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Joint digital pre-distortion model based on Chebyshev expansion

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ABSTRACT

In this paper, a new low complexity model is proposed for the joint digital pre-distortion of in-phase/quadrature-phase (I/Q) imbalance, local oscillator (LO) leakage, and power amplifier nonlinearity in direct-conversion transmitters (DCTs). In this structure, we proposed a set of orthogonal basis functions based on Chebyshev expansion to attenuate the problem of numerical instability created during the conventional model identification method. This robust joint digital pre-distortion (DPD) utilized the indirect learning architecture and updated the coefficients vector based on the recursive least square (RLS) algorithm. To verify the operation and efficiency of the proposed model, an extensive simulation in MATLAB was carried out. The results showed a significant reduction in the conditional number and the coefficient dispersion of the observation matrix. Furthermore, the power of the signal in the adjacent channel decreased by more than 16 dB for the orthogonal frequency division multiplexing (OFDM), 16 QAM input signal. In comparison to the previous digital pre-distorter models, the proposed DPD builds strong numerical stability with the least coefficients.

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1. INTRODUCTION

The direct conversion transceiver due to its simplified and low-cost structure is the most popular architecture and the appropriate choice often used in wireless communication systems [1]. Despite the simplicity and integrability of this structure, there are problems due to its practical implementation of analog components especially in the applications of advanced modulation schemes with high peak to average power ratio (PAPR). The nonlinear nature of the power amplifier on one hand and the in-phase and quadrature-phase asymmetry impairments of the modulator, on the other hand, cause in-band signal distortions and adjacent channel interference and dramatically reduce the performance of the transceiver system [2]. Moreover, the advanced signal waveforms such as orthogonal frequency division multiplexing (OFDM), with wide bandwidth and increased PAPR, cause the negative effects of power amplifier nonlinearity to be worse. Therefore, using linearization techniques to improve the efficiency of the transmission signal is necessary [3]–[5].

The digital pre-distortion (DPD) technique as one of the most effective methods, is typically used to overcome the non-idealities. To utilize this method, it is necessary to model the nonlinear behavior of the system with a proper mathematical model such as the memory polynomials [6]–[8], the Wiener Hammerstein model [9], Volterra series [10], [11], the optimized structures with modified algorithms [12], [13] and orthogonal polynomial [14], [15], which consider the memory effects along with the static nonlinearity. Among the many models that have been proposed in the literature, the memory polynomial model is a true parametric model which represents a simplified case of the Volterra model as only diagonal terms are considered and the others are zero. To estimate the model coefficients, the least-squares (LS) approximation is one of the most widely used approaches. In this method, the inversion of an observation matrix must be calculated, which leads to arise an instability problem, dependent on the size of the matrix and the statistics of the input signal. To prevent this instability problem, Raich and Zhou proposed a set of orthogonal basis functions based on the Laguerre polynomials in [14]. The proposed model worked well only for the signals with Rayleigh distribution. But depending on the signal distribution, the conditioning of the observation matrix drastically deteriorates and decreases the DPD performance. As nonlinear order and memory depth increase, the problem becomes more serious. Aziz *et al*. in [16] proposed a rational-function–based model to jointly attenuate the power amplifier (PA) nonlinearity and in-phase/quadrature-phase (IQ) impairments. The proposed model improved the numerical instability problem of DPD by a reduction in the number of coefficients and so reduced the dispersion coefficients and condition number. In [17] a distributed model using conventional memory polynomials was presented. The proposed model used a two-block structure for DPD, the first block was a memory polynomial model to reduce the nonlinear effects of the PA, and the second one was a mild nonlinear model with memory to compensate the modulator's impairments. It was claimed that by distributing nonlinearity and memory depth, a considerable reduction in matrix conditioning and coefficients dispersion was obtained. The obtained numerical stability due to the reduction in the number of pre-distortion coefficients in [16], [17], is very fragile and as the nonlinear order of the system intensify, the number of pre-distortion coefficients inevitably increase, resulting in the changes in the matrix conditioning and numerical instability arise. An orthogonal polynomial model was introduced by Wang *et al.* in [18] by using the combined least-mean-square (LMS) and recursive least square (RLS) algorithm to model the nonlinearity of PA with memory effect. Although this method improved the DPD performance, it was limited to the input signal values distributed in [0, 1], and the combined LMS and RLS algorithm increased the computational complexity. Manai *et al*. [19] proposed a numerically stable digital pre-distorter based on the Gegenbauer polynomials to linearize the radio frequency (RF) power amplifiers without the IQ impairments compensation. Although the Gegenbauer polynomials were orthogonal and the pre-distorter based on the Gegenbauer basis functions was numerically stable, it worked only on the real-valued variable on the input values of the interval $[-1, 1]$ and only considered the PA nonlinearity while all the other impairments of the transmitter affected on the DPD performance.

In this paper, a set of new orthogonal basis functions is introduced based on the Chebyshev expansion to joint compensate for the non-linearity and the memory effects of the power amplifier as well as frequency-dependent impairments of the direct conversion transmitters. The new DPD structure is intended to solve the numerical instability problem that may arise through the conventional polynomial model approximation, without any limitation on the input signal statistics. Therefore, by increasing the nonlinear order of the system and as a result increasing the number of pre-distortion coefficients, the value of the conditional number remains small and numerical stability is guaranteed. We achieved the best numerical stability merits with the least numbers of coefficients and computational complexity in comparison to the references' literature.

This paper is presented in five sections: section 2, describes the mathematical model of PA nonlinearity, then proposes the Chebyshev pre-distortion method to alleviate all the transmitter imperfections. In section 3, the numerical stability and performance evaluation of this work are analyzed. The simulation results demonstrate the operation of this work in section 4 and finally, the conclusions are given in section 5.

2. MODEL DESCRIPTION

In this section, the memory polynomial (MP) model is presented to model the nonlinearity of PA with memory effect, then the numerical instability problem of the MP model for DPD is explained. The new DPD model based on Chebyshev polynomials is proposed and leads to making the new joint pre-distorter to compensate for the PA nonlinearity and IQ impairments.

2.1. Modeling nonlinearity of power amplifiers regarding the memory effects

Power amplifier as a main element of the transmitter chain has a nonlinear characteristic and distorts the transmission signal which results in spectral regrowth in digitally modulated signals. This phenomenon causes signal interference in neighboring channels. To correct the destructive effects of power amplifier nonlinearity, the behavior of this component must be mathematically modeled. To have a more complete model, the effects of memory must also be considered. For this reason, a memory polynomial model is derived by (1):

$$y(n) = \sum_{p=0}^{P} \sum_{q=0}^{Q} c_{2p+1,q} v(n-q) \, |v(n-q)|^{2p} \tag{1}$$

where *v(n)* is the complex base-band signal at the input of PA, P and Q represent the polynomial order and memory depth of the model respectively, and *y(n)* is nonlinear PA output. The complex-valued coefficients C, are given as follow, which are extracted from an actual Class AB PA [20].

$$\begin{array}{lll} c_{10} = 1.0513 + 0.0904j & c_{11} = -0.0680 - 0.0023j & c_{12} = 0.0289 - 0.0054j \\ c_{30} = -0.0542 - 0.2900j & c_{31} = 0.2234 + 0.2317j & c_{32} = -0.0621 + 0.0932j \\ c_{50} = -0.9657 - 0.7028j & c_{51} = -0.2451 - 0.3735j & c_{52} = 0.1229 + 0.1508j \end{array}$$

2.2. Numerical instability problem in the model identification

The conventional polynomial model of a non-linear system can be expressed as in (2):

$$y(t) = \sum_{k=1}^{K} b_k |x(t)|^{k-1} x(t) \tag{2}$$

where $x(t)$ and $y(t)$ are the baseband input and output of the nonlinear system and b_k is the unknown complex model coefficient. By replacing $\phi_k(x) = |x|^{k-1}x$, the (2) can be rewritten as (3):

$$y(t) = \sum_{k=1}^{K} b_k \phi_k(x(t)) \tag{3}$$

where $\phi_k(x)$ is the conventional polynomial basis function. To extract the model coefficients $b = [b_1, b_2, \ldots, b_K]^T$, N samples of the measured input and output envelopes can be used as N-by-1 input/output data vector:$x = [x(t_1), \ldots, x(t_N)]^T$, $Y = [y(t_1), \ldots, y(t_N)]^T$, then define the following vector notation.

$$\phi_k(x) = [\phi_k(x(t_1)), \ldots, \phi_k(x(t_N))]^T, \quad \Phi = [\phi_1(x) \;\; \phi_2(x) \;\; \cdots \;\; \phi_K(x)] \tag{4}$$

The relation (3) can be written as $Y = \Phi b$ and the least square estimate of the coefficient vector b is

$$b_{LS} = (\Phi^H \Phi)^{-1} \Phi^H \mathrm{Y} \tag{5}$$

where $(\Phi^H \Phi)^{-1}\Phi^H$ is the Moore–Penrose pseudo-inverse of Φ and Φ^H is its Hermitian transpose [14]. The inversion of the K-by-K matrix $\Phi^H\Phi$ can experience a numerical instability problem because the matrix $\Phi^H\Phi$ is often ill-conditioned [14]. To overcome this problem, various methods have been proposed to convert the conventional polynomials to a set of orthogonal basis functions and thus improve the condition number of the observation matrix. But these orthogonal polynomials have some limitations in signal statistics. On the other hand, some classic orthogonal polynomials have very useful properties in solving mathematical and physical problems but are less commonly used in DPD structures. In the next section, we propose a set of orthogonal basis functions based on Chebyshev expansion to attenuate the problem of numerical instability arising during the conventional model identification method.

2.3. Chebyshev polynomials as the pre-distorter model

Digital pre-distortion based on memory polynomial model is one of the popular and efficient models that is used to compensate the PA nonlinearity but can experience the numerical instability problem that reduces the performance of the DPD. To overcome this problem, various methods have been proposed to convert the conventional polynomials to a set of orthogonal basis functions [14], [16]–[19] and thus improve the condition number of the observation matrix. But these orthogonal polynomials have some limitations in signal statistics or some of them are complicated in the calculation [14]. The orthogonal polynomials are a category of polynomials that satisfy the orthogonality relation (the inner product between two terms of the sequence must be zero.) on a specified interval [18]. In this section, an orthogonal polynomial model based on Chebyshev expansion is proposed for the digital pre-distorter modeling. The Chebyshev polynomials of the first kind are defined as in (6):

$$\begin{aligned} &T_0(x) = 1, \; T_1(x) = x, \\ &T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x) \end{aligned} \tag{6}$$

where $T_n(x), \quad n = 0,1,2, \ldots$, is the n^{th}-order term of the polynomial. The original first kind Chebyshev polynomials formula is given as in (7) [21]:

$$T_p(x) = p\sum_{m=0}^{\lfloor p/2 \rfloor}(-1)^m \frac{(p-m-1)!}{m!(p-2m)!} 2^{p-2m-1} x^{p-2m}, \quad p = 1,2,3,\ldots \tag{7}$$

where $\lfloor p/2 \rfloor$ denotes the largest integer number less than or equal to $p/2$ and p is the order of nonlinearity.

As stated earlier, the Chebyshev basis functions are orthogonal and their orthogonality is limited to the real-valued variable in [-1, +1]. Whereas baseband signals used in communication systems are typically complex with an interval $[a, b]$ where $b > a$. So, they cannot be directly used as the basis functions in digital pre-distortion structure. Furthermore, the zero-order term in the Chebyshev polynomial model does not conform to the baseband model of a nonlinear system [14]. To make the proposed Chebyshev polynomial appropriate to any finite range of the communication signals (i.e. $a < z < b$) and apply this orthogonal polynomial to the complex signals as a pre-distorter model, we propose the new variable $\hat{z}$ by applying the linear transformation as $\hat{z} \triangleq 2(r - a/b - a) - 1$ with $r = |z|$ and $a = min(r), b = max(r)$, Then replace thex^{p-2m} in (7) by $(\hat{z})^{p-2m}z$, to define $T'_{p+1}(z)$ as in (8):

$$T'_{p+1}(z) = p\sum_{m=0}^{\lfloor p/2 \rfloor}(-1)^m \frac{(p-m-1)!}{m!(p-2m)!} 2^{p-2m-1} (\hat{z})^{p-2m} z, \ for \quad p = 1,2,3,4,\ldots \tag{8}$$

where $T'_1(z) = z$; z is a complex variable of the input signal and is defined as z = $x + iy$; $x,y \in$ R.

Now, equation (8) defines the modified Chebyshev polynomial which can be used as a nonlinear model to design a new robust pre-distorter in the next section. The closed-form relation of the modified complex Chebyshev polynomial with the memory effects can be written as in (9):

$$\begin{aligned} T'_{p+1,q}(z(\tau)) &= p\sum_{m=0}^{\lfloor p/2 \rfloor}\sum_{q=0}^{Q} \quad (-1)^m \frac{(p-m-1)!}{m!(p-2m)!}) 2^{p-2m-1} (\hat{z}(\tau - q))^{p-2m} z(\tau - q), \\ &\quad for \quad p = 1,2,3,\ldots \\ T'_{1,q}(z(\tau)) &= \sum_{q=0}^{Q} z(\tau - q) \end{aligned} \tag{9}$$

where Q is the memory depth and p is the highest non-linearity order.

2.4. Proposed digital pre-distorter model of PA and IQ modulator

Figure 1 shows the baseband model of a direct conversion transmitter with a joint digital pre-distorter. The schematic of the joint pre-distorter is shown with more details in the mathematic relations. It is based on the Chebyshev polynomial model which can linearize several different nonlinear models with memory. Based on the pre-distorter model in Figure 1, the mismatch compensator block consists of two finite impulse response (FIR) filters that compensate for the frequency-dependent IQ impairments of the modulator [22]. For the joint pre-distorter, we must combine the PA pre-distorter with the mismatch compensator and build a unit structure from both of them. The pre-distorter output is represented as in (10):

$$u_p(n) = \sum_{q=0}^{Q'} a_q^T(n).T'_q(s(n)) \tag{10}$$

where $T'_q(s(n)) = [T'_{1,q}(s(n)), T'_{3,q}(s(n)), \ldots, T'_{2p+1,q}(s(n))]^T$, $a_q(n) = [a_{1,q}(n), a_{3,q}(n), \ldots, a_{2p+1,q}(n)]^T$ and q indicates the memory size.

By converting the pre-distorter output in (10) to a vector form, we can show that equation as a closed-form in the following:

$$u_p = \underline{\mathbf{a}}^T(n).\underline{\mathbf{T}}'(s(n)) \tag{11}$$

where $\underline{\mathbf{T}}'(s(n)) = \left[\mathbf{T}_0'^T(s(n)), \mathbf{T}_1'^T(s(n)), \ldots, \mathbf{T}_{Q'}'^T(s(n))\right]^T$, $\underline{\mathbf{a}}(n) = \left[\mathbf{a}_0^T(n), \mathbf{a}_1^T(n), \ldots, \mathbf{a}_{Q'}^T(n)\right]^T$.

To take into account the modulator mismatches, an obvious form for pre-distorter of the modulator can be obtained by the following signal transformation [22]:

$$x(n) = G_1(n) \otimes u_p(n) + G_2(n) \otimes u_p{}^*(n) + d \tag{12}$$

By using (11) in (13), $x(n)$ is rewritten as in (13):

$$\begin{aligned} x(n) &= G_1(u_p(n)) + G_2(u_p{}^*(n)) + d \\ &= G'_I(u_p(n)) + G'_Q(d\alpha.u_p(n)) + G'_I(u_p^*(n)) + G'_Q(d\beta.u_p^*(n)) + d \end{aligned} \tag{13}$$

where $d\alpha$ and $d\beta$ are constant-coefficient related to the gain and phase mismatch of the modulator. So, after simplification we have:

$$x(n) = G_I^{'}(u_p(n) + u_p^{*}(n)) + G_Q^{'}(d\alpha.u_p(n) + d\beta.u_p^{*}(n)) + d \tag{14}$$

$$G_I^{'}(n) = \sum_{k=0}^{K} c_k x(n-k), \quad G_Q^{'}(n) = \sum_{l=0}^{L} b_l\, x(n-l). \tag{15}$$

Suppose to $L = K$ then $x(n)$ can be rewritten as:

$$x(n) = \sum_{k=0}^{K} d_k(n)\left[\underline{\mathbf{a}}^T(n-k)\underline{\mathbf{T}}'(s(n-k)) + \underline{\mathbf{a}}^H(n-k)\underline{\mathbf{T}}'^{*}(s(n-k))\right] + d \tag{16}$$

where $d_k(n)$ denotes the FIR filter coefficients vector and its order are equal to the number of FIR filter taps. So, the final joint pre-distortion model formulation is written as (17).

$$x(n) = \sum_{kq=0}^{Q'+K} a_{kq}^T(n).T_{kq}(s(n)) + \sum_{kq=0}^{Q'+K} a_{kq}^H(n).T'^{*}{}_{kq}(s(n)) + d \tag{17}$$

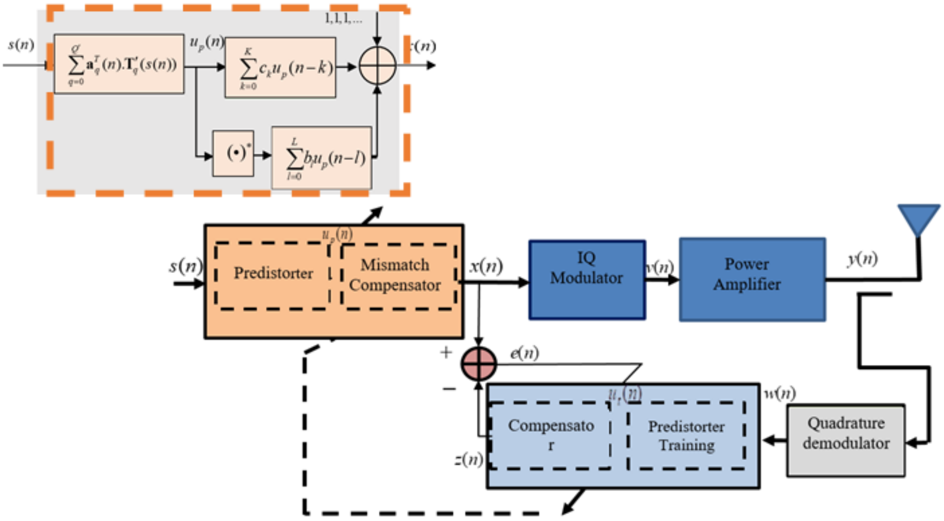


Figure 1. Baseband equivalent model of direct conversion transmitter employing joint pre-distorter

Figure 2 has demonstrated the proposed model. It should be noted that the frequency-dependent *I/Q* impairments are modeled as a linear time-invariant (LTI) system and the memory polynomial PA can be displayed as a two-box structure that contains an LTI system followed by a memory less nonlinear system. To build a joint model, we unified the LTI system of the I/Q modulator with the LTI portion of the power amplifier and obtained one merged LTI system which is connected to a memory less nonlinear system in the joint compensator. After this simplification, we have the final coefficients vector in order of $2(K + Q + 1)(P + 1) + 1$, For example, if $K = 1$, $Q = 2$ and $P = 3$, then the order of joint pre-distorter coefficients will be 33.

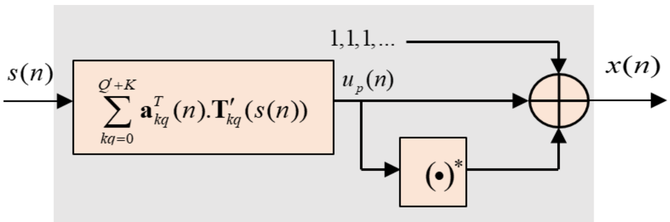


Figure 2. The final schematic of the simplified joint digital pre-distortion model

Joint DPD model coefficients are adaptively corrected using the indirect learning architecture (ILA) [23]. According to the architecture in Figure 1, the coefficients that are estimated for the pre-distorter and compensator training blocks in the feedback direction are equal to those of the pre-distorter and mismatch compensator blocks in the forward direction, therefore the discrete-time output signal of the training block can be written as:

$$z(n) = \underline{\mathbf{b}}_{kq}^{T}(n)\underline{\mathbf{T}}'_{kq}(w(n)) \tag{18}$$

where, $\underline{\mathbf{b}}_{kq}(n) = [\mathbf{b}_{kq}{}^{T}(n), \mathbf{b}_{kq}{}^{H}(n), \mathbf{d}]^{T}$, $\underline{\mathbf{T}}'_{kq}(w(n)) = [\mathbf{T}'_{kq}{}^{T}(w(n)), \mathbf{T}'_{kq}{}^{H}(w(n)), \mathbf{1}]^{T}$.

By using the *least square* cost function, the training block output signal $z(n)$ and joint DPD output signal $x(n)$ will be close together to minimize the differential error:

$$J(n) = \sum_{l=1}^{n} \lambda^{n-l} \left| x(l) - z(l) \right|^{2} = \sum_{l=1}^{n} \lambda^{n-l} \left| x(l) - \underline{\mathbf{b}}_{kq}^{T}(n)\underline{\mathbf{T}}'_{kq}(w(l)) \right|^{2} \tag{19}$$

where λ is named the forgetting factor and its value is a real number in the domain $0<\lambda<1$ (in our calculations $\lambda = 0.988$). In the following, an RLS algorithm has been described [23] to minimize $J(n)$ and estimate the optimal $\underline{b}_{kq}(n)$. After finding the $\underline{b}_{kq}(n)$ at each iteration, they are replaced by $a_{kq}^{T}(n)$ in (13) as the pre-distorter coefficients, and the process will be continued until x(n) be equal to the z(n), see Figure 1.

2.5. RLS algorithm for parameters estimation

The RLS algorithm is defined as following steps:

– Initialization steps:

$$\mathbf{Q}(0) = \delta^{-1}\mathbf{I} \quad , \quad \underline{\mathbf{b}}_{kq}(0) = [1, 0, ..., 0]^{T}$$

Where δ is a small positive constant for initialization (in our work $\delta = 0.008$), I denote the identity matrix.

– N^th iteration*:*

$$\mathbf{k}(n) = \frac{\lambda^{-1}\mathbf{Q}(n-1)\underline{\mathbf{T}}'_{kq}{}^{*}(w(n))}{1 + \lambda^{-1}\underline{\mathbf{T}}'_{kq}{}^{T}(w(n))\mathbf{Q}(n-1)\underline{\mathbf{T}}'_{kq}{}^{*}(w(n))}$$

$$e(n) = x(n) - \underline{\mathbf{b}}_{kq}{}^{T}(n-1)\underline{\mathbf{T}}'_{kq}(w(n))$$

$$\underline{\mathbf{b}}_{kq}(n) = \underline{\mathbf{b}}_{kq}(n-1) + \mathbf{k}(n)e^{*}(n)$$

$$\mathbf{Q}(n) = \lambda^{-1}\mathbf{Q}(n-1) - \lambda^{-1}\mathbf{k}(n)\underline{\mathbf{T}}'_{kq}{}^{T}(w(n))\mathbf{Q}(n-1)$$

3. STABILITY ANALYSIS AND MODEL PERFORMANCE

In this section, at first, the numerical stability and complexity of this method are analyzed. The condition number and the dispersion coefficient are two numerical analysis metrics to evaluate the numerical stability of the proposed model. Then the performance evaluation of the proposed scheme is demonstrated to validate the presented model.

3.1. Numerical stability and complexity analysis

As shown in section 2.2, the coefficients vector of the joint DPD model ($\underline{b}_{kq}$) can be calculated using the Moore-Penrose pseudo-inverse. This pseudo-inverse calculation is significantly influenced by changes in matrix conditioning [24]. The definition of the condition number depends on the norm type, since we used the RLS algorithm based on the norm-2 error cost function, the norm-2 condition number, $\kappa(A)$, is utilized to measure the numerical stability of the proposed model (A is the matrix to be inverted). Therefore, it is defined as the ratio between the highest ($\sigma_{1,A}$) and the smallest ($\sigma_{2,A}$) eigenvalues calculated for the Vandermonde matrix using single value decomposition [24]:

$$\kappa(A) = \|A\|_2 \|A^{-1}\|_2 = \frac{\sigma_{1,A}}{\sigma_{2,A}} \tag{20}$$

In our calculations, $T^{\prime kq}$ is the Vandermonde matrix includes the augmented power of the input signal. Figure 3 shows the condition number of the observation matrix vs. the polynomial order for the proposed Chebyshev DPD model in comparison MP model. As can be seen in the figure, by using the pre-distorter model, we have achieved a significant reduction in the matrix conditioning $\kappa(A)$ for the LTE 16 QAM signal.

Another numerical analysis metric is the dispersion coefficient. The dispersion coefficient specifies the dispersion of the extracted coefficients vector over the domain. Higher dispersion coefficients represent a larger number of bits used to cover the entire range of coefficients in the digital signal processor (DSP) chips [24]. Figures 4 and 5 show the coefficient dispersion and the condition number of the observation matrix for each sample of the input signal during the ILA iterations. As shown in the plot, for the OFDM 16 QAM signal, the condition number has improved more than 80 dB in comparison to the MP model, and the dispersion coefficient parameter has a reduction of around 10-20 dB, which causes a considerable reduction in complexity.

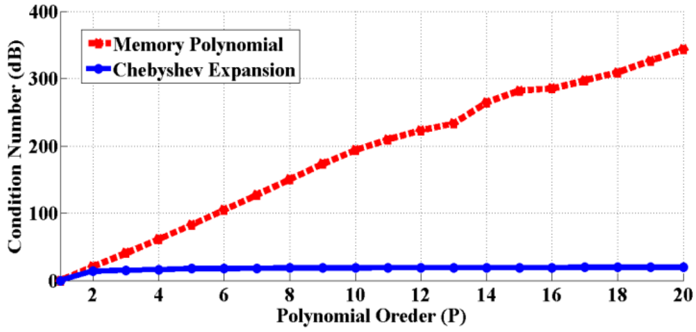


Figure 3. Condition number of the different polynomials' observation matrices for the LTE 16 QAM signal

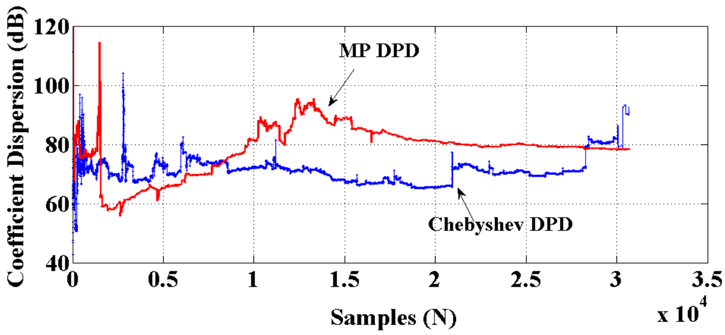


Figure 4. Coefficient dispersion of the different polynomials' observation matrices for the OFDM 16 QAM signal

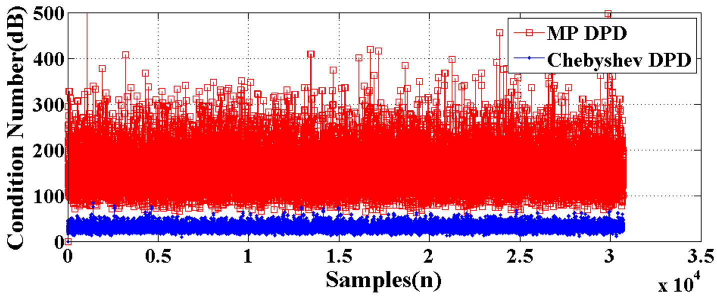


Figure 5. Condition number of the different polynomials' observation matrices for the OFDM 16 QAM signal

The figures of merit for the complexity evaluation of the proposed joint DPD are the number of complex-valued parameters and the number of floating-point operations (FLOPs) required to implement the model. In this work, a new orthogonal Chebyshev DPD has been defined which its polynomial basis functions with odd-order nonlinearity are $T'_{kq}(s(n)) = \left[T'_{1,kq}(s(n)), T'_{3,kq}(s(n)), T'_{5,kq}(s(n)), T'_{7,kq}(s(n)), \dots\right]^T$, $kq = 0,1,2,3$. To calculate the number of required FLOPs, we have driven two relations based on the method used in [17, 25] for the Chebyshev DPD (based on the recursive equation in (2)) and the MP DPD (based on equation (1)). According to the results shown in Table 1, for a joint DPD with 33 complex-valued coefficients ($K = 1$, $Q = 2$ and $P = 3$), we require 356 FLOPs at every step of the computation to implement the model. Although more FLOPs are required for Chebyshev DPD implementation in comparison to MP DPD, the proposed DPD builds strong numerical stability with the least coefficients and complexity.

3.2. Model performance metrics

To evaluate the linearity performance of the system, the normalized mean square error (NMSE) is commonly used [25], [26], which is formulated as follows:

$$NMSE(dB) = 10\, log_{10}\left(\frac{\sum_{n=1}^{N}|y_{desired}(n) - y_{\ measured}(n)|^2}{\sum_{n=1}^{N}|y_{\ desired}(n)|^2}\right) \tag{21}$$

where the *desired* signal is denoted the input signal and the *measured* one is the PA output after the joint digital pre-distortion. The results are shown in Table 1, where the NMSE performance of the transmitted signal after PA linearization and compensation of IQ impairments for the proposed model has improved more than 3 dB in comparison to the MP model.

A principal criterion to compute the power leakage due to the spectral regrowth into the adjacent channel is adjacent channel power ration (ACPR) which is given by [17].

$$ACPR_{(dB)} = -10\, log_{10}\left(\frac{\int_{adj.channel} S(f)}{\int_{inband} S(f)}\right) \tag{22}$$

In the above statement, S(f) defines the power spectral density of signal, adj. channel, and in-band defines the adjacent channel and desired channel, respectively [17]. PA nonlinearity disturbance and I/Q impairments cause excessive distortion of the transmitter output signal, resulting in poor ACPR. However, by performing the proposed pre-distorter, the transmitter impairments are compensated and significant improvement in ACPR, about 16 dB, is achieved. For the input symbol S_{in} and the transmitted symbols S_{Tx}, the error vector magnitude (EVM) between the two is defined as in (23).

$$EVM = \sqrt{\frac{\frac{1}{N}\sum_{i=1}^{N}|S_{in}(i) - S_{Tx}(i)|^2}{\frac{1}{N}\sum_{i=1}^{N}|S_{Tx}(i)|^2}} \times 100\% \tag{23}$$

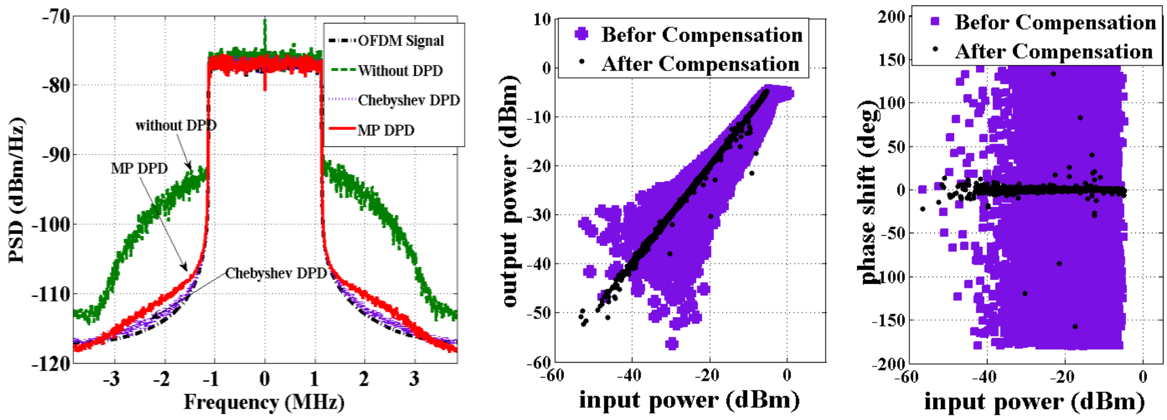
The results are presented in Table 1 and after using the proposed DPD method for the class-AB PA, great improvement for EVM has been obtained.

Table 1. Performance evaluation of Chebyshev DPD in comparison to memory polynomial DPD

DPD Model	No. of Coefficients	No. of Flops	ACPR (dB)	EVM% (before)	EVM% (after)	NMSE (dB)
Chebyshev	33	$(K+Q+1)(2P\times 4+2+8(P+1))+8(K+Q+1)(P+1)-4=356$	-43.2	45.49 %	0.66 %	-36.34 dB
MP	33	$10(K+Q+1)(P+1)+P(K+Q+1)+8(K+Q+1)(P+1)-4=296$	-41	45.49 %	1.3 %	-33.27 dB

4. RESULTS AND DISCUSSION

To demonstrate the performance of the common digital pre-distorter model, a signal transmission system is simulated in MATLAB to model a direct conversion transmitter with impairments. The system is excited by an OFDM 16-QAM single carrier modulation signal with about 4 MHz bandwidth and 7.68 MHz sampling frequency in the digital front-end. A memory polynomial nonlinear model of PA from [20] is simulated which is related to a real class AB power amplifier. An indirect learning architecture is used to model digital pre-distorter structure. The values of the gain and phase imbalance used in the simulations are g=1.07 and $\varphi = 5^{\circ}$, respectively, and a dc offset which is equal to c=0.05+0.01i. The frequency-dependent mismatch is modeled by two filters as h_i=[0.99 -0.1]^T and h_q=[0.98 -0.07]^T, for I and Q branches, respectively. The Chebyshev pre-distorter is employed as three/two order (nonlinear order/memory depth), with filter lengths of 2. We used 30,720 samples in the RLS algorithm to estimate the digital pre-distorter coefficients. Figure 6(a) shows the power spectral density (PSD) of the output signal after applying the proposed pre-distorter. It can be seen that without any pre-distortion, spectral regrowth will occur and causes high distortion in co-channel and adjacent-channel which eventuate a poor ACPR. According to the results, it is clear that by using the proposed pre-distorter, the ACPR has enhanced greatly, and the PSD of the PA's output fully complies with that one which is from the input signal. These simulations are executed for the OFDM 16 QAM input signal and an evaluation of the performance is presented. It can be observed that the proposed joint pre-distortion model was able to compensate for the memory effects and nonlinearity of PA as well as IQ imbalance of modulator in direct-conversion transmitter (DCT). Based on the results, by applying this DPD structure, for high PAPR input signal (about 10.3 dB), in presence of transmitter non-idealities, ACPR more than 43 dB can be obtained which is about 3 dB more than the MP DPD model. When IQ impairments effects on input signal are considered, the PA output has a more complicated nonlinearity, which can be seen as amplitude and phase modulation, known by the amplitude modulation to amplitude modulation (AM/AM) and amplitude modulation to phase modulation (AM/PM) parameters, respectively. The characteristics of the transmitter are shown in Figure 6(b). After linearization, the desired AM/AM and AM/PM characteristics have been obtained with the least model coefficients. In Table 2, we have compared the proposed method with the methods presented in the references to show more the superiority and efficiency of this method. All the table parameters are obtained from the values reported in their reference.



(a) (b)

Figure 6. Output characteristics of the transmitter using 16-QAM OFDM signals (a) PA's output PSD and (b) AM-to-AM and AM-to-PM characteristics for class AB power amplifier with 1.07 dB gain imbalance and 5º phase imbalance.

Table 2. Performance of proposed model in comparison with the other structures in literature.

Ref	[19]	[10]	[16]	[17]	[27]	[28]	This work
Input Signal	LTE	OFDM 16 QAM	WCDMA	WCDMA	LTE	16 QAM	OFDM 16 QAM
PAPR(dB)	11.49	10	-	10.6	9.5	8	10.32
ACPR improvement	16 dB	21 dB	17 dB	14 dB	10 dB	11 dB	16 dB
Type of DPD	Gegenbauer polynomial	Volterra series	Rational Function	Distributed polynomial	Spectral Weighting	Cascaded DPD	Chebyshev polynomial
Coefficients	42	194	76	34	150	38	33
FLOPs	-	-	>400	333	-	-	356
Condition Number(dB)	<30	-	117.8	68.5/48.8	-	-	<20 dB
Component(s) linearization	Just PA	Just PA	PA+IQ mismatch	PA+IQ mismatch	Just PA	PA+DM TX	PA+IQ mismatch

5. CONCLUSION

A low complexity digital pre-distortion model was proposed to estimate and compensate for the defects of direct-conversion transmitters. The proposed model has compensated the nonlinearity of PA, local oscillator (LO) leakage, and frequency-dependent IQ imbalance of transmitter jointly, with the least number of DPD model coefficients and consequently, low complexity and low-cost structure for DSP implementation were obtained. The DPD was based on the orthogonal Chebyshev polynomial model which has been modified to be orthogonal for all real and complex values of the input signal and make the DPD robust to the input signal statistics and numerical instability. The simulation results show the promising performance of the proposed structure and approve its numerical stability in a fixed-point calculation environment. We improved the ACPR of the transmitted signal by about 16 dB for the OFDM-16 QAM excitation signals. Using the new orthogonal polynomial model results the conditional number parameter value reach less than 40 decibels for the OFDM input signal. Therefore, this model can be implemented on field programmable gate array (FPGA) or DSP as a good candidate for the fixed-point calculation environments.

REFERENCES

[1] L. Maurer, R. Stuhlberger, C. Wicpalek, G. Haberpeuntner, and G. Hueber, "Be flexible," *IEEE Microwave Magazine*, vol. 9, no. 2, pp. 83–95, Apr. 2008, doi: 10.1109/MMM.2008.915364.

[2] A. A. Abidi, "CMOS wireless transceivers: the new wave," *IEEE Communications Magazine*, vol. 37, no. 8, pp. 119–124, Feb. 1999, doi: 10.1109/35.783135.

[3] R. Gerzaguet, L. Ros, F. Belvèze, and J.-M. Brossier, "Performance of a digital transmitter leakage LMS-based cancellation algorithm for multi-standard radio-frequency transceivers," *Digital Signal Processing*, vol. 51, pp. 35–46, Apr. 2016, doi: 10.1016/j.dsp.2016.01.009.

[4] J. M. Pardo-Martín and F. J. Ortega-González, "Efficient adaptive compensation of I/Q imbalances using spectral coherence with monobit kernel," *Digital Signal Processing*, vol. 23, no. 5, pp. 1806–1812, Sep. 2013, doi: 10.1016/j.dsp.2013.04.003.

[5] G. C. Tripathi and M. Rawat, "Analysing digital predistortion technique for computation-efficient power amplifier linearisation in the presence of measurement noise," *IET Science, Measurement & Technology*, vol. 15, no. 4, pp. 398–410, Feb. 2021, doi: 10.1049/smt2.12041.

[6] S. Monther Yousif, R. M. Sidek, A. Sabah Mekki, N. Sulaiman, and P. Varahram, "Efficient low-complexity digital predistortion for power amplifier linearization," *International Journal of Electrical and Computer Engineering (IJECE)*, vol. 6, no. 3, pp. 1096–1105, Jun. 2016, doi: 10.11591/ijece.v6i3.pp1096-1105.

[7] S. Cai, X. Tuo, P. Zhan, and K. Qin, "Digital predistortion based on single-feedback method and indirect learning structure," *Analog Integrated Circuits and Signal Processing*, vol. 75, no. 1, pp. 125–131, Apr. 2013, doi: 10.1007/s10470-012-9909-5.

[8] P. N. Landin, K. Barbé, W. Van Moer, M. Isaksson, and P. Händel, "Two novel memory polynomial models for modeling of RF power amplifiers," *International Journal of Microwave and Wireless Technologies*, vol. 7, no. 1, pp. 19–29, Feb. 2015, doi: 10.1017/S1759078714000397.

[9] J. C. Pedro and S. A. Maas, "A comparative overview of microwave and wireless power-amplifier behavioral modeling approaches," *IEEE Transactions on Microwave Theory and Techniques*, vol. 53, no. 4, pp. 1150–1163, Apr. 2005, doi: 10.1109/TMTT.2005.845723.

[10] J. A. Becerra-González, M. J. Madero-Ayora, J. Reina-Tosina, and C. Crespo-Cadenas, "Digital predistortion of power amplifiers using structured compressed-sensing Volterra series," *Electronics Letters*, vol. 53, no. 2, pp. 89–91, Jan. 2017, doi: 10.1049/el.2016.3879.

[11] C. Yu, L. Guan, E. Zhu, and A. Zhu, "Band-limited volterra series-based digital predistortion for wideband RF power amplifiers," *IEEE Transactions on Microwave Theory and Techniques*, vol. 60, no. 12, pp. 4198–4208, Feb. 2012, doi: 10.1109/TMTT.2012.2222658.

[12] J. A. Becerra, A. Perez-Hernandez, M. J. Madero-Ayora, and C. Crespo-Cadenas, "A reduced-complexity direct learning architecture for digital predistortion through iterative pPseudoinverse calculation," *IEEE Microwave and Wireless Components Letters*, vol. 31, no. 8, pp. 933–936, Aug. 2021, doi: 10.1109/LMWC.2021.3079839.

[13] K.-P. Lee, S.-I. Hong, and E.-R. Jeong, "A polynomial digital pre-distortion technique based on iterative architecture," *International Journal of Electrical and Computer Engineering (IJECE)*, vol. 6, no. 1, pp. 106–112, Feb. 2016, doi: 10.11591/ijece.v6i1.9338.

[14] R. Raich and G. T. Zhou, "Orthogonal polynomials for complex gaussian processes," *IEEE Transactions on Signal Processing*, vol. 52, no. 10, pp. 2788–2797, Feb. 2004, doi: 10.1109/TSP.2004.834400.

[15] X. Wu and L. Chen, "2-D orthogonal polynomial model for concurrent dual-band digital predistortion based on complex Gaussian assumption," *AEU - International Journal of Electronics and Communications*, vol. 135, Jun. 2021, doi: 10.1016/j.aeue.2021.153704.

[16] M. Aziz, M. Rawat, and F. M. Ghannouchi, "Rational function based model for the joint mitigation of I/Q imbalance and PA nonlinearity," *IEEE Microwave and Wireless Components Letters*, vol. 23, no. 4, pp. 196–198, Feb. 2013, doi: 10.1109/LMWC.2013.2247751.

[17] M. Aziz, M. Rawat, and F. M. Ghannouchi, "Low complexity distributed model for the compensation of direct conversion transmitter's imperfections," *IEEE Transactions on Broadcasting*, vol. 60, no. 3, pp. 568–574, Sep. 2014, doi: 10.1109/TBC.2014.2343071.

[18] D. Wang, J. Wang, and W. Wu, "Orthogonal polynomial-based digital predistortion for wideband power amplifiers," in *2015 Asia-Pacific Microwave Conference (APMC)*, Feb. 2015, pp. 1–3, doi: 10.1109/APMC.2015.7413524.

[19] M. Manai, H. Chenini, A. Harguem, N. Boulejfen, and F. M. Ghannouchi, "Robust digital predistorter for RF power amplifier linearisation," *IET Microwaves, Antennas & Propagation*, vol. 14, no. 7, pp. 649–655, Jun. 2020, doi: 10.1049/iet-map.2018.5680.

[20] L. Ding *et al.*, "A robust digital baseband predistorter constructed using memory polynomials," *IEEE Transactions on Communications*, vol. 52, no. 1, pp. 159–165, Feb. 2004, doi: 10.1109/TCOMM.2003.822188.

[21] M. Abramowitz, I. A. Stegun, and D. Miller, "Handbook of mathematical functions with formulas, graphs and mathematical

tables (National Bureau of standards applied mathematics series No. 55)," *Journal of Applied Mechanics*, vol. 32, no. 1, pp. 239–239, Mar. 1965, doi: 10.1115/1.3625776.
[22] L. Anttila, M. Valkama, and M. Renfors, "Frequency-selective I/Q mismatch calibration of wideband direct-conversion transmitters," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 55, no. 4, pp. 359–363, Feb. 2008, doi: 10.1109/TCSII.2008.919500.
[23] R. Marsalek, P. Jardin, and G. Baudoin, "From post-distortion to pre-distortion for power amplifiers linearization," *IEEE Communications Letters*, vol. 7, no. 7, pp. 308–310, Feb. 2003, doi: 10.1109/LCOMM.2003.814714.
[24] M. Rawat, F. M. Ghannouchi, and K. Rawat, "Three-layered biased memory polynomial for dynamic modeling and predistortion of transmitters with memory," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 60, no. 3, pp. 768–777, Mar. 2013, doi: 10.1109/TCSI.2012.2215740.
[25] A. S. Tehrani, Haiying Cao, S. Afsardoost, T. Eriksson, M. Isaksson, and C. Fager, "A comparative analysis of the complexity/accuracy tradeoff in power amplifier behavioral models," *IEEE Transactions on Microwave Theory and Techniques*, vol. 58, no. 6, pp. 1510–1520, Jun. 2010, doi: 10.1109/TMTT.2010.2047920.
[26] O. Hammi, M. Younes, and F. M. Ghannouchi, "Metrics and methods for benchmarking of RF transmitter behavioral models with application to the development of a hybrid memory polynomial model," *IEEE Transactions on Broadcasting*, vol. 56, no. 3, pp. 350–357, Feb. 2010, doi: 10.1109/TBC.2010.2052408.
[27] P. L. Gilabert, D. Lopez-Bueno, and G. Montoro, "Spectral weighting orthogonal matching pursuit algorithm for enhanced out-of-band digital predistortion linearization," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 66, no. 7, pp. 1277–1281, Feb. 2019, doi: 10.1109/TCSII.2018.2878581.
[28] L. Chen *et al.*, "Linearization of a directional modulation transmitter using low-complexity cascaded digital predistortion," *IEEE Transactions on Microwave Theory and Techniques*, vol. 67, no. 11, pp. 4467–4478, Nov. 2019, doi: 10.1109/TMTT.2019.2921346.

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