.21}{2899.7} + \frac{0.19}{2900.3},\ B_{11} = \frac{0.79}{2899.7} + \frac{0.81}{2900.3} + \frac{0.22}{2931.8},\ B_{12} = \frac{0.58}{2952.6} + \frac{0.70}{2948.7} + \frac{0.78}{2931.8},\ B_{13} = \frac{0.42}{2952.6} + \frac{0.30}{2948.7} + \frac{0.54}{2987},\ B_{14} = \frac{0.46}{2987} + \frac{0.38}{3026.02},\ B_{15} = \frac{0.79}{3046} + \frac{0.62}{3026.02} + \frac{0.16}{3067.4},\ B_{16} = \frac{0.21}{3046} + \frac{0.84}{3067.4} + \frac{0.92}{3075.6} + \frac{0.43}{3092.4} + \frac{0.11}{3103.3},\ B_{17} = \frac{0.08}{3075.6} + \frac{0.57}{3092.4} + \frac{0.89}{3103.3},\ B_{20} = \frac{0.36}{3235.8},\ B_{21} = \frac{0.64}{3235.8} + \frac{0.44}{3269.1},\ B_{22} = \frac{0.56}{3269.1} + \frac{0.45}{3305.4},\ B_{23} = \frac{0.55}{3305.4},$$

Example. The actual rubber production data in 2001 is 1723.3. Subsequently, the grades of membership were obtained for data point 1723.3 as 0.05 and 0.95, respectively, corresponding to the triangular fuzzy set A_6 and A_7. This data has reached maximum membership in the triangular fuzzy set A_7. Therefore, the rubber production datum in 2001 is fuzzified by a triangular fuzzy set A_7. Table 5 showed the fuzzification of the time series data of rubber production and land area by triangular fuzzy sets.

FLR was developed in cross-relation. Table 6 showed the four FLRs namely HSAFLR, HLAFLR, HSCAFLR, and HLCAFLR. Furthermore, the result of the forecast is displayed in Table 7. Four FLRs are constructed for forecasting, hence four predictions $\{x^*_{2003_S},\ x^*_{2003_L},\ x^*_{2003_{SC}}, x^*_{2003_{LC}}\}$ are calculated, and the final prediction x^*_{2003} can be obtained. The detailed process of forecasting is described in the following steps:

Step 1: calculate $x^*_{2003_S}$ from the available *HSAFLRs* in *S*. The fuzzified values of $x_{2000}, x_{2001}, x_{2002}$ of the main factor time series are A_2, A_7, A_3 respectively. The available *HSAFLRs* found in *S*, whose premises are A_2, A_7, A_3:

$$A_2, A_7, A_3 \rightarrow A_5(1)$$

By using these FLRs, the prediction $x^*_{2003_S}$ is given as

$$x^*_{2003_S} = \frac{1\times\bar{x}(5)}{1} = 1381.41$$

where

$$\begin{aligned}\bar{x}(5) &= \frac{0.5m(5-1)+Q(5)+0.5m(5+1)}{2}\\ &= \frac{0.5m(4)+Q(5)+0.5m(6)}{2}\\ &= \frac{0.5(1315.452)+1315.452+0.5(1579.277)}{2} = 1381.41\end{aligned}$$

Step 2: $x^*_{2003_L}$ is calculated from the available HLAFLRs in L. The fuzzified values of $x_{1999}, x_{2000}, x_{2001}$ of the main factor time series are NA, A_2, A_7 respectively. The available HLAFLRs found in L, whose premises are NA, A_2, A_7:

$$NA, A_2, A_7 \rightarrow A_5(1)$$

By using these FLRs, the prediction $x^*_{2003_L}$ is given as

$$x^*_{2003_L} = \frac{1\times\bar{x}(5)}{1} = 1381.41$$

where

$$\begin{aligned}\bar{x}(5) &= \frac{0.5m(5-1)+Q(5)+0.5m(5+1)}{2}\\ &= \frac{0.5m(4)+Q(5)+0.5m(6)}{2}\\ &= \frac{0.5(1315.452)+1315.452+0.5(1579.277)}{2} = 1381.41\end{aligned}$$

Step 3: $x^*_{2003_{SC}}$ is calculated from the available HSCAFLRs in SC. The fuzzified values of $x_{2000}, x_{2001}, x_{2002}$ of the influence factor time series are B_{15}, B_9, B_3 respectively. The available HSCAFLRs found in SC, whose premises are B_{15}, B_9, B_3:

$$B_{15}, B_9, B_3 \rightarrow A_5(1)$$

By using these FLRs, the prediction $x^*_{2003_{SC}}$ is given as

$$x^*_{2003_{SC}} = \frac{1\times\bar{x}(5)}{1} = 1381.41$$

where

$$\begin{aligned}\bar{x}(5) &= \frac{0.5m(5-1)+Q(5)+0.5m(5+1)}{2}\\ &= \frac{0.5m(4)+Q(5)+0.5m(6)}{2}\\ &= \frac{0.5(1315.452)+1315.452+0.5(1579.277)}{2} = 1381.41\end{aligned}$$

Step 4: calculate $x^*_{2003_{LC}}$ from the available HLCAFLRs in LC. The fuzzified values of x_{1999} , x_{2000}, x_{2001} of the influence factor time series are NA, B_{15}, B_9 respectively. The available HLCAFLRs found in LC, whose premises are NA, B_{15}, B_9:

$NA, B_{15}, B_9 \rightarrow A_5(1)$

The use of these FLRs gives a prediction $x^*_{2003_{LC}}$ as

$$x^*_{2003_{LC}} = \frac{1\times\bar{x}(5)}{1} = 1381.41$$

where

$$\bar{x}(5) = \frac{0.5m(5-1)+Q(5)+0.5m(5+1)}{2}$$
$$= \frac{0.5m(4)+Q(5)+0.5m(6)}{2}$$
$$= \frac{0.5(1315.452)+1315.452+0.5(1579.277)}{2} = 1381.41$$

The final prediction for $x^*_{2003} = \frac{1381.41+1381.41+1381.41+1381.41}{4} = 1381.41$. Analog for next observation.

Table 5. Fuzzification

Year	Actual rubber production data	Fuzzified rubber production	Actual land area data	Fuzzified land area
2000	1125.2	A_2	3046	B_{15}
2001	1723.3	A_7	2838.4	B_9
2002	1226.6	A_3	2825.5	B_3
2003	1396.2	A_5	2772.5	B_7
2004	1662.0	A_7	2747.9	B_6
⋮	⋮	⋮	⋮	⋮
2020	2533.5	A_{10}	3305.4	B_{23}

Table 6. Third-order cross-relation FLR

Year	HSAFLR	HLAFLR	HSCAFLR	HLCAFLR
2000	NA	NA	NA	NA
2001	NA	NA	NA	NA
2002	NA	NA	NA	NA
2003	$A_2, A_7, A_3 \rightarrow A_5$	$NA, A_2, A_7 \rightarrow A_5$	$B_{15}, B_9, B_3 \rightarrow A_5$	$NA, B_{15}, B_9 \rightarrow A_5$
2004	$A_7, A_3, A_5 \rightarrow A_7$	$A_2, A_7, A_3 \rightarrow A_7$	$B_9, B_3, B_7 \rightarrow A_7$	$B_{15}, B_9, B_3 \rightarrow A_7$
2005	$A_3, A_5, A_7 \rightarrow A_8$	$A_7, A_3, A_5 \rightarrow A_8$	$B_3, B_7, B_6 \rightarrow A_8$	$B_9, B_3, B_7 \rightarrow A_8$
⋮	⋮	⋮	⋮	⋮
2020	$A_{13}, A_{13}, A_{13} \rightarrow A_{11}$	$A_{12}, A_{13}, A_{13} \rightarrow A_{11}$	$B_{17}, B_{21}, B_{22} \rightarrow A_{11}$	$B_{17}, B_{17}, B_{21} \rightarrow A_{11}$

Table 7. Defuzzification

Year	HSAFLR (x^*_S)	HLAFLR (x^*_L)	HSCAFLR (x^*_{SC})	HLCAFLR (x^*_{LC})	Forecasting
2000	NA	NA	NA	NA	NA
2001	NA	NA	NA	NA	NA
2002	NA	NA	NA	NA	NA
2003	1381.41	1381.41	1381.41	1381.41	1381.41
2004	1658.46	1658.46	1658.46	1658.46	1658.46
2005	1817.176	1817.176	1817.176	1817.176	1817.176
⋮	⋮	⋮	⋮	⋮	⋮
2020	2390.408	2390.408	2390.408	2390.408	2390.408

After this, the forecast value is determined using AFER. The last step involves comparing this method with existing ones as shown in Table 8. According to Table 8, the proposed method had a smaller error than existing ones, with an AFER value of 4.77%.

Table 8. AFER of the proposed method with existing ones

Evaluated criteria	Huarng [46]	Lee [48] with triangular fuzzy set	Li [50]	Proposed method of third-order
AFER	10.99%	5.10%	5.06%	4.77%

5. CONCLUSION

This research modified Li's method in the partition of the universe of discourse and determining the two arbitrary positive numbers using intervals ratio. The proposed method was applied in third-order (h=3) with long relation h+1. Furthermore, this modification has a smaller error than previous methods with an AFER value of 4.77% < 10%, hence good forecasting criteria. Furthermore, the coefficient values of D_1 and D_2 were automatically obtained from the intervals ratio algorithm. The future works used the frequency density to modify partitioning the universe of discourse to eliminate unused partition intervals.

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BIOGRAPHIES OF AUTHORS

Etna Vianita received her Bachelor Degree in Mathematics in 2018 from Universitas Diponegoro, Semarang, Indonesia. In 2019 she received a scholarship from The Indonesia Endowment Funds for Education (LPDP). She is currently continuing her Master Degree in Universitas Diponegoro, Semarang, Indonesia. Her interest is in the fields of applied mathematics, especially in applied fuzzy. She can be contacted at email: etnavianita@student.undip.ac.id.

Muhammad Sam'an received his Bachelor Degree from Universitas Negeri Semarang and Master Degree from Universitas Diponegoro in Mathematics 2010 and 2016 respectively and now, he is studying in the postgraduate student, Faculty of Technology Management and Business, Universiti Tun Hussein Onn Malaysia (UTHM). His research interests are in optimization, fuzzy mathematics, and computational mathematics. He can be contacted at email: muhammad.92sam@gmail.com.

A. Nafis Haikal received his Bachelor Degree in Mathematics in 2019 from Universitas Airlangga, Surabaya, Indonesia. He is currently continuing his Master Degree in Universitas Diponegoro, Semarang, Indonesia. The scope of research he is currently working on is in the fields of Applied Mathematics, especially in Applied Fuzzy. He can be contacted at email: anafishaikal@students.undip.ac.id.

Ina Salamatul Mufaricha received her Bachelor Degree in Mathematics in 2017 from Universitas Islam Negeri Sunan Kalijaga, Yogyakarta, Indonesia. In 2019 she received a scholarship from The Indonesia Endowment Funds for Education (LPDP). She is currently continuing her Master Degree in Mathematics at the Universitas Gadjah Mada, Sleman, Yogyakarta, Indonesia. She is very keen on game theory, control theory, and optimization theory. She can be contacted at email: faricha295@gmail.com.

Redemtus Heru Tjahjana received his B.Sc. and M.Sc. in Mathematics from Universitas Gadjah Mada, Sleman, Yogyakarta, Indonesia in 1998 and 2001 respectively. He got his Ph.D. in Mathematics from Institut Teknologi Bandung in 2010. Since 2000 until now, he joined Mathematics, Universitas Diponegoro to be lecturer. He is currently the Head of Master Degree Program in Mathematics, Department of Mathematics, Faculty of Sains and Mathematics, Universitas Diponegoro, Semarang, Indonesia. His research interests include fuzzy, system and control. He can be contacted at: redemtusherutjahjana@lecturer.undip.ac.id.

Titi Udjiani received her B.Sc. in Mathematics from Universitas Diponegoro, Semarang, Indonesia in 1998. She acquired her M.Sc. and Ph.D in Mathematics from Universitas Gadjah Mada, Sleman, Yogyakarta, Indonesia in 2001 and 2016 respectively. She is currently the Secretary of the Undergraduate Mathematics Study Program, Department of Mathematics, Faculty of Sains and Mathematics, Universitas Diponegoro, Semarang, Indonesia. Her research interests in algebra and fuzzy. She can be contacted at email: udjianititi@lecturer.undip.ac.id.