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Modeling and simulation of the two-tank system within a hybrid framework

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ABSTRACT

Most real-world dynamical systems are often involving continuous behaviors and discrete events, in this case, they are called hybrid dynamical systems (HDSs). To properly model this kind of systems, it is necessary to consider both the continuous and the discrete aspects of its dynamics. In this paper, a modeling framework based on the hybrid automata (HA) approach is proposed. This hybrid modeling framework allows combining the multi-state models of the system, described by nonlinear differential equations, with the system's discrete dynamics described by finite state machines. To attest to the efficiency of the proposed modeling framework, its application to a two-tank hybrid system (TTHS) is presented. The TTHS studied is a typical benchmark for HDSs with four operating modes. The MATLAB Simulink and Stateflow tools are used to implement and simulate the hybrid model of the TTHS. Different simulations results demonstrate the efficiency of the proposed modeling framework, which allows us to appropriately have a complete model of an HDS.

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1. INTRODUCTION

Performance requirements for modeling dynamic systems are proving to be more and more severe. Modeling properly a complex system is a difficult task, particularly, in the case of a system that involves continuous dynamics and discrete events. The continuous dynamic is usually described by differential/difference equations and the discrete dynamic is described by switching laws. This kind of system, called the hybrid dynamical system (HDS), is very popular because of its relevance to different applications and domains such as computer sciences, control systems, mathematics, and automatics.

In this work, our attention is focused on the modeling and the simulation of HDSs. In order to model efficiently an HDS, in a common modeling framework the continuous dynamics, discrete events, and their interaction must be described. In the literature, mainly, three approaches for modeling HDSs exist: i) hybrid automata (HA) approach introduced by Lynch *et al.* [1], which is particularly well suited for modeling HDSs. The HA complements finite automata by time-dependent continuous variables. While the system is in a certain discrete state, these continuous variables evolve according to differential equations called flows [2]; ii) approach consists of taking discrete formalisms, such as finite automata and Petri nets, and extending them by continuous variables which evolve according to differential equations associated with discrete states. Resulting frameworks of this kind are timed automata [3] or hybrid Petri nets [4]; and iii) an approach that directly combines the continuous with its discrete variables through an interface was introduced by

Bemporad and Morari [5]. This approach describes a discrete-time hybrid system. It is called discrete hybrid automata (DHA), whose continuous dynamics are described by linear difference equations and whose discrete dynamics are described by finite state machines. The DHA system can be translated into a form denoted as the mixed logical dynamical (MLD) form [6].

Not only that modeling HDSs is a complex task, but also their numerical simulation is another challenge faced by researchers. It consists of the numerical integration of very complex systems and the synchronized execution of the continuous and discrete model dynamics. In that field, some studies have been reported in the literature [7].

The objective of this study is to build a hybrid framework that efficiently allows the modeling, simulation, and analysis HDSs. The HA approach proved to be the most adequate for this task. Since it is a general modeling formalism for the formal specification, modeling, and algorithmic analysis of HDSs. A set of analysis approaches is available for HA formalism, including reachability analysis, stability analysis, and optimal control methods. A two-tank hybrid system (TTHS) is considered to apply the proposed modeling approach. The TTHS is represented by combining multi-state models (continuous dynamics) with discrete jumps (discrete dynamics). The simulation is conducted using Simulink and Stateflow in MATLAB.

The rest of the paper is organized as: in the section 2, a brief presentation of HDSs is given first, then the modeling approach adopted is detailed. The section 3 represents an application of the proposed approach to a TTHS. In this section, the simulation results are highlighted and discussed. We conclude the paper in section 4.

2. HYBRID AUTOMATA MODELING APPROACH FOR HDS

2.1. Hybrid dynamical systems

HDS is defined as a system that explicitly and simultaneously involves continuous phenomena and discrete events [8]. Since continuous systems are dynamic processes with continuous behavior, their evolution over time can be described by continuous functions or ordinary differential equations. While discrete systems are systems with a countable number of states. The hybrid term refers to the coupling of the continuous phenomena and the discrete ones in a dynamic system as shown in Figure 1 [9]. Generally, HDS represents the interdependence of continuous/discrete dynamic elements in the classical sense of differential subject to discrete event systems.

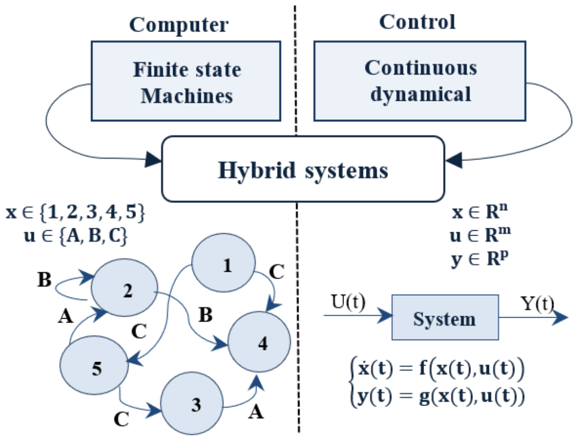

Figure 1. Coupling of continuous and discrete phenomena in an HDS

HDS can be seen as the aggregation of a discrete event system (DES), continuous systems (CS), and an interface that manages the interactions between the two evolutions. An HDS exhibits a behavior characterized by several operating modes. According to their action and the way they are triggered, four types of hybrid phenomena are distinguished [1]: autonomous switching of the dynamics [10], controlled switching of the dynamics [11], autonomous state jumps [12], and controlled state jumps [13].

An example of an HDS within four modes is illustrated in Figure 2. In each mode, the evolution of the continuous states of the system is described by a proper differential equation. The HDS switches between different modes when a particular event occurs. The links are used to indicate the events causing changes between the different dynamics of the HDS.

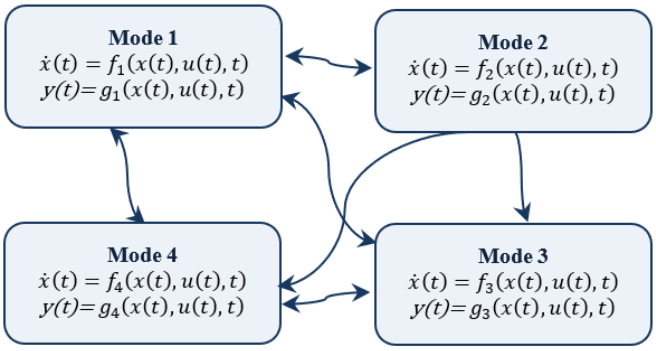

Figure 2. Example of an HDS with four operating modes

2.2. Hybrid automata formalism

A hybrid automaton is a mathematical model that allows describing exactly the systems in which digital processes and analog physical processes interact. It is a transition system that is extended with continuous dynamics. HA are the generalized finite state machines, which can be used for modeling and analysis systems involving mixed continuous and discrete evolutions of the variables. It is the most popular model adopted for HDSs in computer science. It has acquired a particular interest in research for the study of this type of system, in this context, many HA models have been presented in the literature [14]–[17]. Various definitions of hybrid automaton exist, the more comprehensive one is presented by Lygeros *et al.* [18]. The definition is pinpointed in Figure 3.

Definition 2.1 (Hybrid automaton) A hybrid automaton H is a collection $H = (Q, X, f, Init, D, E, G, R)$:
Q finite set of discrete variables,
X finite set of continuous variables,
$f\colon Q \times X \to TX$ vector field,
$Init \subseteq Q \times X$ set of initial states,
$D\colon Q \to P(X)$ assigns to each $q \in Q$ a domain,
$E \subseteq Q \times Q$ collection of discrete transitions,
$G : E \to P(X)$ assigns to each $e = (q, q') \in E$ a guard, and
$R : E \times X \to P(X)$ assigns to each $e = (q, q') \in E$ and $x \in X$ a reset relation.
Recall that $P(X)$ denotes the power set (set of all subsets) of X. We refer to $(q, x) \in Q \times X$ as the state of H.

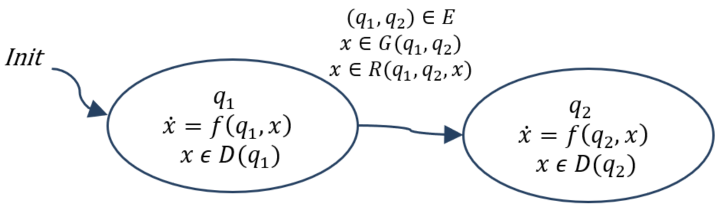

Figure 3. Hybrid automaton

The state of hybrid automaton H can be then defined as a pair $(q, x(t)) \in Q \times \mathbb{R}^n$, where $x(t) \in X$ and $q \in Q$. H is represented in a graphical form such as an oriented graph, individual members of a set of discrete modes Q represented as graph nodes and individual possible transitions denoted as edges. Each node is determined by an initial set, a domain, and a continuous dynamic. An edge can be determined with a member of a reset map and a guard set. Figure 4 represents a water tank system and its hybrid automaton model, represented in Figure 5, is defined as:
- $Q = \{q_1, q_2\}$ (Two discrete states, inflow going left and inflow going right),
- $X = \mathbb{R}^2$ (Two continuous states, the level of water in the two tanks),
- $f(q_1, x) = \begin{bmatrix} w - v_1 \\ -v_2 \end{bmatrix}$ and $f(q_2, x) = \begin{bmatrix} -v_1 \\ w - v_2 \end{bmatrix}$,

- $\mathrm{Init} = \{q_1, q_2\} \times \{x \in \mathbb{R}^2 | x_1 \geq r_1 \wedge x_2 \geq r_2\}$,
- $D(q_1) = \{x \in \mathbb{R}^2 \mid x_2 \geq r_2\}$ and $D(q_2) = \{x \in \mathbb{R}^2 \mid x_1 \geq r_1\}$,
- $E = \{(q_1, q_2), (q_2, q_1)\}$,
- $G(q_1, q_2) = \{x \in \mathbb{R}^2 \mid x_2 \leq r_2\}$ and $G(q_2, q_1) = \{x \in \mathbb{R}^2 \mid x_1 \leq r_1\}$,
- $R(q_1, q_2, x) = R(q_2, q_1, x) = \{x\}$.

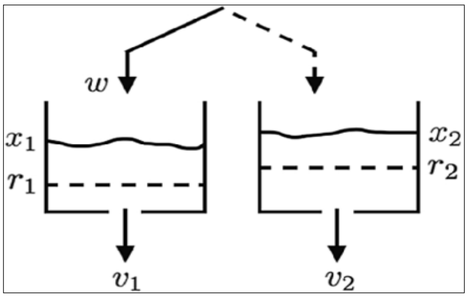

Figure 4. The water tank system

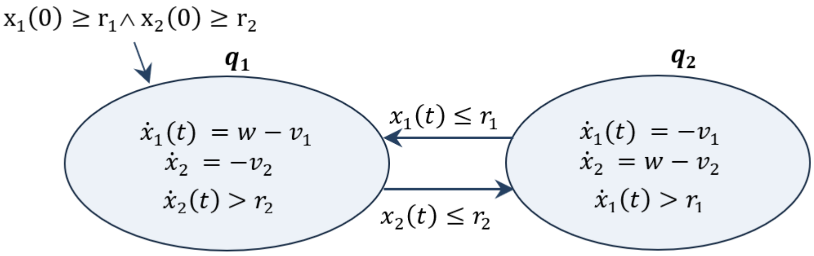

Figure 5. Hybrid automaton model of the water tank system

2.3. Numerical simulation of hybrid automata

The most used technique for simulating HDSs is the computer simulation, via an association of discrete-event simulation and differential algebraic equation (DAE) solvers. Various computer simulation tools are presented for HDSs [7]. Table 1 presents tools and environments available for the simulation of a hybrid automaton model for HDSs. Simulink and Stateflow, which are interactive tools of the very popular technical computing environment MATLAB, are extensively used for modeling and simulating hybrid automata [19]–[21]. Simulink is a tool for the modeling and simulation of nonlinear dynamic systems, and with Stateflow, it is possible to model complex hierarchical state machines. Our proposed simulation approach consists of combining Simulink and Stateflow blocks to build a common framework carrying out a complete system specification of discrete events dynamic system as well as a continuous dynamic system.

Table 1. Various Tools and environments for HA model simulation

Name	Brief description
Modelica	Object-oriented multi-domain acausal modeling language for hybrid systems with DAE dynamics.
SHIFT	Modeling and simulation environment for dynamically changing networks of hybrid automata.
Simulink	MATLAB-based graphical multi-domain modeling environment for nonlinear dynamic systems.
Stateflow	Simulink-based modeling environment for discrete-event reactive systems that implements an extended version of the Statecharts formalism.
Hytech	A tool that allows the step-by-step computation of the state spaces of linear hybrid systems.

3. RESULTS AND DISCUSSION

3.1. Description of the TTHS

Tank systems have proven to be a typical class of HDSs, and many kinds of research have been conducted in this field [22]–[26]. The TTHS depicted in Figure 6 consists of two cylindrical tanks T_1 and T_2 connected by a pipe located at the bottom and by another located at a height h_v from the bottom. Both two tanks have the same cross-sectional area S and are located at the same level. The liquid inflow Q_p to the first tank T_1 is governed by a pump and the flow Q_3 from the second tank T_2 is a permanent leak. Flows of liquid between the tanks can be controlled by the valves V_1 and V_2. The model parameters of the TTHS are given in Table 2.

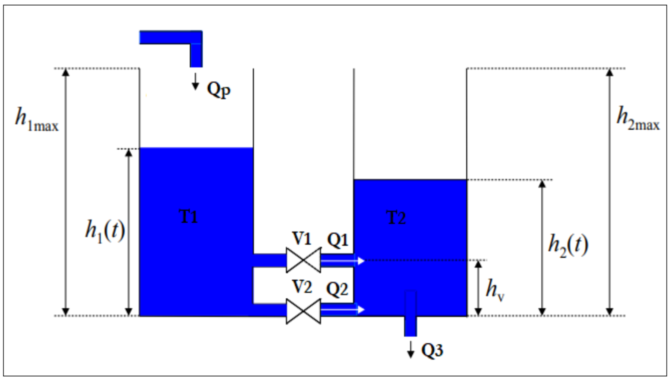

Figure 6. Two-tank hybrid system (TTHS)

Table 2. Model parameters of the TTHS

Parameter	Value
Maximum input flow Q_p	0.1e-3 (m3/s)
Maximum liquid height in Tank1 and Tank2 h_1max and h_2max	0.62 (m)
Upper and Lower pipe distance h_v	0.3 (m)
Area of each Tank S	0.0143 (m)
Cross section of valves S_c	10e-6 (m)

A mathematical model of the TTHS can be obtained by applying the mass conservation law as (1) and (2).

$$\dot{h}_1(t) = \frac{1}{S}(Q_P - Q_1V_1 - Q_2V_2) \tag{1}$$

$$\dot{h}_2(t) = \frac{1}{S}(Q_1V_1 + Q_2V_2 - Q_3) \tag{2}$$

The flows Q_1, Q_2 and Q_3 are given by (3)-(4):

$$Q_1 = \alpha\ sign(h_1 - h_V)\sqrt{|h_1 - h_v|} \tag{3}$$

$$Q_2 = \alpha\ sign(h_1 - h_2)\sqrt{|h_1 - h_2|} \tag{4}$$

$$Q_3 = \alpha\sqrt{h_2} \tag{5}$$

with, $\alpha = S_c\sqrt{2g}$, S_c is the scross-section of valves and g is the gravity acceleration.

The liquid level $h_1(t)$ of tank T_1 depends on the input flow $Q_p(t)$ and the outflow $Q_1(t)$ and $Q_2(t)$ from tank T_1. The level $h_2(t)$ of tank T_2, depends on the level $h_1(t)$ and the outflow $Q_3(t)$ from tank T_2. The dynamical model of the TTHS has two continuous state variables $h_1(t)$ and $h_2(t)$. The manipulated variables of the system are the liquid inflow Q_p, which can change continuously, and the position of valve V_2 which can be either fully open ($V_2 = 1$) or fully closed ($V_2 = 0$). The TTHS is subject to constraints on liquid inflow and heights of liquid levels in both tanks. It shows clear characteristics of HDS. The dynamics change depending on the following operating conditions: if the level in the first tank is above the level h_v there is a direct flow from tank T_1 to tank T_2 through valve V_1, and the system input valve V_1, which is in fact on-off switches.
Assumption 1. There is no input flow from tank T_2 to tank T_1, resulting in the condition that the liquid level $h_1(t)$ is always greater than $h_2(t)$.
Assumption 2. If valve V_1 is closed the system acts as a normal two-tank interacting system, so it is always assumed to be open.

3.2. Hybrid automaton of the TTHS

The nonlinear TTHS considered in this work can be operated in four different modes. The discrete modes and the continuous dynamics are given in Table 3. The hybrid automaton, given in Figure 7, represents the system's switching between different operating modes. The two corresponding actions of the valve V_2 correspond in the following to events e_1 (V_2 is open) and $\bar{e}_1$ (V_2 is closed).

The TTHS with the respective heights $h_1(t)$, $h_2(t)$ and the discrete modes, which they belong to, is illustrated in Figure 8. The system discrete modes can be described as: i) Mode I-the system behaves like two isolated tanks, no liquid flows between tanks; ii) Mode II-the liquid flows from the first tank T_1 into the second tank T_2 via valve V_1; iii) Mode III-varies from Mode II in terms of the opening valve V_2, the liquid flows from the first tank T_1 into the second tank T_2 via valve V_1 and valve V_2; iv) Mode IV-the height of the level $h_1(t)$ is lower than h_v, the liquid flows from the first tank T_1 into the second tank T_2 via valve V_2.

Table 3. Discrete modes and continuous dynamics of the TTHS

Modes	Condition for $h_1(t)$	Condition for V_2	Continuous dynamics
Mode I	$h_1 < h_v$	closed	$\dot{h}_1(t) = \frac{1}{S}(Q_p)$
Mode II	$h_1 \geq h_v$	closed	$\dot{h}_1(t) = \frac{1}{S}(Q_p - Q_1)$
Mode III	$h_1 \geq h_v$	open	$\dot{h}_1(t) = \frac{1}{S}(Q_p - Q_1)$
Mode IV	$h_1 < h_v$	open	$\dot{h}_1(t) = \frac{1}{S}(Q_p - Q_2)$

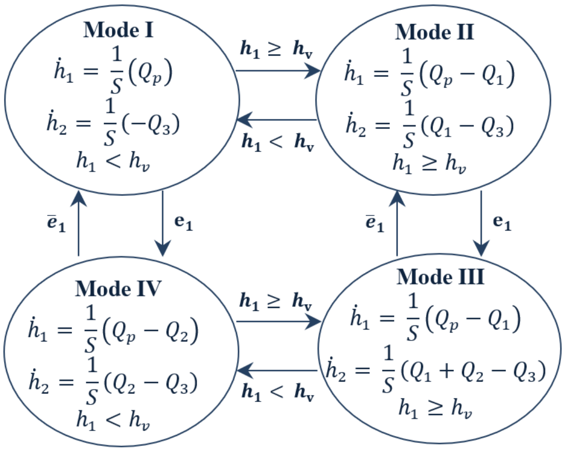

Figure 7. Hybrid automaton of TTHS

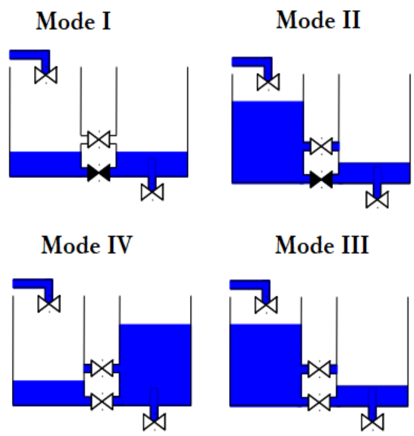

Figure 8. Graphical depiction of the TTHS

3.3. Simulations results and discussion

The hybrid model of the TTHS described above is simulated using Stateflow [27] interfaced with the Simulink as shown in Figure 9. The system is simulated for 500 seconds and operates under the following initial conditions: $h_{10} = 0.5\,m$, $h_{20} = 0\,m$. The Stateflow model of the TTHS, which is presented in Figure 10, graphically describes the system switches between the operating modes. In each mode, the continuous evolution of the system is given by a specified differential equation. The continuous dynamics switch between four discrete modes $q(t)$ depending on the state of the valve V_2 and whether or not the liquid level h_1 exceeds the height $h_v = 0.3$.

The manipulated inputs (the liquid inflow Q_p and the valve V_2) are shown in Figures 11 and 12. The evolutions of the liquid levels $h_1(t)$ and $h_2(t)$, which are the two continuous outputs of the TTHS, are presented in Figure 13. In this figure, the red dotted line shows the height h_v at that the valve V_1 is positioned. The orange lines represent the time at that a transition between discrete modes happens. Changing the opening logical state of the valve V_2 is highlighted by the horizontal black lines. Figure 14 shows the discrete modes in which the system is located. According to the open-loop response of the TTHS with valve V_2 opening/closing and variable input flow Q_p, it is very clear that the levels $h_1(t)$ and $h_2(t)$ in the two tanks are dependent on each other.

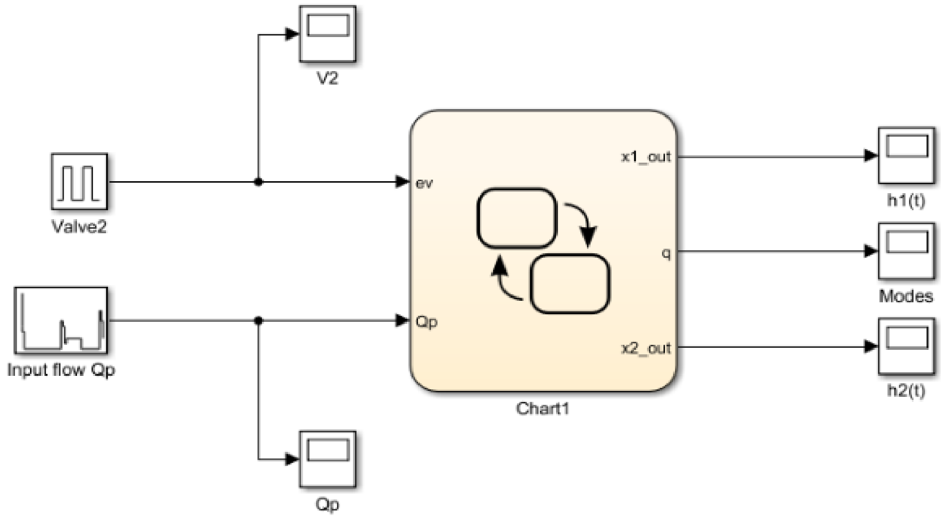

Figure 9. Model of the two-tank hybrid system with Simulink/Stateflow

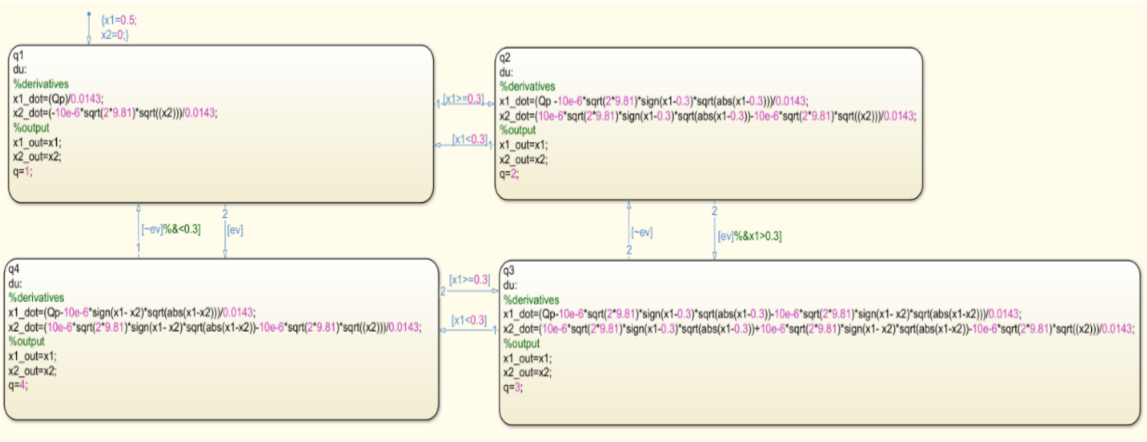

Figure 10. The stateflow model of the two-tank hybrid system

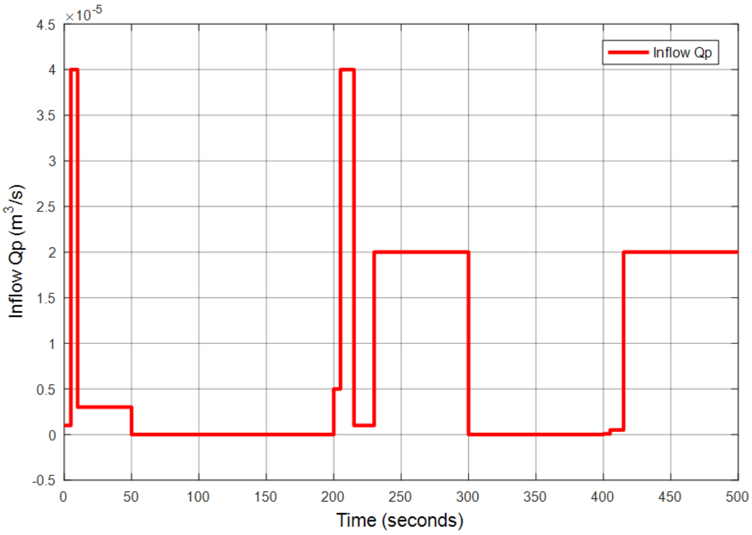

Figure 11. Inflow in tank T_1 (Q_p)

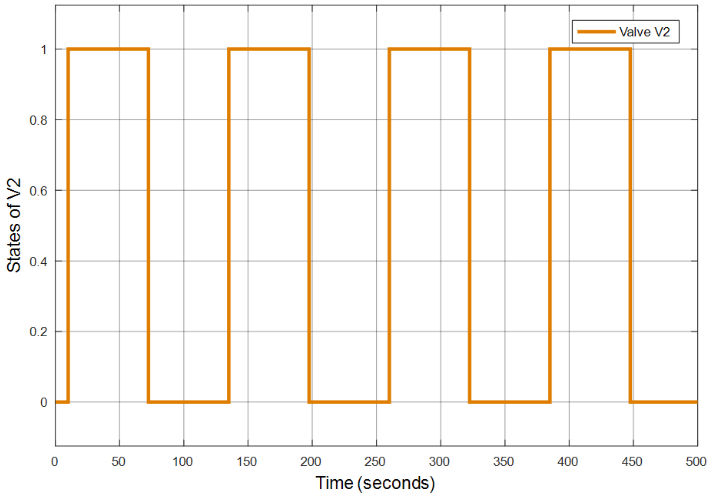

Figure 12. States (On/Off) of the valve V_2

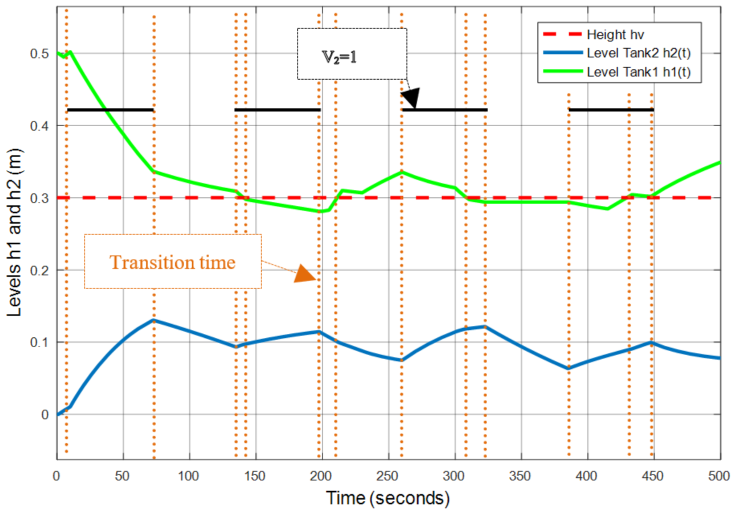

Figure 13. Dynamical evolutions of the two levels $h_1(t)$ and $h_2(t)$

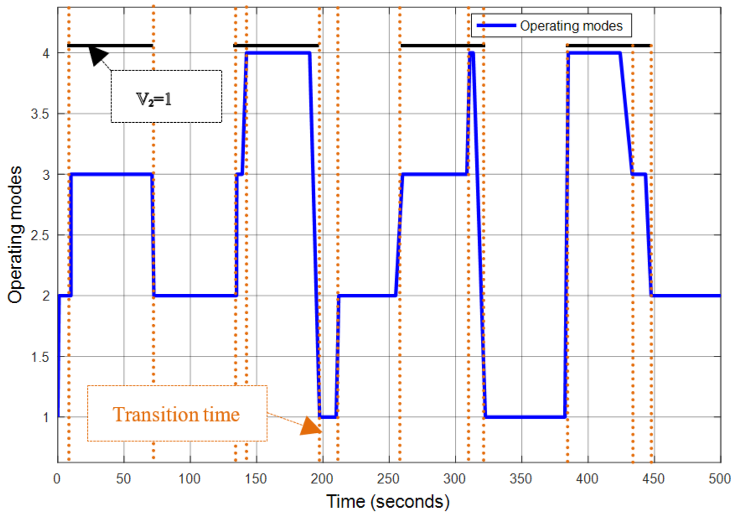

Figure 14. Operating modes of the TTHS

As expected according to conditions illustrated in Table 3, the transition between discrete modes occurs when the height of a tank level $h_1(t)$ exceeds or not the height h_v, or after the change of valve V_2 logical state. A striking observation to emerge from Figure 13 is that the temporal evolutions of two levels $h_1(t)$ and $h_2(t)$ follow the transitions between discrete modes with the given inputs. That confirms that the liquid levels $h_1(t)$ and $h_2(t)$ depend on the state of valve V_2 and whether or not the value of the level $h_1(t)$ exceeds the height h_v.

A second essay is presented where the valve V_2 is completely open. It can be seen in Figure 15 that the transitions between modes depend only on whether or not the liquid level $h_1(t)$ exceeds the height h_v. In this case, is apparent from Figure 16 that the operating modes of the system are Mode III and Mode IV.

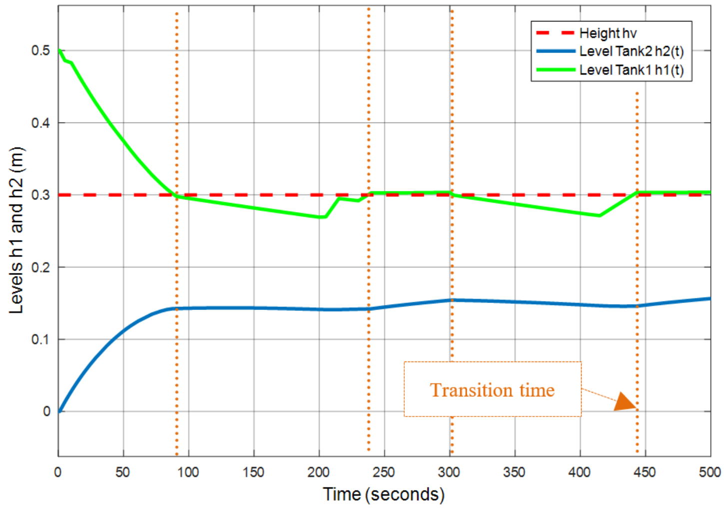

Figure 15. Dynamical evolution of two levels $h_1(t)$ and $h_2(t)$ with V_2 =1

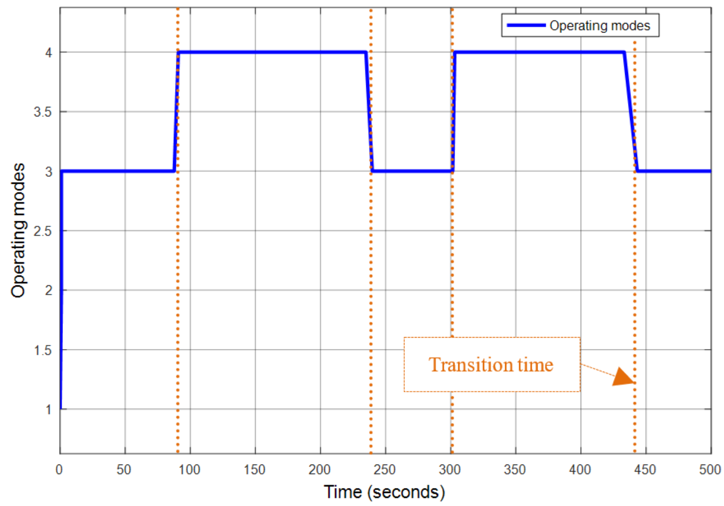

Figure 16. Operating modes of the TTHS with V_2=1

A last simulation was performed assuming that there are no interactions between the two tanks T_1 and T_2 ($V_2 = 0$) is shown in Figure 17. When the interaction valve V_2 is closed, then the system is operating only in one operating mode which is Mode II as shown in Figure 18. The TTHS thus operates as a continuous system.

The various simulations presented above prove that the proposed hybrid modeling framework is very effective for the modeling, simulating, and analysis HDSs. This modeling approach gives a complete model for an HDS and represents it under all its operating mode cases. These simulations are very useful in the construction of the diagnosis which is based on the temporal knowledge of the process such as the opening time of the valves. From Figure 12, the transition times of the valve V_2 are detailed in Table 4.

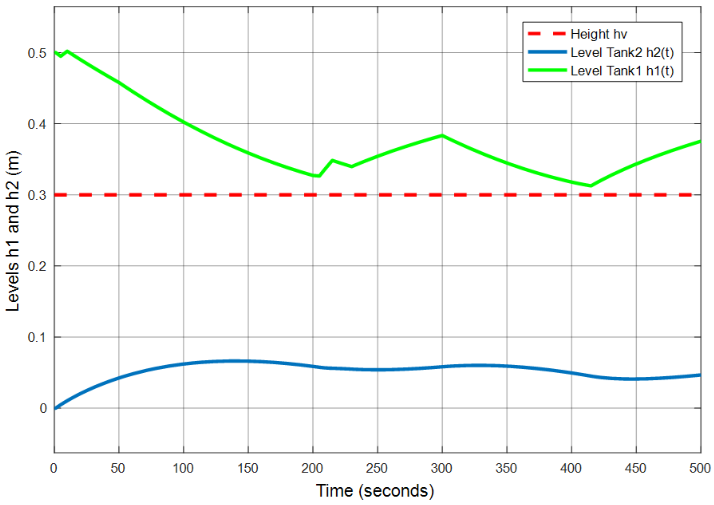

Figure 17. Dynamical evolution of the two levels $h_1(t)$ and $h_2(t)$ with V_2 =0

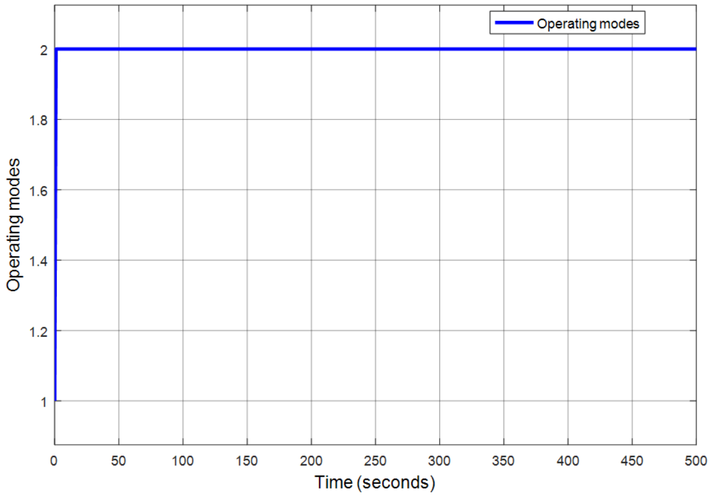

Figure 18. Operating modes of the TTHS with V_2 =0

Table 4. Opening time of the valve V_2

Actions	Time in second	Interval of time
Opening of the valve V_2	10	[10,72.5]
Closing of the valve V_2	72.5	[72.5,135]
Opening of the valve V_2	135	[135,197.5]
Closing of the valve V_2	197.5	[197.5,260]
Opening of the valve V_2	260	[260,322.5]
Closing of the valve V_2	322.5	[322.5,385]
Opening of the valve V_2	385	[322.5, 447.5]
Closing of the valve V_2	447.5	[447.5,500]

4. CONCLUSION

Typically, the mathematical model of a physical system is given by differential equations that relate both inputs and outputs. In the case of HDS, for the reason to correctly represent the dynamic evolution of the system, this is not adequate. Therefore, a hybrid modeling framework for this kind of system must be used. In this paper, a modeling framework based on the HA approach is proposed and a TTHS is considered for the application. First, the HDS is introduced and an illustration of how it can be modeled compositionally as products of HA is given. Then, the modeling of the considered TTHS is developed using Stateflow and Simulink in MATLAB. The simulations results prove the usefulness of the developed modeling framework for HDSs. The proposed formalism model represents the most general class of HDSs. Even if this model is not directly tailored to control system design purposes, it provides a good understanding of the modeling capabilities of HDSs. To further our research, approaches like model checking using HA models will be established.

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