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A new three-term conjugate gradient method with application to regression analysis

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ABSTRACT

Conjugate gradient (CG) method is well-known for its ability to solve unconstrained optimization (UO.) problems. This article presenting a new CG method with sufficient descent conditions which improves the former method developed by Rvaie, Mustafa, Ismail and Leong (RMIL). The efficacy of the proposed method has been demonstrated through simulations on the Kijang Emas pricing regression problem. The daily data between January 2021 to May 2021 were obtained from Malaysian Ministry of Health and Bank Negara Malaysia. The dependent variable for this study was the Kijang Emas price, and the independent variables were the coronavirus disease (COVID-19) measures (i.e., new cases, R-naught, death cases, new recovered). Data collected were analyzed on its correlation and coefficient determinant, and the influences of COVID-19 on Kijang Emas price was examined through multiple linear regression model. Findings revealed that the suggested technique outperformed the existing CG algorithms in terms of computing efficiency.

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1. INTRODUCTION

The conjugate gradient (CG) is one of the optimization techniques using gradient based methods for parameter learning. It is one of the well-known methods to solve unconstrained optimization (UO.) problems. The UO. problems usually arise directly in some applications which includes the least-squares problem for data-fitting or regression analysis [1]. The structure of the CG method consists of four crucial procedures:

– Computation of a search direction, d_k from x_k using (1),

$$d_k = \begin{cases} -g_k, & \text{if} \quad k = 0, \\ -g_k + \beta_k d_{k-1}, & \text{if} \quad k \geq 1. \end{cases} \tag{1}$$

– Ensure that the direction d_k denotes a descent direction to guarantee reduction on the objective function $f(x)$.
– Computation of step-size, α_k such that $f(x_k + \alpha_k d_k) < f(x_k)$ is satisfied. The step-size can be computed using an exact or inexact line search known as strong Wolfe [2] specified by the rules as in (2) and (3):

$$f(\mathrm{x}_k + \alpha_k \mathrm{d}_k) \leq f(\mathrm{x}_k) + 0.0001\alpha_k \mathrm{g}_k^T \mathrm{d}_k, \tag{2}$$

$$\mathrm{g}|(\mathrm{x}_k + \alpha_k \mathrm{d}_k)^T \mathrm{d}_k| \leq -0.1\mathrm{g}_k^T \mathrm{d}_k. \tag{3}$$

– Updating the new iterative point, $\mathrm{x}_{k+1} = \mathrm{x}_k + \alpha_k \mathrm{d}_k$. The iteration will be terminated when stopping criteria have been satisfied.

The coefficient β_k in (1) is called the CG coefficient and plays a vital role in the convergence behavior. Some of the β_k proposed in the literature were suggested by Hestenes and Stiefel [3], Fletcher [4], Polak and Ribiere [5], Polyak [6], Daniel [7], Fletcher [8], Liu and Storey [9], Dai and Yuan [10]. However, these classical CG failed to satisfy the sufficient descent condition (S.D.C.) [11] for some inexact line search, which guarantee global convergence. The S.D.C. can be represented by (4).

$$\mathrm{g}_k^T \mathrm{d}_k \leq -c\|\mathrm{g}_k\|^2,\ k \geq 0, c > 0. \tag{4}$$

There are various attempts by researchers to rectify this shortcoming, including research by Rivaie, *et al.* [12] who proposed new β_k defined as (5).

$$\beta_k^{\mathrm{RMIL}} = \frac{\mathrm{g}_k^T(\mathrm{g}_k - \mathrm{g}_{k-1})}{\mathrm{d}_{k-1}^T(\mathrm{d}_{k-1} - \mathrm{g}_k)}. \tag{5}$$

In the same year, the same authors also proposed a similar beta coefficient known as RMIL* in [13].

$$\beta_k^{\mathrm{RMIL*}} = \frac{\mathrm{g}_k^T \mathrm{y}_{k-1}}{\|\mathrm{d}_{k-1}\|^2}. \tag{6}$$

Although RMIL/RMIL* has global convergence properties in correspondence to exact line search, its numerical output lacks the original method of Hestenes and Stiefel (HS) and Polak-Ribiére-Polyak (PRP). Besides, RMIL/RMIL* was less efficient and robust under inexact line search as compared to exact line search [14]. Therefore, some variant RMIL/RMIL* have been introduced, for example in [14]–[23]. One of the most efficient versions known as RMIL+ method was introduced in research [24] and later analyzed in research [25] defined as (7).

$$\beta_k^{\mathrm{RMIL+}} = \begin{cases} \frac{\mathrm{g}_k^T(\mathrm{g}_k - \mathrm{g}_{k-1})}{\|\mathrm{d}_{k-1}\|^2}, & \text{if} \quad 0 \leq \left|\mathrm{g}_\mathrm{k}^\mathrm{T}\mathrm{g}_{\mathrm{k}-1}\right| \leq \|\mathrm{g}_\mathrm{k}\|^2, \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

However, it was also agreed that the three-term CG algorithms are more efficient, reliable, and resilient than traditional CG algorithms [26]. In addition, some three-term CG algorithms may be capable of providing reasonable directions that ensure global convergence [27]. The beneficial effects of three-term CG have encouraged more investigations of its application to the RMIL method. The following are some of the three-term versions of RMIL methods termed as ttRMIL+, ttRMIL and ttRMIL* proposed in [28]–[30], respectively.

$$\mathrm{d}_k^{\mathrm{ttRMIL+}} = \begin{cases} -\mathrm{g}_k & \text{if } \mathrm{k} = 0, \\ -\mathrm{g}_k + \beta_k^{\mathrm{RMIL+}} \mathrm{d}_{k-1} + \frac{\mathrm{g}_k^T \mathrm{d}_{k-1}}{\|\mathrm{d}_{k-1}\|^2} \mathrm{g}_{k-1}, & \text{if } \mathrm{k} \geq 1. \end{cases} \tag{8}$$

$$\mathrm{d}_k^{\mathrm{ttRMIL}} = \begin{cases} -\mathrm{g}_k & \text{if } \mathrm{k} = 0, \\ -\mathrm{g}_k + \beta_k^{\mathrm{RMIL}} \mathrm{d}_{k-1} - \frac{\mathrm{g}_k^T \mathrm{d}_{k-1}}{\mathrm{d}_{k-1}^T(\mathrm{d}_{k-1} - \mathrm{g}_k)} \mathrm{y}_{k-1}, & \text{if } \mathrm{k} \geq 1. \end{cases} \tag{9}$$

$$\mathrm{d}_k^{\mathrm{ttRMIL*}} = \begin{cases} -\mathrm{g}_k & \text{if} \quad \mathrm{k} = 0, \\ -\mathrm{g}_k + \beta_k^{\mathrm{RMIL*}} \mathrm{d}_{k-1} - \frac{\mathrm{g}_k^T \mathrm{d}_{k-1}}{\|\mathrm{d}_{k-1}\|^2} \mathrm{y}_{k-1}, & \text{if} \quad \mathrm{k} \geq 1. \end{cases} \tag{10}$$

In CG methods, where the focus is often limited to their successive iteration formulation, the usefulness of the initial direction is yet to be proved. The review paper by [11] pointed out that it is necessary to take the direction of negative gradient as the initial search direction. Saha and Nath [31] proved that convergence may also be assured by adding parameter $0 < \omega < 1$ to the negative gradient in the following manner:

$$\mathrm{d}_k = \begin{cases} -\mathrm{g}_k + \omega \mathrm{g}_k, & \text{for} \quad \mathrm{k} = 0 \\ -\mathrm{g}_k + \beta_k \mathrm{d}_{k-1}, & \text{for} \quad \mathrm{k} \geq 1 \end{cases} \tag{11}$$

Since ω is a scalar, then the direction of d_0 still follows the direction of the negative gradient. Thus, this research proposed further improvement on RMIL method [12] by adopting the three-term parameter and a scaled initial direction. The complete definition of the proposed method is:

$$\mathrm{d}_k^{\mathrm{NEWRMIL}} = \begin{cases} -\mathrm{g}_k + \omega \mathrm{g}_k, & \text{if} \mathrm{k} = 0 \text{ and } \omega \in (0,1), \\ -\mathrm{g}_k + \beta_k^{\mathrm{RMIL}} \mathrm{d}_{k-1} + \frac{\mathrm{g}_k^T \mathrm{d}_{k-1}}{\mathrm{d}_{k-1}^T (\mathrm{d}_{k-1} - \mathrm{g}_k)} \mathrm{g}_{k-1}, & \text{if} \mathrm{k} \geq 1. \end{cases} \tag{12}$$

To be classified as efficient, a CG technique must at the very least satisfy the S.D.C. and have convergence properties. The following theorem shows that our proposed CG method named as NEWRMIL, satisfies the S.D.C. NEWRMIL referred to the modified RMIL method.

Theorem 1.1 (Sufficient descent under exact line search) *Assume that a CG method with search direction is generated by the NEWRMIL method for all* $k \geq 0$ *under exact line search. Then the proposed method satisfies the S.D.C.*

$$\mathrm{g}_k^T \mathrm{d}_k \leq -c \|\mathrm{g}_k\|^2, \qquad k \geq 0, c > 0.$$

Proof. If $k = 0$, then the initial direction takes $\mathrm{d}_0 = -\mathrm{g}_0 + \omega \mathrm{g}_0 = -(1-\omega)\mathrm{g}_0$ where $\omega \in (0,1)$. Next, multiply with g_0^T,

$$\begin{aligned} \mathrm{g}_0^T \mathrm{d}_0 &= -(1-\omega)\mathrm{g}_0^T \mathrm{g}_0 \\ &= -(1-\omega)\|\mathrm{g}_0\|^2. \end{aligned} \tag{13}$$

Since $(1-\omega) > 0$, then the S.D.C. is satisfied for $k = 0$. The proof is ensued for $k \geq 1$ by induction approach where $\mathrm{d}_k = -\mathrm{g}_k + \beta_k^{\mathrm{RMIL}} \mathrm{d}_{k-1} - \frac{\mathrm{g}_k^T \mathrm{d}_{k-1}}{\|\mathrm{d}_{k-1}\|^2} \mathrm{g}_{k-1}$. Next, multiply with g_k^T,

$$\begin{aligned} \mathrm{g}_k^T \mathrm{d}_k &= -\mathrm{g}_k^T \left(\mathrm{g}_k + \beta_k^{\mathrm{RMIL}} \mathrm{d}_{k-1} - \frac{\mathrm{g}_k^T \mathrm{d}_{k-1}}{\|\mathrm{d}_{k-1}\|^2 - \mathrm{g}_k^T \mathrm{d}_{k-1}} \mathrm{g}_{k-1} \right) \\ &= -\|\mathrm{g}_k\|^2 - \beta_k^{\mathrm{RMIL}} \mathrm{g}_k^T \mathrm{d}_{k-1} + \mathrm{g}_k^T \left(\frac{\mathrm{g}_k^T \mathrm{d}_{k-1}}{\|\mathrm{d}_{k-1}\|^2 - \mathrm{g}_k^T \mathrm{d}_{k-1}} \mathrm{g}_{k-1} \right) \end{aligned} \tag{14}$$

For exact line search, $\mathrm{g}_k^T \mathrm{d}_{k-1} = 0$. Thus, $\mathrm{g}_k^T \mathrm{d}_k = -\|\mathrm{g}_k\|^2$, which implies d_k is sufficient descent direction. Hence, $\mathrm{g}_k^T \mathrm{d}_k \leq -c\|\mathrm{g}_k\|^2$ holds true. The proof is complete.

The remaining parts of this article are arranged as follow: The case problems for this research are discussed in Section 2, and the research method are presented in Section 3. Some numerical experiments have been done to demonstrate the efficacy of the new method, and results are discussed in Section 4. Finally, the conclusion is summarized in Section 5.

2. DESCRIPTION OF PROBLEM

The 25th of January 2020 marked the first case of coronavirus disease (COVID-19) in Malaysia involving three Chinese citizens and the first local case was detected on the 4th of February 2020. On 30th of January 2020, COVID-19 virus was declared as public health concern by World Health Organization (WHO). On 11th of March 2020, it was acknowledged as a worldwide pandemic [32]. The spread of COVID-19 cases accelerated globally, with high risk of human fatalities, infections, and health issues. The rate of disease spread was measured by the index of R-naught, R_0. Governments, publics and mass media all over the world frequently utilize R_0 to raise the alarm, and the R_0 higher than 1 is a significant source of worry [33].

This highly contagious pandemic also harms the global economy [34]. The worldwide uncertainties regarding the COVID-19 outbreaks have affected the market dynamics of major commodities, including gold. For instance, prior to the current pandemic crisis, the highest gold spot price was USD 60.9807074 or RM181.36 per gram on 5th of September 2011 [35]. Meanwhile, throughout the pandemic, the highest gold spot price hit USD 66.286 per gram (RM 278.24) on 7th of August 2020. The price continued to fluctuate, reaching a drop to USD 52.233 per gramme (RM 227.61) on 30th of March 2020.

There were many studies and discussions on determining the asset property of gold. Baur and Lucey [36] investigated the role of gold among investors based on the daily price data of stock markets and closing spot gold from the 30th of November 1995 to the 30th of November 2005. They provided evidences that gold has a short haven time, which was estimated to be around 15 trading days. They concluded that the safe-haven property of gold was only valid after extreme adverse stock market shocks bonds but not during the rising of stock markets. Salisu *et al.* [37] also confirmed the hedging effectiveness of gold with respect to risks associated with crude oil, notably during the pandemic era, since gold is a hedge towards geopolitical and economic problems. Gold investment during the COVID-19 pandemic appears to be a wise financial option, as it will shield investors from market fluctuations in stock and oil markets [38]. The gold market also serves as a haven and performs better than U.S. stocks during the pandemic [39]. The empirical study by Atri *et al.* [34] employing the autoregressive distributed lag (ARDL) approach from 23rd of January 2020 to the 23rd of June 2020 concluded that COVID-19 current media coverage, deaths, and infections positively affect the gold price.

The outbreak of the pandemic also increased the volatility of gold return. The COVID-19 impact on daily gold spot price from January 2020 to May 2020 was proven in the study carried out by Yousef and Shehadeh [40]. The researchers utilized a generalized autoregressive conditional heteroskedasticity (GARCH) model with a single dependent variable, the US dollars' daily gold spot price. The logarithms of the cumulative total number of daily cases and the number of fresh possibilities were the independent variables. Jianyi *et al.* [41] investigated the effect of COVID-19 on the gold price by developing a multiple linear regression model. They used the Python third-party machine learning algorithm as a framework for multivariate linear fitting. The data covers the period from 22nd of January 2020 to 23rd of September 2020, consisting of the COVID-19 measures (number of confirmed cases, deaths, and cured). The value of R square obtained was roughly around 0.926, indicating that the model fits the data well. Motivated by the strong potential of gold in protecting hedging properties and its ability in providing a strong financial safeguard during any crisis, Malaysian State Government has produce various gold coins to encourage interests towards gold investments among the people [42]. Malaysia was the 12th country worldwide to produce its first gold coin, the Kijang Emas (KE), introduced by Malaysia's then-Prime Minister, Tun Dr Mahathir Mohamad, on 17th July 2001.

The goal of this study is to apply the CG algorithm to the KE regression model pricing. The gold price model provided by Jianyi *et al.* [41] was adopted in our KE model, with an extra R-naught reproductive index number as the independent variable. As the results, a data set was generated with four independent variables linked to COVID-19: new cases (NC), R-naught (R_0), death cases (ND), and new recovered (NR). The suggested CG and several current CG techniques were used to generate the regression models. The performance of each CG algorithm was compared using error analysis, which includes mean squared error (MSE), root mean square error (RMSE), mean absolute error (MAE), and R squared.

3. RESEARCH METHOD

The method used in this study is visualized in Figure 1, consisting of three main stages: i) data collection, ii) correlation analysis and calculation of coefficient of determination to investigate relationship among variables, and iii) multiple linear regression model produced by applying the CG method together with the least square approach. The data collection period was done from January to May 2021 and includes all independent variables obtained from the ministry of health's (MOH) daily data. It should be noted that these are not the only factors affecting KE pricing, but for this current research, the considered variables were limited to the factors related to COVID-19 circumferences. This study's KE gold bullion price data was derived from Bank Negara Malaysia, and it includes the selling price of one gram of KE. To maintain the integrity of the five-time series, the discordant sequences caused by weekends or holidays were removed, hence yielding a total of 470 sets of data. Table 1 outlines the database's descriptive analysis of each data sets' minimum, maximum, and mean values. The data was then transformed using Z-score standardization method as defined by (15) [43].

$$\mathrm{Z-score} = \frac{X - \mathrm{Mean}(X)}{\mathrm{Standarddeviation}(X)}. \tag{15}$$

The standardized data helps produce dimensionless regression coefficients, especially when dealing with iterative optimization algorithms. Based on the conceptual framework depicted in Figure 2, the dependent variable KE is modeled through a linear combination of the four independent variables. The approximated $\overline{K}E$ is given by the following multiple linear regression (MLR) model:

$$\overline{K}E_i = a_0 + a_1(R_0)_i + a_2NC_i + a_3NR_i + a_4ND_i + E_i. \tag{16}$$

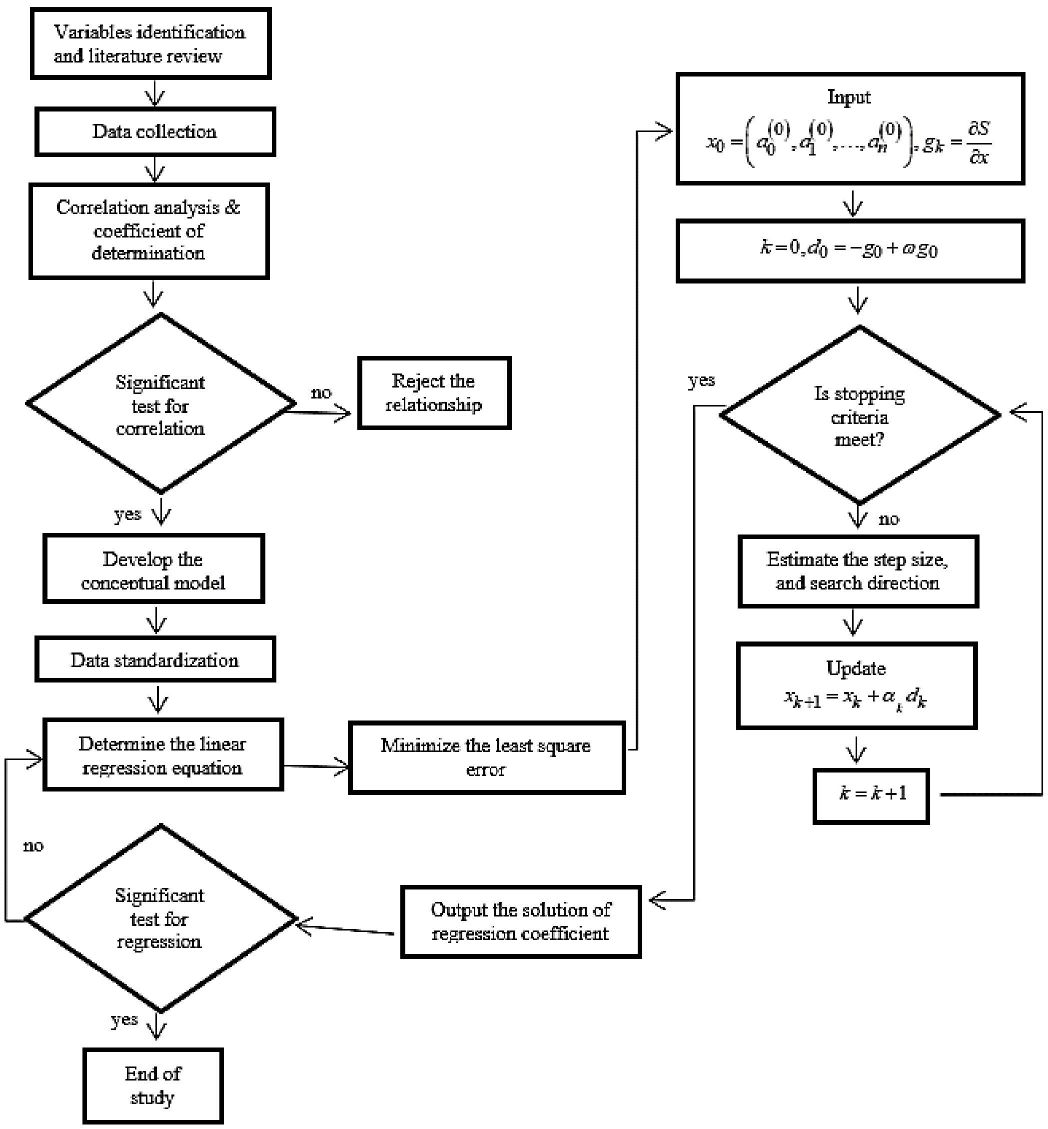


Figure 1. Research methodology

Table 1. Data description

Variable	Minimum	Maximum	Mean
KE (RM per gram)	234.69	268.01	250.54
R_0	0.81	1.2	1.03
NC	941	8290	2891
NR	994	5718	2513
ND	1	61	14

Next was the determination of regression parameters a_0, a_1, a_2, a_3, a_4 through least squares method involving the sum of the squares of residuals, E_i:

$$S_r = \sum_{i=1}^{N} E_i^2 = \sum_{i=1}^{N} (KE_i - a_0 - a_1(R_0)_i - a_2NC_i - a_3NR_i - a_4ND_i)^2. \quad (17)$$

Furthermore, the least-squares approach could be transformed into unconstrained optimization (UO.) problem with S_r as the objective function:

$$\min_{(a_0,a_1,a_2,a_3,a_4)} S_r. \quad (18)$$

The CG method is prominent in solving UO. problems and the flowchart explaining the procedures is presented in Figure 1. The CG method is one of the nonlinear optimization routines starting with a guess for the parameters $x_0 = \{a_0^{(0)}, a_1^{(0)}, a_2^{(0)}, a_3^{(0)}, a_4^{(0)}\}$. The complete definition of g_k corresponding to S_r for this paper are:

$$\begin{aligned}
S_r a_0 &= -2\sum (KE_i - a_0 - a_1(R_0)_i - a_2NC_i - a_3NR_i - a_4ND_i) \\
S_r a_1 &= -2\sum (R_0)_i(KE_i - a_0 - a_1(R_0)_i - a_2NC_i - a_3NR_i - a_4ND_i) \\
S_r a_2 &= -2\sum NC_i(KE_i - a_0 - a_1(R_0)_i - a_2NC_i - a_3NR_i - a_4ND_i) \\
S_r a_3 &= -2\sum NR_i(KE_i - a_0 - a_1(R_0)_i - a_2NC_i - a_3NR_i - a_4ND_i) \\
S_r a_4 &= -2\sum ND_i(KE_i - a_0 - a_1(R_0)_i - a_2NC_i - a_3NR_i - a_4ND_i).
\end{aligned} \tag{19}$$

Consequently, the optimal value of the regression parameters is obtained when $\|g_k\| \leq 10^{-6}$ where $\|.\|$ is the Euclidean norm and the value 10^{-6} corresponds to the tolerance of accuracy.

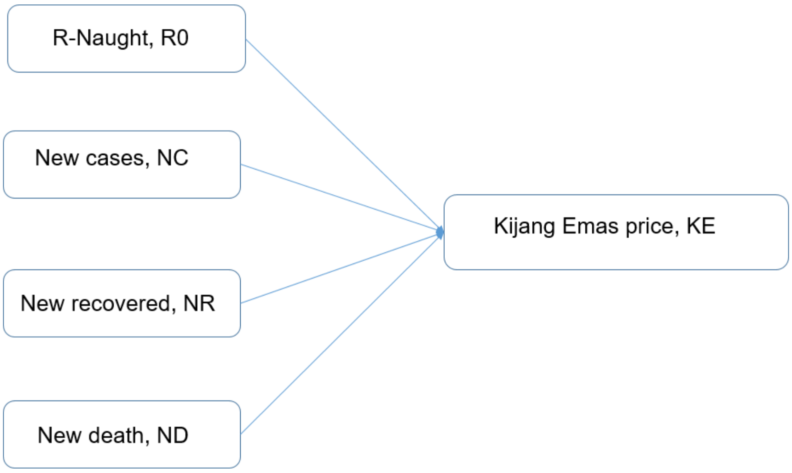


Figure 2. Conceptual framework

All CG algorithms were evaluated on the same machine using a strong Wolfe line search to provide unbiased numerical results. The test problems that cannot be solved using any methods were labeled as ’FAIL,’ which indicates that the NOI and central processing unit (CPU) time values were unavailable. The process was usually halted due to two conditions: it takes a longer time to discover a positive value of step-size when iteration is more than 1000 or the method cannot be computed by strong Wolfe. The iteration was stopped when it reached the accuracy up to six (6) decimal places, which implies that iterates were already adequately close to the solution points.

4. RESULT AND DISCUSSION

Simulations were executed in this section to evaluate the performance of the proposed CG technique by comparing it with existing CG methods and also the SPSS software using the methodology as described in Section 3. The multiple regression model was obtained by employing our proposed three-term CG method, named NEWRMIL, compared to RMIL+, ttRMIL+, ttRMIL, and ttRMIL* methods. The parameter value for NEWRMIL is set to $\omega = 0.3$. These algorithms were carried out using MATLAB R2020b subroutine program on AMD Ryzen 5 3550H, Radeon Vega Mobile Gfx, 2,100 Mhz, 4 Core(s), 8 logical processor(s).

Correlation analysis describes the relations of direct influences between the variables. This paper uses the Spearman’s Rank correlation coefficient, $-1 \leq r \leq 1$ [44] to dictate the strength among variables such that :

$$r = \begin{cases} \leq 0.35 & \text{means weak correlations;} \\ \geq 0.68 & \text{means strong correlations;} \\ \text{otherwise} & \text{means moderate correlations.} \end{cases} \tag{20}$$

In addition, $r = 1$ means a perfect relation, and $r = 0$ means no relation.

The result in Table 2 shows the relationship between $KE \leftrightarrow NC$ (0.756771) and $NC \leftrightarrow ND$ (0.88064) are strong positive relationship. $KE \leftrightarrow R_0$ (0.6590331), $KE \leftrightarrow ND$ (0.653945), $R_0 \leftrightarrow NC$ (0.558109), $NR \leftrightarrow ND$ (0.562716), and $NR \leftrightarrow NC$ (0.594737) exhibit moderate positive relationship. In contrast, $R_0 \leftrightarrow ND$ (0.389516) shows weak positive relationship. To conclude, all relationships are significant (p-value < 0.05).

Apart from that, R^2 was analyzed to evaluate how much variation the independent variables explain the dependent variable, KE. Table 2 shows that the variation in KE is explained by R_0 (43.4%), NC (57.3%), NR (9.4%), and ND (42.8%), respectively. The variation in R_0 is explained by ND (15.2%), and NC (31.1%), respectively. The variation in NC is explained by ND (77.6%), whereas the variation in NR is explained by ND (31.7%) and NC (35.4%).

Table 2. Correlation and coefficient of determination analysis

Relationship	Spearman's Rank Correlation Coefficient (r)	Significant (p value)	Strength	R^2
$KE \leftrightarrow R_0$	0.6590331	5.155E-13	moderate	0.4343246
$KE \leftrightarrow NC$	0.756771	1.12E-18	strong	0.572702
$KE \leftrightarrow NR$	0.307447	0.002576	weak	0.094524
$KE \leftrightarrow ND$	0.653945	8.91E-13	moderate	0.427644
$R_0 \leftrightarrow ND$	0.389516	0.000104	weak	0.151723
$R_0 \leftrightarrow NC$	0.558109	5.09E-09	moderate	0.311486
$NC \leftrightarrow ND$	0.88064	1.34E-31	strong	0.775527
$NR \leftrightarrow ND$	0.562716	3.57E-09	moderate	0.31665
$NR \leftrightarrow NC$	0.594737	2.61E-10	moderate	0.353712

As shown in Table 2, all the relationships are significant (p-value < 0.05). Thus, the regression analysis was carried on to measure the direct effect of the independent variables upon the dependent variable. The complete multiple regression output from SPSS is depicted in Table 3. Based on the significant values (p-value< 0.05), only R_0 and NC were considered to have impact on KE.

Table 3. Regression analysis using SPSS

Variable	Unstandardized parameter	Standard error	Standardized parameter	t	Sig.
Constant	-1.72E-11	.062		.000	1.000
R_0	.349	.095	.349	3.685	.000
NC	.480	.178	.480	2.705	.008
NR	-.019	.095	-.019	-.203	.840
ND	.016	.136	.106	.780	.437

Therefore, another linear model was developed using MLR model of Kijang Emas by dropping the NR and ND to (16). The complete equation is shown in (21).

$$\bar{K}E_i = a_0 + a_1(R_0)_i + a_2NC_i. \tag{21}$$

The results of linear regression to demonstrate the benefits of the proposed algorithm is displayed in Table 4. By using ten different initial points, the tests which achieved the solution point were considered success. Therefore, the outcome of this research will be concluded based on the success percentage of problem solved. Results in Table 4 shows that ttRMIL+ failed to solve one problem, but ttRMIL, ttRMIL*, and RMIL+ was unable to solve three, four, and ten problems, respectively. However, NEWRMIL has been able to solve almost all problems

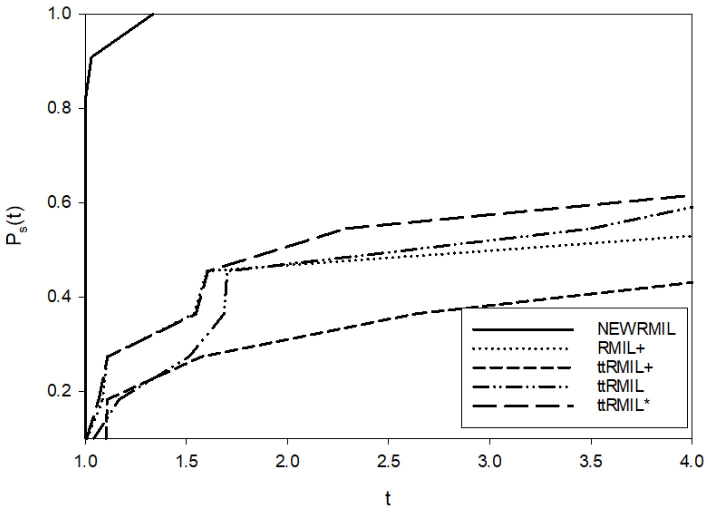
The detailed comparison of the efficiency and robustness of all methods was made using the performance profile method of Dolan and Moré [45] which is a practical methodology for standardizing the comparison of algorithms. The performance criteria used in this research focused on the speed of convergence, measured in the number of iterations (NOI), CPU time, and accuracy. Figure 3 shows the performance profile of this study based on the output displayed in Table 4. Figure 3(a) and 3(b) specifically summarize the plots of performance profile due to NOI and CPU time, respectively. The proportion P of test problems for which each method is within a factor t of the best are also shown. The proportion of test problems for which a method is the fastest are shown on the left side of the graph. At the same time, the percentage of test problems successfully solved by each method, which is also an indicator of their effectiveness, are shown on the right side. The curved shape of NEWRMIL is the most top compared to other methods, suggested its effectivity in outperforming all the CG methods in terms of speed and degree of robustness.

The model's performance was then evaluated in terms of mean squared error (MSE), root mean square error (RMSE), mean absolute error (MAE) and R squared (R^2). The performance of the method was observed through the one with the least MSE, RMSE, MEA with the R^2 close to 1. Interestingly, the value of MSE, RMSE, MAE and R^2 for all CG methods and the SPSS were $MSE \approx 0.342262873$, $RMSE \approx 0.585032369$, and $MAE \approx 0.426035347$. All models produced $R^2 \approx 0.654056881$, which translated to 65% of KE explained by the COVID-19 measures (R_0, NC).

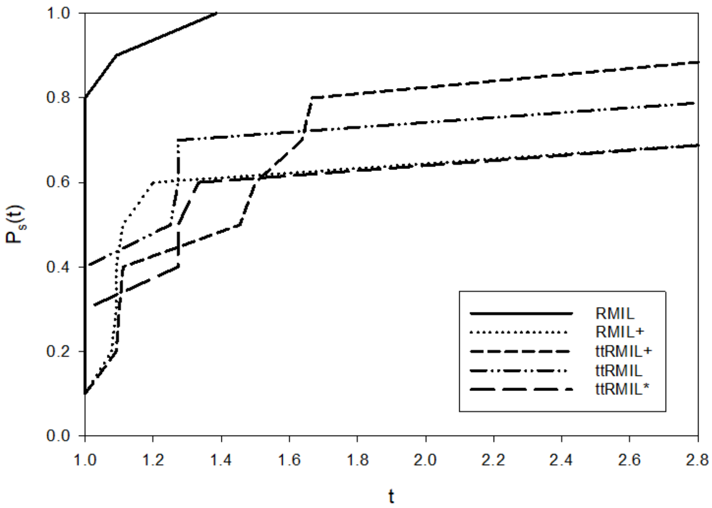
Table 4. Regression coefficients approximates for (21)

	Initial point	a_0	a_1	a_2	CPU time	NOI
SPSS	NIL	-1.64E-11	0.343743	0.564925		
NEWRMIL	(5, 1, 0)	-1.4916E-09	0.343743421	0.564924647	0.0003495	9
RMIL+		FAIL				
ttRMIL+		-1.14937E-10	0.343743429	0.564924643	0.0027048	15
ttRMIL		FAIL				
ttRMIL*		FAIL				
NEWRMIL	(4, 3, -2)	-3.67718E-10	0.343743421	0.564924647	0.0003403	9
RMIL+		-2.25769E-10	0.343743426	0.564924645	0.0003854	10
ttRMIL+		-2.251E-10	0.343743426	0.564924645	0.0004039	10
ttRMIL		-1.94631E-09	0.343743424	0.564924645	0.000364	9
ttRMIL*		-1.94632E-09	0.343743424	0.564924645	0.0003956	9
NEWRMIL	(3, 3, -1)	-5.25E-10	0.34374342	0.564924648	0.0003863	11
RMIL+		4.62E-09	0.343743422	0.564924646	0.0005053	12
ttRMIL+		-2.67E-11	0.343743422	0.564924647	0.0014332	16
ttRMIL		-1.62E-09	0.343743426	0.564924644	0.0005956	14
ttRMIL*		-1.62E-09	0.343743426	0.564924644	0.0004407	14
NEWRMIL	(2, 2, -1)	-5.57267E-10	0.343743419	0.564924649	0.0003861	11
RMIL+		FAIL				
ttRMIL+		-1.30976E-09	0.343743424	0.564924647	0.0020928	18
ttRMIL		2.50173E-10	0.343743426	0.564924647	0.0003834	11
ttRMIL*		2.51673E-10	0.343743426	0.564924647	0.0003621	11
NEWRMIL	(1, 1, 5)	8.67E-10	0.343743417	0.564924651	0.0004388	13
RMIL+		7.67E-11	0.343743421	0.564924648	0.000769	14
ttRMIL+		FAIL				
ttRMIL		1.57E-09	0.34374342	0.56492465	0.0004625	13
ttRMIL*		1.57E-09	0.34374342	0.56492465	0.0005417	13
NEWRMIL	(-1, -1, 4)	8.37E-10	0.343743421	0.564924647	0.0004226	12
RMIL+		-2.32E-11	0.343743421	0.564924647	0.0003932	12
ttRMIL+		-1.90E-12	0.343743421	0.564924648	0.0020654	18
ttRMIL		5.90E-11	0.343743419	0.564924647	0.000479	15
ttRMIL*		-1.25E-09	0.343743417	0.564924649	0.0007025	16
NEWRMIL	(-2, 0, 4)	-2.48611E-09	0.343743415	0.564924653	0.0009874	18
RMIL+		FAIL				
ttRMIL+		-1.77212E-10	0.343743416	0.56492465	0.0004706	13
ttRMIL		FAIL				
ttRMIL*		FAIL				
NEWRMIL	(-3, -2, 3)	-1.67511E-11	0.343743422	0.564924647	0.0003677	10
RMIL+		-2.58514E-12	0.343743416	0.56492465	0.0005915	12
ttRMIL+		2.0645E-09	0.343743419	0.564924648	0.0004075	11
ttRMIL		FAIL				
ttRMIL*		FAIL				
NEWRMIL	(-4, -1, 3)	6.10535E-10	0.343743422	0.564924646	0.0004766	11
RMIL+		1.96786E-12	0.343743421	0.564924647	0.00042	12
ttRMIL+		2.68306E-12	0.343743421	0.564924647	0.0004491	12
ttRMIL		6.5898E-10	0.34374342	0.564924649	0.0006077	14
ttRMIL*		6.59261E-10	0.34374342	0.564924649	0.0004684	14
NEWRMIL	(-5, -1, 1)	-2.94735E-09	0.343743417	0.56492465	0.0004612	12
RMIL+		FAIL				
ttRMIL+		3.36917E-10	0.343743415	0.564924651	0.0058446	33
ttRMIL		2.33855E-10	0.34374342	0.564924648	0.0005962	11
ttRMIL*		FAIL				

The overall F statistic is 86.02451223 and the $p-\text{value} = 1.05856E - 21 < 0.05$. Meanwhile, t-statistics for KE, R_0 and NC are significant at the 0.05 level. Based on this justification, only R_0 and NC impacts KE. This indicates that as the rate of R_0 and the number of new COVID-19 cases rises, the price of KE will increase as well. The related regression output is depicted in Table 5.



(a)



(b)

Figure 3. Performance profile based on (a) NOI, and (b) CPU time

Table 5. Regression analysis using SPSS for two independent variables

	Coefficients	Standard Error	t	Sig.
Constant	-4.18748E-15	0.061328056	-6.828E-14	1
R-naught	0.343743421	0.07430623	4.626037682	1.22988E-05
new cases	0.564924647	0.07430623	7.602655183	2.52577E-11

5. CONCLUSION

The primary emphasis of this paper is to present a new CG algorithm, NEWRMIL, and its applicability in solving linear regression models. The current pandemic crisis directed the author's interests to study the direct impacts of COVID-19 on the price of Kijang Emas (KE) through a multiple linear regression model. Estimation of parameters in linear regression has been carried out using the least- squares method and CG algorithm. The NEWRMIL method was compared with a set of well-known variants of RMIL: RMIL+, ttRMIL+, ttRMIL, and ttRMIL* methods. Numerical analysis proves that the NEWRMIL method has the highest NOI and CPU time efficiency among the CG methods used in this study. Therefore, the NEWRMIL method is deemed to be efficient, theoretically and practically. The regression analysis results revealed that R0 and the number of COVID-19 daily cases significantly affected the price of Kijang Emas. Furthermore, the results from the proposed CG method are comparable to SPSS with good performance of MSE, MAE, AND R2. In a nutshell, NEWRMIL method is considered as reliable, with regard to its practicality, useful estimates and reasonable accuracies.

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REFERENCES

[1] I. M. Sulaiman and M. Mamat, "A new conjugate gradient method with descent properties and its application to regression analysis," *Journal of Numerical Analysis, Industrial and Applied Mathematics*, vol. 12, no. 1–2, pp. 25–39, 2020

[2] P. Wolfe, "Convergence conditions for ascent methods. II: some corrections," *SIAM Review*, vol. 13, no. 2, pp. 185–188, Apr. 1971, doi: 10.1137/1013035.

[3] M. R. Hestenes and E. Stiefel, "Methods of conjugate gradients for solving linear systems," *Journal of Research of the National Bureau of Standards*, vol. 49, no. 6, Dec. 1952, doi: 10.6028/jres.049.044.

[4] R. Fletcher, "Function minimization by conjugate gradients," *The Computer Journal*, vol. 7, no. 2, pp. 149–154, Feb. 1964, doi: 10.1093/comjnl/7.2.149.

[5] E. Polak and G. Ribiere, "Note on the convergence of conjugate direction methods (in French)," *French journal of computer science and operational research. red series*, vol. 3, no. 16, pp. 35–43, May 1969, doi: 10.1051/m2an/196903R100351.

[6] B. T. Polyak, "The conjugate gradient method in extremal problems," *USSR Computational Mathematics and Mathematical Physics*, vol. 9, no. 4, pp. 94–112, Jan. 1969, doi: 10.1016/0041-5553(69)90035-4.

[7] J. W. Daniel, "The conjugate gradient method for linear and nonlinear operator equations," *SIAM Journal on Numerical Analysis*, vol. 4, no. 1, pp. 10–26, Mar. 1967, doi: 10.1137/0704002.

[8] R. Fletcher, *Practical Methods of Optimization*, 2nd ed. Chichester: John Wiley & Sons, Ltd, 2000., doi: 10.1002/9781118723203.

[9] Y. Liu and C. Storey, "Efficient generalized conjugate gradient algorithms, part 1: Theory," *Journal of Optimization Theory and Applications*, vol. 69, no. 1, pp. 129–137, Apr. 1991, doi: 10.1007/BF00940464.

[10] Y. H. Dai and Y. Yuan, "A nonlinear conjugate gradient method with a strong global convergence property," *SIAM Journal on Optimization*, vol. 10, no. 1, pp. 177–182, Jan. 1999, doi: 10.1137/S1052623497318992.

[11] W. Hager and H. Zhang, "A survey of nonlinear conjugate gradient methods," *Pacific Journal of Optimization*, vol. 2, no. 1, 2006

[12] M. Rivaie, M. Mamat, W. J. Leong, and I. Mohd, "A new conjugate gradient coefficient for large scale nonlinear unconstrained optimization," *International Journal of Mathematical Analysis*, vol. 6, no. 23, pp. 1131–1146, 2012

[13] M. Rivaie, M. Mamat, L. W. June, and I. Mohd, "A new class of nonlinear conjugate gradient coefficients with global convergence properties," *Applied Mathematics and Computation*, vol. 218, no. 22, pp. 11323–11332, Jul. 2012, doi: 10.1016/j.amc.2012.05.030.

[14] N. b. Z. Pakkal, *New conjugate gradient methods using strong Wolfe line search for estimating dividend rate*. Thesis, Universiti Sultan Zainal Abidin, 2016.

[15] M. Hamoda, M. Mamat, M. Rivaie, and Z. Salleh, "A conjugate gradient method with strong Wolfe-Powell line search for unconstrained optimization," *Applied Mathematical Sciences*, vol. 10, pp. 721–734, 2016, doi: 10.12988/ams.2016.56449.

[16] N. S. Mohamed, M. Mamat, F. S. Mohamad, and M. Rivaie, "A new coefficient of conjugate gradient methods for nonlinear unconstrained optimization," *Jurnal Teknologi*, vol. 78, no. 6–4, Jun. 2016, doi: 10.11113/jt.v78.8988.

[17] N. S. Mohamed, M. Mamat, M. Rivaie, and S. M. Shaharuddin, "Global convergence of a new coefficient nonlinear conjugate gradient method," *Indonesian Journal of Electrical Engineering and Computer Science (IJEECS)*, vol. 11, no. 3, pp. 1188–1193, Sep. 2018, doi: 10.11591/ijeecs.v11.i3.pp1188-1193.

[18] M. Rivaie, M. Mamat, and A. Abashar, "A new class of nonlinear conjugate gradient coefficients with exact and inexact line searches," *Applied Mathematics and Computation*, vol. 268, pp. 1152–1163, Oct. 2015, doi: 10.1016/j.amc.2015.07.019.

[19] N. S. Mohamed, M. Mamat, M. Rivaie, and S. M. Shaharudin, "A new hyhbrid coefficient of conjugate gradient method," *Indonesian Journal of Electrical Engineering and Computer Science (IJEECS)*, vol. 18, no. 3, pp. 1454–1463, Jun. 2020, doi: 10.11591/ijeecs.v18.i3.pp1454-1463.

[20] M. Malik, M. Mamat, S. S. Abas, I. M. Sulaiman, and Sukono, "A new coefficient of the conjugate gradient method with the sufficient descent condition and global convergence properties," *Engineering Letters*, vol. 28, no. 3, 2020

[21] A. O. Umar, M. Mamat, I. M. Sulaiman, M. Y. Waziri, and A. M. V., "A new modification of conjugate gradient parameter with efficient line search for nonconvex function," *International Journal of Scientific and Technology Research*, vol. 9, no. 3, pp. 6572–6575, 2020.

[22] A. V Mandara, M. Mamat, M. Y. Waziri, and M. A. Mohamed, "A class of conjugate gradient parameters and its global convergence for solving unconstrained optimization," *Far East Journal of Mathematical Sciences (FJMS)*, vol. 106, no. 1, pp. 43–58, Sep. 2018, doi: 10.17654/MS106010043.

[23] A. V. Mandara, M. Mamat, M. Y. Waziri, M. A. Mohamed, and U. A. Yakubu, "A new conjugate gradient coefficient with exact line search for unconstrained optimization," *Far East Journal of Mathematical Sciences (FJMS)*, vol. 105, no. 2, pp. 193–206, Jul. 2018, doi: 10.17654/MS105020193.

[24] Z. Dai, "Comments on a new class of nonlinear conjugate gradient coefficients with global convergence properties," *Applied Mathematics and Computation*, vol. 276, pp. 297–300, Mar. 2016, doi: 10.1016/j.amc.2015.11.085.

[25] O. O. O. Yousif, "The convergence properties of RMIL+ conjugate gradient method under the strong Wolfe line search," *Applied Mathematics and Computation*, vol. 367, Feb. 2020, doi: 10.1016/j.amc.2019.124777.

[26] B. Baluch, Z. Salleh, A. Alhawarat, and U. A. M. Roslan, "A new modified three-term conjugate gradient method with sufficient descent property and its global convergence," *Journal of Mathematics*, vol. 2017, pp. 1–12, 2017, doi: 10.1155/2017/2715854.

[27] Y. Narushima, H. Yabe, and J. A. Ford, "A three-term conjugate gradient method with sufficient descent property for unconstrained optimization," *SIAM Journal on Optimization*, vol. 21, no. 1, pp. 212–230, Jan. 2011, doi: 10.1137/080743573.

[28] I. M. Sulaiman, M. Mamat, A. E. Owoyemi, P. L. Ghazali, M. Rivaie, and M. Malik, "The convergence properties of some descent conjugate gradient algorithms for optimization models," *Journal of Mathematics and Computer Science*, vol. 22, no. 03, pp. 204–215, Jul. 2020, doi: 10.22436/jmcs.022.03.02.

[29] W. Khadijah, M. Rivaie, and M. Mamat, "A three-term conjugate gradient method under the strong-Wolfe line search," in *AIP Conference Proceedings*, 2017, 40056, doi: 10.1063/1.4995888.

[30] J. K. Liu, Y. M. Feng, and L. M. Zou, "Some three-term conjugate gradient methods with the inexact line search condition," *Calcolo*, vol. 55, no. 2, p. 16, Jun. 2018, doi: 10.1007/s10092-018-0258-3.

[31] S. Saha and B. Nath, "A new initial search direction for nonlinear conjugate gradient method," *International Journal of*

Mathematics Research, vol. 6, no. 2, pp. 183–190, 2014

[32] D. Cucinotta and M. Vanelli, “WHO declares COVID-19 a pandemic,” *Acta bio-medica : Atenei Parmensis*, vol. 91, no. 1, pp. 157–160, 2020, doi: 10.23750/abm.v91i1.9397.

[33] R. M. Shetty, N. C. Achaiah, and S. B. Subbarajasetty, “R0 and Re of COVID-19: can we predict when the pandemic outbreak will be contained?,” *Indian Journal of Critical Care Medicine*, vol. 24, no. 11, pp. 1125–1127, Dec. 2020, doi: 10.5005/jp-journals-10071-23649.

[34] H. Atri, S. Kouki, and M. imen Gallali, “The impact of COVID-19 news, panic and media coverage on the oil and gold prices: An ARDL approach,” *Resources Policy*, vol. 72, Aug. 2021, doi: 10.1016/j.resourpol.2021.102061.

[35] T. H. Van Hoang, Z. Zhu, B. Xiao, and W. Wong, “The seasonality of gold prices in China does the risk-aversion level matter?,” *Accounting and Finance*, vol. 60, no. 3, pp. 2617–2664, Sep. 2020, doi: 10.1111/acfi.12396.

[36] D. G. Baur and B. M. Lucey, “Is gold a hedge or a safe haven? an analysis of stocks, bonds and gold,” *Financial Review*, vol. 45, no. 2, pp. 217–229, May 2010, doi: 10.1111/j.1540-6288.2010.00244.x.

[37] A. A. Salisu, X. V. Vo, and A. Lawal, “Hedging oil price risk with gold during COVID-19 pandemic,” *Resources Policy*, vol. 70, 101897, Mar. 2021, doi: 10.1016/j.resourpol.2020.101897.

[38] O. B. Adekoya, J. A. Oliyide, and G. O. Oduyemi, “How COVID-19 upturns the hedging potentials of gold against oil and stock markets risks: Nonlinear evidences through threshold regression and markov-regime switching models,” *Resources Policy*, vol. 70, Mar. 2021, doi: 10.1016/j.resourpol.2020.101926.

[39] A. A. Salisu, I. D. Raheem, and X. V. Vo, “Assessing the safe haven property of the gold market during COVID-19 pandemic,” *International Review of Financial Analysis*, vol. 74, Mar. 2021, doi: 10.1016/j.irfa.2021.101666.

[40] I. Yousef and E. Shehadeh, “The impact of COVID-19 on gold price volatility,” *International Journal of Economics and Business Administration*, vol. 8, no. 4, pp. 353–364, Oct. 2020, doi: 10.35808/ijeba/592.

[41] Y. Jianyi, W. Chenyang, H. Yupeng, and L. Zicheng, “Research on the relationship between Covid-19 epidemic and gold price trend based on Linear Regression Model,” in *2020 IEEE 9th Joint International Information Technology and Artificial Intelligence Conference (ITAIC)*, Dec. 2020, pp. 1796–1798, doi: 10.1109/ITAIC49862.2020.9338828.

[42] M. Peredaryenko, “Gold dinar as a consumer product: Modified version of the theory of planned behaviour,” *International Journal of Economics, Management and Accounting*, vol. 27, no. 2, 2019

[43] D. T. Larose, *Data mining and predictive analytics*, 2nd Edition. 2015.

[44] R. Taylor, “Interpretation of the correlation coefficient: a basic review,” *Journal of Diagnostic Medical Sonography*, vol. 6, no. 1, pp. 35–39, Jan. 1990, doi: 10.1177/875647939000600106.

[45] E. D. Dolan and J. J. Moré, “Benchmarking optimization software with performance profiles,” *Mathematical Programming*, vol. 91, no. 2, pp. 201–213, Jan. 2002, doi: 10.1007/s101070100263.

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