Reasoning in inconsistent prioritized knowledge bases: an argumentative approach

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ABSTRACT

A study of query answering in prioritized ontological knowledge bases (KBs) has received attention in recent years. While several semantics of query answering have been proposed and their complexity is rather well-understood, the problem of explaining inconsistency-tolerant query answers has paid less attention. Explaining query answers permits users to understand not only what is entailed or not entailed by an inconsistent description logic DL-Lite$_{\mathcal{R}}$ KBs in the presence of priority, but also why. We, therefore, concern with the use of argumentation frameworks to allow users to better understand explanation techniques of querying answers over inconsistent DL-Lite$_{\mathcal{R}}$ KBs in the presence of priority. More specifically, we propose a new variant of Dung's argumentation frameworks, which corresponds to a given inconsistent DL-Lite$_{\mathcal{R}}$ KB. We clarify a close relation between preferred subtheories adopted in such prioritized DL-Lite$_{\mathcal{R}}$ setting and acceptable semantics of the corresponding argumentation framework. The significant result paves the way for applying algorithms and proof theories to establish preferred subtheories inferences in prioritized DL-Lite$_{\mathcal{R}}$ KBs.

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1. INTRODUCTION

Ontologies have been remarkably successful in a specific domain such as modelling biomedical knowledge, policies and semantic web [1]-[4]. In oder to represent and reason over ontologies, the focus has been placed on logical formalisms such as description logics [5] and rule-based languages (also called Datalog$^{\pm}$) [6]. Description logic DL-Lite is a family of tractable description logics (DLs) where the ontological view (i.e. TBox) is used to reformulate asked queries to offer better exploitation of assertions (i.e ABox), since its expressiveness and decidability results [5].

In many real applications, there exist assertions in several conflicting sources having reliability levels. Indeed, sets of assertions with different reliability levels in given sources are gathered to build a prioritized assertional base (i.e. a prioritized ABox). In order to reason in such assertional bases, variants of the inconsistency-tolerant semantics (also called repairs) have been considered, in which attention has restricted to the most preferred repairs based upon weight, cardinality or a stratification of the assertional base in DL KBs [7], [8]. One of the potential approaches is to utilize a notion of preferred subtheories (which is used in prioritized logic setting [9]) to generate preferred maximal consistent subsets instead of calculating all maximal consistent subsets.

While the study of inconsistency-tolerant querying in prioritized knowledge bases (KBs) is rather well-understood, less interest has been paid to the issue of explaining query answers under such semantics. There are some existing approaches concerning with the explanation of standard reasoning and query entailment [10]-[16]. The earlier approaches mainly introduce a proof-theoretic approach with explanation services [10], [11]. The main idea of these approaches is to identify explanations based on subsets of axioms in the ontology, known as axiom pinpointing, that are responsible for an entailment. It should emphasize that the works on axiom pinpointing focus on classical reasoning and the associated types of entailment. To date, the only works in explaining inconsistency-tolerant querying are given for the DL-Lite family of languages [12], [16] and Datalog$^{\pm}$ [13]-[15]. The idea of these works is to determine a set of assertions as an explanation that will lead to the answer of a given query entailing under inconsistency-tolerant semantics. Specifically, Bienvenu *et al.* [12] considered the work in explaining query answers under inconsistency-tolerant semantics for the DL-Lite family of languages , where explanations are defined as causes that are sets of assertions under three semantics (intelligent augmented reality (IAR), brave and AR semantics). Lukasiewicz *et al.* [15] defined explanations for conjunctive queries in different formalisms based on existential rules, and provided a thorough complexity analysis under different complexity measures. An argumentation framework is a prominent approach for representation and reasoning in inconsistent KBs, with an aim to improve explanation techniques of querying answers [14], [16]. The authors explored the explanation techniques for query answering under the intelligent character recognition (ICR) semantics in rule-based languages.

The analyses illustrate that the problem of explanation techniques for inconsistency-tolerant query answering are not studied in the context where an ABox is prioritized. Therefore, we concern ourself with the development of a framework for explaining inconsistency-tolerant querying in DL-Lite$_{\mathcal{R}}$ KBs associated with priorities. The main propose of this study is that we propose a new prioritized argumentation framework corresponding to an inconsistent DL-Lite$_{\mathcal{R}}$ KB with a prioritized ABox. By considering the use of argumentation framework, explanations of query answers allow users to naturally understand why a query is (not) entailed by a DL-Lite$_{\mathcal{R}}$ KB in the presence of the priority. In our work, the information expressed by the argument allows for tracking the provenance of data employed to imply querying answers and the attack relations show which pieces of information are incomparable, i.e. inconsistent information indicates erroneous data. Compared with the former approaches, such as [12], [14], [15], [17], the result allows a user to better understand explanation techniques of inconsistency-tolerant semantics in the presence of priority. Moreover, we clarify a relation between semantics from inconsistent prioritized KB query answering (preferred subtheories) and semantics from the corresponding argumentation framework (preferred, stable semantics). The significant result, therefore, paves the way for applying algorithms and argument-game proof theories to establish preferred subtheories inferences in inconsistent DL-Lite$_{\mathcal{R}}$ KB with the priority ABox.

The remaining sections of the paper contain: We discusses the preliminaries on description logics (DLs) and abstract argumentation framework (AAF) in the second section. Section 3 introduces a logical instantiating argumentation framework that corresponds to a given prioritized DL-Lite$_{\mathcal{R}}$ KB. Section 4 discusses other works from the literature. Finally, we summarize and set out some future works.

2. PRELIMINARIES ON AAF, DL

2.1. Abstract argumentation framework

In this section, we recall definitions of an abstract argumentation framework (AAF). The AAF is introduced by Dung in [18]. In particular, we briefly introduce the definition of AAF, extensions and acceptability semantics.

2.1.1. Abstract argumentation

An abstract argumentation framework (AAF) is a tuple of $\mathcal{AF} = \langle Arg, Re \rangle$. The AAF includes two components that are argument and attack relations. In particular, Arg is a set of arguments and $Re \subseteq Arg \times Arg$ is a binary attack relation between arguments.

2.1.2. Extensions

Let $\mathcal{AF} = \langle Arg, Re \rangle$ be an AAF, and k be an argument in Arg and $\mathcal{R} \subseteq Arg$. Argument k is acceptable w.r.t set $\mathcal{R}$ iff for any argument s in Arg, s attacks k and there exists argument t in $\mathcal{R}$ s.t. t attacks s. $\mathcal{R}$ is conflict free if there are no arguments k and s in $\mathcal{R}$ such that k attacks s. $\mathcal{R}$ is admissible if $\mathcal{R}$ is conflict free and for all argument k in $\mathcal{R}$, k is acceptable w.r.t $\mathcal{R}$. $\mathcal{R}$ is said to be a complete extension if k is acceptable w.r.t $\mathcal{R}$ implies argument k in $\mathcal{R}$. $\mathcal{R}$ is said to be a preferred extension if $\mathcal{R}$ is a maximal (w.r.t. set inclusion) complete extension. $\mathcal{R}$ is said to be a grounded extension if $\mathcal{R}$ is a minimal (w.r.t. set inclusion) complete extension. $\mathcal{R}$ is said to be a stable extension iff $\mathcal{R}$ is conflict-free and there is no argument in $\mathcal{R}$ such that it is attacked by other argument in $\mathcal{R}$. In AF, the output is determined by the set of conclusions that are emerged into all extensions under given semantics. We distinguish the following three acceptability states.

2.1.3. Acceptability semantics

Let $\mathcal{AF} = \langle Arg, \mathcal{R} \rangle$ be an AAF. For an argument $x \in Arg$ and $ex \in \{g, s, p\}$, the argument x is sceptically accepted w.r.t semantic ex iff x is in all extensions under ex, argument x is credulously accepted w.r.t semantic ex if argument x is in at least on extensions under ex, the argument x is rejected if argument x is not in any extension under ex, where g, s, p stand for a grounded, a stable and a preferred semantic, respectively.

2.2. Description logics

We give a brief overview of description logic (DL) KBs. We will consider the DL-Lite$_\mathcal{R}$ of the DL-Lite family through this paper [5]. In partcular, we introduce syntax, semantics and queries, respectively.

2.2.1. Syntax

A DL-Lite$_\mathcal{R}$ KB is a pair of TBox and ABox, written $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$, which are built from a concept name set N_C (unary predicates), a role name set N_R (binary predicates), an individual set N_I (constants). In DL-Lite$_\mathcal{R}$, $\mathcal{A}$ includes a finite set of concept assertions expressed by the form $A(c)$ and role assertions expressed by the form $S(c, d)$, in which $A \in N_C$, $S \in N_R$, $c, d \in N_I$. In DL-Lite$_\mathcal{R}$, $\mathcal{T}$ contains a set of axioms. Axioms in $\mathcal{T}$ are concept inclusions of the form $C \sqsubseteq D$ and role inclusions of the form $R \sqsubseteq Q$, which are formulated by the syntax: $C := A \mid \exists R \quad D := C \mid \neg C \quad R := S | S^- \quad Q := R \mid \neg R$, where $A \in N_C$ are atomic concepts, $S \in N_R$ are atomic roles, S^- is the inverse of an atomic role. A basic concept is denoted by C where C is either an atomic concept or a concept of the form $\exists R$. A basic role is denoted by R where R is either an atomic role or the inverse of an atomic role. A (general) concept is denoted D where D is either a basic concept or its negation. A (general) role is denoted by Q where Q is a basic role or its negation. ATBox axiom is formulated by $C_1 \sqsubseteq C_2$ or $R_1 \sqsubseteq R_2$, which is called positive inclusions. A TBox axiom is formed by $C_1 \sqsubseteq \neg C_2$ or $R_1 \sqsubseteq \neg R_2$, which is called a negative inclusion.

2.2.2. Semantics

An interpretation $\mathcal{I} = (\Delta^\mathcal{I}, .^\mathcal{I})$ comprehends a non-empty set $\Delta^\mathcal{I}$ and an interpretation function. $^\mathcal{I}$ that maps each individual c to each element $c^\mathcal{I} \in \Delta^\mathcal{I}$ with $c^\mathcal{I} \neq d^\mathcal{I}$ for $c \neq d$ (Note that $c^\mathcal{I} \neq d^\mathcal{I}$ is known as unique names assumption (UNA)), each concept A to a subset $A^\mathcal{I} \subseteq \Delta^\mathcal{I}$ and each role S to a set $S^\mathcal{I} \subseteq \Delta^\mathcal{I} \times \Delta^\mathcal{I}$. For complex concepts and roles, the interpretation function $.^\mathcal{I}$ is extended as following: $A^\mathcal{I} \subseteq \Delta^\mathcal{I}$, $S^\mathcal{I} \subseteq \Delta^\mathcal{I} \times \Delta^\mathcal{I}$, $(S^-)^\mathcal{I} = \{(d, c) \in \Delta^\mathcal{I} \times \Delta^\mathcal{I} | (c, d) \in S^\mathcal{I}\}$, $(\exists Q)^\mathcal{I} = \{c \in \Delta^\mathcal{I} | \exists c' \in \Delta^\mathcal{I} s.t. (c, c') \in Q^\mathcal{I}\}$, $(\neg C)^\mathcal{I} = \Delta^\mathcal{I} \setminus C^\mathcal{I}$, $(\neg Q)^\mathcal{I} = \Delta^\mathcal{I} \times \Delta^\mathcal{I} \setminus Q^\mathcal{I}$. An interpretation $\mathcal{I}$ satisfies a concept (resp. role) inclusion axiom, denoted by $\mathcal{I} \models C_1 \sqsubseteq C_2$ (resp. $\mathcal{I} \models Q_1 \sqsubseteq Q_2$), if $C_1^\mathcal{I} \subseteq C_2^\mathcal{I}$ (resp. $Q_1^\mathcal{I} \subseteq Q_2^\mathcal{I}$). $\mathcal{I}$ satisfies a concept (resp. role) assertion, denoted by $\mathcal{I} \models A(c)$ (resp. $\mathcal{I} \models S(c, d)$) if $c^\mathcal{I} \in A^\mathcal{I}$(resp. $(c^\mathcal{I}, d^\mathcal{I}) \in S^\mathcal{I}$). We say that $\mathcal{I}$ is a model of $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ if $\mathcal{I}$ satisfies all axioms in $\mathcal{T}$ and assertions in $\mathcal{A}$. $\mathcal{K}$ is consistent if it has a model; otherwise it is inconsistent. If $\mathcal{K}$ is the consistent KB the ABox $\mathcal{A}$ is $\mathcal{T}$-consistent. For our work, we are interested in querying DL KBs. A considered query class is a class of conjunctive queries (CQs) or unions of conjunctive queries (UCQs). In a place, we present a specific query language, namely first-order queries (FOL-queries).

2.2.3. Queries

A FOL-query is a first-order logic formula whose atoms are constructed using concepts and roles of $\mathcal{K}$ (and variables and constants from N_I). We denote a (CQ) by $Q(\overrightarrow{u}) = \exists \overrightarrow{v}.\delta(\overrightarrow{u}, \overrightarrow{v})$, where δ is a conjunction of atoms with atoms of the form $A(s)$ or $S(s, t)$ whose terms are either individuals or variables from $\overrightarrow{u} \cup \overrightarrow{v}$. If CQ includes a single tom, then the CQ is called an instance query (IQ). If CQ has no free variables, then the CQ is said to be a Boolean query. Let $Q(\overrightarrow{u})$ be a query with free variables $\overrightarrow{u} = (u_1, ..., u_m)$ and $\overrightarrow{s} = (s_1, ..., s_m)$ be a tuple of individuals, we say that Q has arity m and use $Q(\overrightarrow{s})$ to denote the Boolean query resulting from substituting s_i for each u_i. Given a FOL query $Q(\overrightarrow{u})$, a tuple of individuals $\overrightarrow{s}$ is an answer to $Q(\overrightarrow{u})$ of an interpretation $\mathcal{I}$, denoted by $\mathcal{I} \models Q(\overrightarrow{s})$, iff $\overrightarrow{s}$ has the same arity as Q and the Boolean query $Q(\overrightarrow{s})$ is satisfiable in $\mathcal{I}$ following standard first-order logic semantics. For a CQ Q, a tuple $\overrightarrow{s}$ is said to be a certain answer to Q w.r.t $\mathcal{K}$, denoted by $\mathcal{K} \models Q(\overrightarrow{s})$, iff it is an answer to $Q(\overrightarrow{u})$ in every model of $\mathcal{K}$.

2.3. Prioritized DL-Lite$_\mathcal{R}$ knowledge base

In many applications, there is some information from multiple sources that have different reliability levels. To represent such information, we introduce a notion of a prioritized DL-Lite$_\mathcal{R}$ KB where all assertions of an ABox have different reliability levels.

Definition 1 (Prioritized DL KB) Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ be a prioritized DL-Lite$_\mathcal{R}$ KB where the ABox $\mathcal{A}$ is partitioned into n strata $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n)$ such that:

- The strata are pairwise disjoint, i.e. $\mathcal{A}_i \cap \mathcal{A}_j = \emptyset$, for any $\mathcal{A}_i \neq \mathcal{A}_j$.
- The priority of assertions in $\mathcal{A}_i$ have the same level.
- The priority of assertions in $\mathcal{A}_i$ are higher than the priority of ones in $\mathcal{A}_j$ where $i < j$. Consequently, the assertions of $\mathcal{A}_1$ are the most crucial ones and the assertions of $\mathcal{A}_n$ are the least crucial ones.

Example 1 Given $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ in which $\mathcal{T} = \{G \sqsubseteq \neg H\}$ and assume that assertions of $\mathcal{A}$ provided by distinct sources $\mathcal{A} = \{\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3\}$ such that: $\mathcal{A}_1 = \{G(a_2)\}$, $\mathcal{A}_2 = \{H(a_1), G(a_1)\}$, and $\mathcal{A}_3 = \{H(a_2), H(a_3)\}$. In this example, $\mathcal{A}_1$ consists of the most reliable assertions while $\mathcal{A}_3$ contains the least reliable ones.

2.4. Inconsistency-tolerant semantics

In this section, we present definitions related to the problem of inconsistency in KBs. Note that the problem of inconsistency in KBs is considered w.r.t some assertional bases (i.e. ABoxes) and considered queries are Boolean queries.

Definition 2 (Inconsistency) A DL-Lite$_\mathcal{R}$ KB $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ is said to be inconsistent if $\mathcal{K}$ does not have any model. Otherwise, $\mathcal{K}$ is said to be consistent. Next, we introduce a concept of a conflict set, which is a minimal inconsistent subset of assertions w.r.t. the TBox.

Definition 3 (Conflict) Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ be a DL-Lite$_\mathcal{R}$ KB. A subset $\mathcal{C} \subseteq \mathcal{A}$ is a minimal conflict subset in $\mathcal{K}$ iff $\langle \mathcal{T}, \mathcal{C} \rangle$ is inconsistent and $\forall c \in \mathcal{C}, \langle \mathcal{T}, \mathcal{C} \setminus \{c\} \rangle$ is consistent. We denote $\mathcal{C}(\mathcal{A})$ is a set of conflicts in $\mathcal{A}$.

From definition 3, any assertion c from $\mathcal{C}$ is eliminated to restore to the consistency of $\langle \mathcal{T}, \mathcal{C} \rangle$. In the case of the coherent TBox, a conflict set consists of exactly two assertions having either the same priority level or the different priority level can be deduced from it.

Example 2 Consider $\mathcal{K}'' = \left\langle \mathcal{T}'', \mathcal{A}'' \right\rangle$ with $\mathcal{T}'' = \{G \sqsubseteq \neg H\}$, $\mathcal{A}'' = \{H(a_1), G(a_2), G(a_1), H(a_3)\}$. Then by definition 3, we have the set of conflicts in $\mathcal{A}$ yielding: $\mathcal{C}(\mathcal{A}) = \{H(a_1), G(a_1)\}$.

In a flat KB, inconsistency-tolerant semantics (also called repairs) have been studied to obtain significantly answer over the inconsistent KB [19].

Definition 4 (Repair) Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ be a flat DL-Lite$_\mathcal{R}$ KB. $RE \subseteq \mathcal{A}$ is called a *repair* w.r.t $\mathcal{K}$ iff $\langle \mathcal{T}, RE \rangle$ is consistent and $\forall RE_1 \supset RE$, $RE_1 \subseteq \mathcal{A}$, $\langle \mathcal{T}, RE_1 \rangle$ is inconsistent.

The above notion of repair can be extended when all ABox assertions have been partitioned into priority levels with the coherent TBox$\mathcal{T}$. In such case, the repairs are also computed in the scene of the term ”flat ABox”. So from now on, we shall use the notation $\mathcal{A} = (\mathcal{A}_1 \cup \cdots \cup \mathcal{A}_n)$ for $\mathcal{A} = (\mathcal{A}_1, \ldots, \mathcal{A}_2)$ to express the prioritized ABox. To exploit the priorities of the assertions, we only consider some maximal consistent subsets (not all of them), which are preferred maximal consistent subsets - called *preferred subtheories* used in classical logic setting [9]. We now introduce a version of preferred subtheories for the prioritized ontological KB.

Definition 5 (Preferred Subtheory) Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ be a prioritized DL-Lite$_\mathcal{R}$ KB with $\mathcal{A} = (\mathcal{A}_1 \cup \mathcal{A}_2 \cup \cdots \cup \mathcal{A}_n)$. $\mathcal{P} = \mathcal{P}_1 \cup \cdots \cup \mathcal{P}_n$ is a *preferred subtheory (PS)* w.r.t $\mathcal{K}$ iff $\forall k \in (1, \ldots, n)\, \mathcal{P}_1 \cup \cdots \cup \mathcal{P}_k$ is a maximal (w.r.t. set inclusion) consistent subset of $\mathcal{A}_1 \cup \cdots \cup \mathcal{A}_k$. We denote $Prs(\mathcal{A})$ the set of preferred theories w.r.t. $\mathcal{K}$.

In order to compute a preferred subtheory of $\mathcal{A}$ w.r.t $\mathcal{T}$, we first determine the maximal consistent subset of $\mathcal{A}_1$, then enlarge this maximal consistent subset as much as possible with assertions from $\mathcal{A}_2$ while preserving consistency and continuing this process for $\mathcal{A}_3, \ldots, \mathcal{A}_n$.

Example 3 (Example 1 Continued) Consider $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$. We get the set of conflicts and the set of PS: $\mathcal{C}(\mathcal{A}) = (\{G(a_2), H(a_2)\}, \{G(a_1), H(a_1)\})$, $\mathcal{P}_1 = \{H(a_1), G(a_2), H(a_3)\}$, $\mathcal{P}_2 = \{G(a_2), G(a_1), H(a_3)\}$. Indeed, either the assertion $X(A)$ is ignored, then the remaining assertions $\mathcal{P}_1 = \mathcal{A}_1 \cup (\mathcal{A}_2 \setminus G(a_1)$ is consistent with $\mathcal{T}$. Or the assertions $G(a_1)$ is kept and the assertions $H(a_1)$ is removed, then we get $\mathcal{P}_2$ in this case. Since the assertion $G(a_2) \in \mathcal{A}_1$ has a higher priority than the assertion $Y(a_2) \in \mathcal{A}_3$, then $H(a_2)$ is ignored. Thus, the remaining assertions $\mathcal{P}_1 = \mathcal{A} \setminus \{G(a_1), H(a_2)\}$ is consistent with $\mathcal{T}$. Computing $\mathcal{P}_2$ is similar to computing $\mathcal{P}_1$.

We next introduce notions of consistent entailment under PSs in the KB. *SPS-entailments* consider a query that is entailed by every preferred subtheory. *CPS-entailments* evaluate a query that is entailed by some preferred subtheories. In our context, the accepted query has either "Yes" answer (entailed) or "No" answer (not entailed). Note that the SPS-entailments (resp. the CPS-entailments) extend the definition of the AR semantics (resp. the brave semantics) proposed in [19] when we consider the case of the prioritized ABox. According to definition 6, $\models$ denotes a standard entailment used in consistent and flat $DL - Lite_{\mathcal{R}}$ KBs, namely $\langle \mathcal{T}, \mathcal{P} \rangle \models Q$ iff all models of $\langle \mathcal{T}, \mathcal{P} \rangle$ are also models of Q [5].

Definition 6 Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ be a prioritized $DL - Lite_{\mathcal{R}}$ KB and Q be a Boolean query. Then,

- Q is said to be a SPS-entailment w.r.t. $\mathcal{K}$, written $\mathcal{K} \models_{SPS} Q$, iff $\langle \mathcal{T}, \mathcal{P} \rangle \models Q$ for every $\mathcal{P} \in Prs(\mathcal{A})$,
- Q is said to be a CPS-entailment w.r.t. $\mathcal{K}$, written $\mathcal{K} \models_{CPS} Q$, iff $\langle \mathcal{T}, \mathcal{P} \rangle \models Q$ for some $\mathcal{P} \in Prs(\mathcal{A})$.

We remark a relation among the above semantics as follows: $\mathcal{K} \models_{SPS} Q \Rightarrow \mathcal{K} \models_{CPS} Q$.

Example 4 Consider a prioritized KB $\mathcal{K}_{ani} = \langle \mathcal{T}_{ani}, \mathcal{A}_{ani} \rangle$ where $\mathcal{T}_{ani} = \{jaguar \sqsubseteq animal, leopard \sqsubseteq animal, jaguar \sqsubseteq \neg leopard\}$, $\mathcal{A}_{ani} = \{\mathcal{A}_1, \mathcal{A}_2\}$ with $\mathcal{A}_1 = \{jaguar(m)\}$ and $\mathcal{A}_2 = \{leopard(m)\}$. The set of conflicts in $\mathcal{A}$: $\mathcal{C}(\mathcal{A}) = \{jaguar(m), leopard(m)\}$. The set of preferred subtheories: $\mathcal{P} = \{jaguar(m)\}$. We consider a Boolean query $Q_1 = animal(m)$. It can see that $\langle \mathcal{T}_{ani}, \mathcal{P} \rangle \models animal(m)$. By definition 6, $\mathcal{K}_{ani} \models_{SPS} animal(m)$. Clearly, we also get $\mathcal{K}_{ani} \models_{CPS} animal(m)$ and $\mathcal{K}_{ani} \models_{IPS} animal(m)$.

3. INSTANTIATING ABSTRACT ARGUMENTATION FRAMEWORKS WITH PRIORITIZED KNOWLEDGE BASES

This section focuses on the use of argumentation framework to deal with the issue of explaining answers for prioritized DL-Lite$_{\mathcal{R}}$ KBs under preferred repair semantics. The idea is that we propose a new prioritized argumentation framework (PAF) that corresponds to a given prioritized KB. By using a form of arguments and extensions of the PAF, we show that the support set of argument is a minimal subset of $\mathcal{A}$ that will lead to the answer (i.e. the consequence of arguments) holding under preferred repair semantics. Attack relations between the arguments explain what has been stated in the KB that causes the inconsistency.

3.1. Prioritized argumentation framework

We now introduce a new prioritized argumentation framework, which includes arguments and attack relations between the arguments. In the PAF, an argument consists of two elements: a support of the argument (also called a hypothesis) is a set of assertion of $\mathcal{A}$ and a consequence is entailed from the hypothesis. Before formalizing the notion of the argument, we discuss a closure of χ ($\chi \subseteq \mathcal{A}$) with respect to $\mathcal{T}$, denoted $Cl_{\mathcal{T}}(\chi)$, is repeatedly calculated by possible applications of all rules (positive inclusion assertions) in the TBox $\mathcal{T}$ over χ until reaching a fixed point. We employ a definition of restricted chase to calculate $Cl_{\mathcal{T}}(\chi)$ [20]. In this paper, the consequence of the argument is an assertion or a set of assertions.

Definition 7 (Argument) Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ be a prioritized KB. An argument x is a tuple (Φ, α) such that: (1) $\Phi \subseteq \mathcal{A}$ and $Cl_{\mathcal{T}}(\Phi)$ is $\mathcal{T}$-consistent. (2) $\alpha = \{\alpha_0, ..., \alpha_k\}$ is an assertion or a set of assertions s.t. $\{\alpha_0, ..., \alpha_k\} \subseteq Cl_{\mathcal{T}}(\Phi)$ (entailment). (3) $\nexists \Phi^{'} \subset \Phi, Cl_{\mathcal{T}}(\Phi^{'}) \vDash \alpha$.

For any argument, we observe that its support is the set of assertions induced for the entailment of consequence α in the KB $\mathcal{K}$. We emphasize that there are no positive or negative inclusion assertions in the conclusion or the support of argument. In the above definition, the first statement guarantees that the support of argument is consistent [21]. The next one guarantees that the consequence α of the argument is entailed from the support Φ. The third one ensures that the support is minimal.

Notation 1 for an argument $x = (\Phi, \alpha)$, its support is denoted by $Sup(x) = \Phi$ and its consequence is denoted by $Con(x) = \alpha$. Let us denote $Arg_{\mathcal{K}}$ the set of arguments built from the DL-Lite$_{\mathcal{R}}$ KB $\mathcal{K}$.

A second element of PAF is attack relations between the arguments. We recall a notion of direct undercut attack (known as assumption attack) as a concept of argumentative attacks to express the conflicts of the assertions in a given DL-Lite$_{\mathcal{R}}$ KB. The attack relations are not symmetric.

Definition 8 for two arguments $x, z \in Arg_{\mathcal{K}}$, x is said to attack z on the argument $z' = (\{\beta\}, \beta)$ (abusing notion we may wirte "*x attack z on β*") iff for $z' = (\{\beta\}, \beta)$, $\beta \in Sup(z)$, $Cl_{\mathcal{T}}(\{Con(x), \beta\})$ is $\mathcal{T}$-inconsistent.

Since our prioritized argumentation framework is built from a given prioritized DL-Lite$_{\mathcal{R}}$ KB, there exist preference relations between arguments in such argumentation setting. Note that for the prioritized KB, $\forall \phi \in \mathcal{A}_i, \forall \mu \in \mathcal{A}_j : \phi$ is more important than μ, denoted by $\phi \geq \mu$, iff $rank(\phi) \leq rank(\mu)$. We formally define preference relations. Definition 9 (Preference relation) Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ be a prioritized DL-Lite$_\mathcal{R}$ KB with $\mathcal{A} = (\mathcal{A}_1, \ldots, \mathcal{A}_n)$.

- For an assertion $\mu \in \mathcal{A}$, $rank(\mu) = k$ iff $\mu \in \mathcal{A}_i, \forall i \in (1, \ldots n)$. For an argument $x \in Arg(\mathcal{A})$, $rank(x) = min_{\mu \in Sup(x)} rank(\mu)$.
- Let x and y be arguments in $Arg_\mathcal{K}$. x is preferred to y, denoted by $x \succeq_{rank} y$, iff $rank(x) \leq rank(y)$.

We now introduce a definition of $\mathcal{R}_{\succeq_{rank}}$*-attack relation*, which means that the attack relation succeeds if the attacking argument is more preferred than the one attacked. We remark that $\mathcal{R}_{\succeq_{rank}}$-attack relation is not symmetric, irreflexive.

Definition 10 ($\mathcal{R}_{\succeq_{rank}}$-attack relation) Let x,y be two arguments in $Arg_\mathcal{K}$ and $\succeq_{rank}$ be a preference relation on $Arg_\mathcal{K}$. x $\mathcal{R}_{\succeq_{rank}}$-attacks y, denoted by $x\mathcal{R}_{\succeq_{rank}}y$, iff x attacks y on y' s.t. $x \succeq_{rank} y'$.

Next, we introduce a prioritized argumentation framework.

Definition 11 (Prioritized argumentation framework) Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ be a prioritized DL-Lite$_\mathcal{R}$ KB. A prioritized argumentation framework for $\mathcal{K}$ is a triple $\mathcal{AF}^{\succeq_{rank}} = \langle Arg_\mathcal{K}, \mathcal{R}_{\succeq_{rank}} \rangle$ such that $Arg_\mathcal{R}$ is a set of arguments. $\mathcal{R}_{\succeq_{rank}} \subseteq Arg_\mathcal{R} \times Arg_\mathcal{R}$, where $\mathcal{R}_{\succeq_{rank}}$ is an attack relation of $\mathcal{AF}^{\succeq_{rank}}$. We write $x\mathcal{R}_{\succeq_{rank}}y$, i.e, x $\mathcal{R}_{\succeq_{rank}}$-attacks y.

Notation 2 Let $\mathcal{AF}^{\succeq_{rank}} = \langle Arg_\mathcal{K}, \mathcal{R}_{\succeq_{rank}} \rangle$ be a PAF and $\mathcal{D} \subseteq Arg_\mathcal{K}$ be a set of arguments constructed from a given prioritized KB. The notations are used through this paper:

- $Args(\mathcal{A}') = \{x \in \mathcal{D} | Sup(x) \subseteq \mathcal{A}'\}$, where $\mathcal{A}' \subseteq \mathcal{A}$ be a set of assertions in a given prioritized KB. We call $Arg(\mathcal{A}')$ is a set of arguments built from the set of assertions $\mathcal{A}'$. We denote $Ext_{ex}(\mathcal{AF}^{\succeq_{rank}})$ a set of extensions under ex semantics.
- $Base(\mathcal{D}) = \bigcup_{x \in \mathcal{D}} Sup(x)$. Let $Base(\mathcal{D})$ denote a base of arguments such that it includes the supports of arguments in $\mathcal{D}$. $Cons(\mathcal{D}) = \{Con(x) \mid x \in \mathcal{D}\}$. We denote $Cons(\mathcal{D})$ the set of conclusions of arguments in $\mathcal{D}$. $Output(\mathcal{AF}_{\succeq_{rank}}) = \bigcap_{\mathcal{X} \in Ext_{ex}(\mathcal{AF}^{\succeq_{rank}})} Cons(\mathcal{X})$. We denote $Output(\mathcal{AF}_{\succeq_{rank}})$ the output of $\mathcal{AF}_{\succeq_{rank}}$ under ex semantics.

3.2. The results for characterizing PAF

Now that we have translated prioritized DL-Lite$_\mathcal{R}$ KBs into prioritized argumentation frameworks. Next, we shows the main results of the paper are that: (1) We clarify a relation between preferred subtheories of the prioritized KB are equal to stable/ preferred extensions of its corresponding PAF. (2) We show the use of PAF to explain the query answering problem.

Proposition 1 Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ be a prioritized DL-Lite$_\mathcal{R}$ KB, $\mathcal{AF}^{\succeq_{rank}} = \langle Arg_\mathcal{K}, \mathcal{R}_{\succeq_{rank}} \rangle$ be its corresponding argumentation framework. Then:

(a) If $\mathcal{P} \in Prs(\mathcal{A})$ is a preferred subtheory in $\mathcal{K}$ then $Args(\mathcal{P})$ is a stable extension of $\langle Arg_\mathcal{K}, \mathcal{R}_{\succeq_{rank}} \rangle$.

(b) If $\mathcal{X} \in Ext_s(\mathcal{AF}^{\succeq_{rank}})$ is a stable extension of $\langle Arg_\mathcal{K}, \mathcal{R}_{\succeq_{rank}} \rangle$ then $Base(\mathcal{X})$ is a preferred subtheory in $\mathcal{K}$.

Proof. 1) Firstly, we prove that $Args(\mathcal{P})$ is conflict-free. Assume that the contrary that $Args(\mathcal{P})$ is not conflict-free. From definition 10 for $\mathcal{R}_{\succeq_{rank}}$-attack, let $x, y \in Args(\mathcal{P})$ so that $x\mathcal{R}_{\succeq_{rank}}y$, then there exists $\beta \in Sup(y)$ s.t $Cl_\mathcal{T}(\{Con(x), \beta\})$ is $\mathcal{T}$-inconsistent. Thus $Cl_\mathcal{T}(Sup(x) \cup \{\beta\})$ is $\mathcal{T}$-inconsistent. $\mathcal{P}$ is hence not consistent. Contradiction. It can be concluded that $Args(\mathcal{P})$ must be conflict-free. We prove now that $Args(\mathcal{P})$ attacks each argument not belong to itself. Let $y \in Args(\mathcal{A}) \setminus Args(\mathcal{P})$, $\beta \in Sup(y)$ so that $\beta \notin \mathcal{P}$. Consider $x = (\mathcal{P}, \mathcal{P})$, we have $\beta \notin \mathcal{P}$, and $\mathcal{P}$ is also the set inclusion maximality of preferred theories, then $Cl_\mathcal{T}(\{Con(x), \beta\})$ is $\mathcal{T}$-inconsistent. By construction, $\mathcal{P} = \mathcal{P}_1 \cup \cdots \cup \mathcal{P}_n$ is a preferred subtheory s.t. $\forall k = 1 \ldots n, \mathcal{P}_1 \cup \cdots \cup \mathcal{P}_k$ is a maximal consistent subset of $\mathcal{A}_1 \cup \cdots \cup \mathcal{A}_k$. Therefore, we assume $\beta \in \mathcal{A}_j$ for some $j = 1 \ldots n$, then $\{\beta\} \cup P_1 \cup \cdots \cup \mathcal{P}_j$ is inconsistent subset. Since $\beta \in \mathcal{A}_j$ and the supports of argument x are in $\mathcal{P}_k$, $k \leq j$; i.e every support in x is greater or equal to β, then $rank(Sup(x)) \leq rank(Sup(y))$, and so by the definition of the attack relation $x\mathcal{R}_{\succeq_{rank}}y$.

2) Next, we show that $Base(\mathcal{X}) = \bigcup_{x \in \mathcal{X}} Con(x)$ must be consistent. By contradiction, we suppose that $Base(\mathcal{X})$ is inconsistent. Let $\{\alpha_1, \ldots, \alpha_n\}$, denoted by $\mathcal{M}$, be a minimal inconsistent subset of $Base\,(\mathcal{X})$. Let $x \in \mathcal{X}$ be an argument s.t $\alpha_n \in Sup\,(x)$. Let $x' = (\{\mathcal{M} \setminus \{\alpha_n\}, \{\alpha_1, \ldots, \alpha_{n-1}\}\})$, then $Cl_\mathcal{T}(\{Con(x'), \alpha_n\})$

is $\mathcal{T}$-inconsistent and $rank(x') \leq rank(x)$. Therefore $x'\mathcal{R}_{\succeq_{rank}}x$. Because $\mathcal{X}$ is conflict-free, then $x' \notin \mathcal{X}$. Since $\mathcal{X}$ is also a stable extension, $\exists y \in \mathcal{X}$ s.t $y\mathcal{R}_{\succeq_{rank}}x'$. It is clear that y attack x'. Thus, there exists $k \in \{1,\ldots,n-1\}$ such that $Cl_{\mathcal{T}}(\{Con(y),\alpha_k\})$ is $\mathcal{T}$-inconsistent. Since $\alpha_k \in Base(\mathcal{X})$, $\exists z \in \mathcal{X}$ such that $\alpha_k \in Sup(z)$ and $rank(y) \leq rank(z)$. Thus, $y\mathcal{R}_{\succeq_{rank}}z$, contradiction.Therefore, $Base(\mathcal{X})$ must be consistent.

Next, we show that $Base(\mathcal{X}$ is a preferred subtheory. By means of contradiction, we assume that $Base(\mathcal{X})$ is not a preferred subtheory, which means that $\exists k \in \{1,\ldots n\}$ such that $\mathcal{P}_1\cup\cdots\cup\mathcal{P}_k$ is not a maximal consistent subset in $\mathcal{A}_1\cup\cdots\cup\mathcal{A}_k$. Thus, there exist $\beta \notin Base(\mathcal{X})$ s.t $\beta \in \mathcal{A}_k$ and $\mathcal{P}_1\cup\cdots\cup\mathcal{P}_k\cup\{\beta\}$ is consistent. Let $x' = (\{\beta\},\beta)$. Because $\mathcal{X}$ is a stable extension, $\exists x \in \mathcal{X}$, $x\mathcal{R}_{\succeq_{rank}}x'$. Since $\mathcal{P}_1\cup\cdots\cup\mathcal{P}_k\cup\{\beta\}$ is consistent, no argument in $\mathcal{X}$ having level at most k cannot attack x'. This means that there exists no $x \in \mathcal{X}$ s.t $x\mathcal{R}_{\succeq_{rank}}x'$, then $\mathcal{X}$ is not stable extension. Contradicting. Thus, $Base(\mathcal{X})$ must be a preferred subtheory. Let us indicate that if $\mathcal{X} \subseteq Arg_{\mathcal{K}}$ is a stable extension of $\langle Arg_{\mathcal{K}},\mathcal{R}_{\succeq_{rank}}\rangle$, then $\mathcal{X} = Args(Base(\mathcal{X}))$. Suppose the contrary. For any $\mathcal{X} \subseteq Args(\mathcal{A})$, $\mathcal{X} \subseteq Args(Base(\mathcal{X}))$. Hence, it is easily be seen that the case $\mathcal{X} \subsetneq Args(Base(\mathcal{X}))$ is not possible.

The next theorem shows the main result of this section: the relation between acceptable semantics (sceptical, credulous semantics) from the PAF and entailments (SPS, CPS entailments) from the inconsistent prioritized DL-Lite$_{\mathcal{R}}$ KB . A query Q is sceptically accepted (resp. credulously accepted) w.r.t semantics ex iff it is a logical consequence over all extensions (at least one extension) with regards to stable/preferred semantics ex.

Theorem 1 Let $\mathcal{K} = \langle\mathcal{T},\mathcal{A}\rangle$ be a prioritized DL-Lite$_{\mathcal{R}}$ KB, $\mathcal{AF}^{\succeq_{rank}} = \langle Arg_{\mathcal{K}},\mathcal{R}_{\succeq_{rank}}\rangle$ be its corresponding PAF. For a Boolean query Q and $ex \in \{s,p\}$. Then:

(a) $\mathcal{K} \models_{SPS} Q$ iff Q is sceptically accepted w.r.t semantics ex.

(b) $\mathcal{K} \models_{CPS} Q$ iff Q is credulously accepted w.r.t semantics ex.

Proof. We invoke proposition 1 to deduce that $Ext_{ex}\left(\mathcal{AF}^{\succeq_{rank}}\right) = \{Arg(\mathcal{P}) \mid \mathcal{P} \in Prs(\mathcal{A})\}$. Evidently, the function Arg is a bijection between $Prs(\mathcal{A})$ and $Ext_{ex}\left(\mathcal{AF}^{\succeq_{rank}}\right)$. It is easily seen that for every preferred subtheory $\mathcal{P} \in Prs(\mathcal{A})$, we have that $Cl_{\mathcal{T}}(\mathcal{P}) \models Q$ iff $Cons(Arg(\mathcal{P})) \models Q$. From those two facts, the results of the proposition yield: (1) For query Q, $\mathcal{K} \models_{SPS} Q$ iff for every preferred subtheory $\mathcal{P} \in Prs(\mathcal{A})$, $Cl_{\mathcal{T}}(\mathcal{P}) \models Q$ iff for every extension $\mathcal{X} \in Ext_{ex}\left(\mathcal{AF}^{\succeq_{rank}}\right)$, $Cons(\mathcal{X}) \models Q$ iff Q is sceptically accepted. (2) For query Q, $\mathcal{K} \models_{CPS} Q$ iff at least one preferred subtheory $\mathcal{P} \in Prs(\mathcal{A})$, $Cl_{\mathcal{T}}(\mathcal{P}) \models Q$ iff at least one extension $\mathcal{X} \in Ext_{ex}\left(\mathcal{AF}^{\succeq_{rank}}\right)$, $Cons(\mathcal{X}) \models Q$ iff Q is credulously accepted.

The next corollary follows theorem 1 and definition 7. Corollary 1 states that the use of characterizing arguments provides explanations for the query in prioritized KBs.

Corollary 1 Let $\mathcal{K} = \langle\mathcal{T},\mathcal{A}\rangle$ be a prioritized DL-Lite$_{\mathcal{R}}$ KB, $\mathcal{AF}^{\succeq_{rank}} = \langle Arg_{\mathcal{K}},\mathcal{R}_{\succeq_{rank}}\rangle$ be its corresponding argumentation framework. For query Q and $\Psi \subseteq \mathcal{A}$, $A = (\Psi,\alpha)$ is an argument s.t. $\{\alpha\} \subseteq Q$ holding under semantics ex in $\mathcal{AF}^{\succeq_{rank}}$ iff Ψ is an explanation for α under preferred repairs in $\mathcal{K}$.

The next example illustrates the use of PAF to explain the query answering problem.

Example 5 (Example 4 continued) Reconsider the KB $\mathcal{K}_{ani} = \langle\mathcal{T}_{ani},\mathcal{A}_{ani}\rangle$. The set of arguments: $x_1 = (\{jaguar(m)\},\{jaguar(m)\})$, $x_2 = (\{leopard(m)\},\{leopard(m)\})$, $x_3 = (\{jaguar(m)\},\{animal(m)\})$, $x_4 = (\{leopard(m)\},\{animal(m)\})$. The set of stable extensions $Ext_s(\mathcal{AF}^{\succeq_{rank}})$: $\mathcal{X} = \{x_1,x_3\}$. The outputs of PAF: $output(\mathcal{AF}^{\succeq_{rank}}) = Cons(\mathcal{X}) = \{jaguar(m), animal(m)\}$.

Consider $Q_1 = animal(m)$, it is clear that $\mathcal{K}_{ani} \models_{SPS} animal(m)$ and $\mathcal{K}_{ani} \models_{CPS} animal(m))$. The explanations of Q_1: $\{jaguar(m)\}$ in x_3. The causes are $C_{x_3} = \{x_4,x_2\}$. The example shows that a user receives the explanations that lead to the answer for $\mathcal{K}_{ani} \models_{SPS} animal(m)$ and (one or more) causes that lead to the conflicts of Q_1 (i.e. the set of attacked arguments). Thus, PAF allows users to ask why a given query is (not) entailed in KB (in which case, the set of attacked arguments can be showed). Observe that the answers of query Q_1 are similar to the results in example 4. Moreover, the example explicitly illustrates the relation between the preferred subtheories of prioritized KB and the acceptable semantics of the corresponding PAF.

3.3. Rationality postulates

We now demonstrate that our framework satisfies the rationality postulates in [21]. Definition 12 Let $\mathcal{K} = \langle\mathcal{T},\mathcal{A}\rangle$ be a prioritized DL-Lite$_{\mathcal{R}}$ KB, $\mathcal{AF}^{\succeq_{rank}} = \langle Arg_{\mathcal{K}},\mathcal{R}_{\succeq_{rank}}\rangle$ be its corresponding PAF. For every $\mathcal{X} \in Ext(\mathcal{AF}^{\succeq_{rank}})$ and an arbitrary argument $x \in \mathcal{S}$. The postulates are defined as follows: i) Closure of extensions: For each $\mathcal{X} \in Ext(\mathcal{AF}^{\succeq_{rank}}), Cons(\mathcal{X}) = Cl_{\mathcal{T}}(Cons(\mathcal{X}))$; ii) Closure under Sub-argument: For all $\mathcal{X} \in Ext(\mathcal{AF}^{\succeq_{rank}})$, if $x \in \mathcal{X}$ then $Suba(x) \in \mathcal{X}$, where $Suba(x)$ is a sub-arguments

set of an argument x; iii) Weak Closure under sub-arguments: For all $\mathcal{X} \in Ext(\mathcal{AF}^{\succeq_{rank}})$, if $x \in \mathcal{X}$, $y \in Suba(x)$ and $x \succeq_{rank} y$, then $y \in Ext(\mathcal{AF}^{\succeq_{rank}})$; iv) Consistency: For all $\mathcal{X} \in Ext(\mathcal{AF}^{\succeq_{rank}})$, then $Cons(\mathcal{X})$ and $Base(\mathcal{X})$ are consistent; v) Exhaustiveness: For all $\mathcal{X} \in Ext(\mathcal{AF}^{\succeq_{rank}})$, for all $x \in \mathcal{S}$, if $Sup(x) \cup \{Con(x)\} \subseteq Cons(\mathcal{X})$, then $x \in \mathcal{X}$; and vi) Free precedence: For all $\mathcal{X} \in Ext(\mathcal{AF}^{\succeq_{rank}})$, $Arg(Free(\mathcal{K})) \subseteq E$ where $Free(\mathcal{K}) = \mathcal{A} \setminus \bigcup_{C \text{ is a minimal conflict}} C$.
In the PAF, the preference relation has some interesting properties, namely *Minimality* and *And,* as stated in [21]. Basing on the fact that the preference relation $\succeq_{rank}$ satisfies "Minimality" for set inclusion, one can see that if y is a sub-argument of x (which means that $Sup(y) \subseteq Sup(x)$) then $y \succeq_{rank} x$. We shall consider these postulates under the following assumptions: i) The preference relation $\succeq_{rank}$ is left monotonic: If $x \succeq_{rank} y$ and $Sup(y) \subseteq Sup(y')$ then $x \succeq_{rank} y'$ and ii) We will consider the preference relation $\succeq_{rank}$ on the sets of arguments: $x \succeq_{rank} y$ iff $Sup(x) \succeq_{rank} Sup(y)$.
Proposition 2 Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ be a prioritized DL-Lite$_\mathcal{R}$ KB, $\mathcal{AF}^{\succeq_{rank}} = \langle Arg_\mathcal{K}, \mathcal{R}_{\succeq_{eli}} \rangle$ be its corresponding PAF. $\mathcal{AF}^{\succeq_{eli}}$ satisfies closure of extensions, Weak closure under sub-argument, consistency, exhaustiveness and free precedence.

Proof. We prove each postulate in proposition 2:

(a) Closure of extensions: From the definition of closure of extensions, for any $\mathcal{X} \in Ext(\mathcal{AF})$, $Cons(\mathcal{X}) \subseteq Cl_\mathcal{T}(Cons(\mathcal{X}))$. Next, we shall show that $Cl_\mathcal{T}(Cons(\mathcal{X})) \subseteq Cons(\mathcal{X})$. Let $\phi \in Cl_\mathcal{T}(Cons(\mathcal{X}))$. Since $\mathcal{X}$ is a stable extension, theorem 1 implies that $\exists \mathcal{E}, \mathcal{E} \subseteq Base(\mathcal{X})$ so that $\mathcal{X} = Arg(\mathcal{E})$. Hence $\mathcal{X} = Arg(Base(\mathcal{X}))$. Since the supports of arguments in $\mathcal{X}$ include the assertions from $\mathcal{E}$, it follows that $\phi \in Cl_\mathcal{T}(\mathcal{E})$. Consequently, $\exists x \in \mathcal{X}$ s.t $Con(x) = \phi$.

(b) Weak closure under sub-argument: Let $y \in \mathcal{X}$ and y' be a sub-argument of y s.t $y' \in \mathcal{X}$ and $y' \succeq_{rank} y$. Assume the contrary, that $y' \notin \mathcal{X}$. Since $\mathcal{X}$ is a stable extension, $\exists z, z \in \mathcal{X}$ s.t z $\mathcal{R}_{\succeq_{rank}}$-attacks y', which means $\exists \phi \in Sup(z)$ s.t $Cl_\mathcal{T}(\{Con(z), \phi\})$ is $\mathcal{T}$-inconsistent and $z \succeq_{rank} y'$. Since y' is a sub-argument of y, $Sup(y') \subseteq Sup(y)$ then $\phi \in Sup(y)$. In addition, since $z \succeq_{rank} y'$ and $z' \succeq_{rank} y$, then $y \succeq_{rank} y$. Clearly, $Cl_\mathcal{T}(\{Con(z), \phi\})$ is $\mathcal{T}$-inconsistent and $z \succeq_{rank} y$, which implies $z\mathcal{R}_{\succeq_{rank}}$-attacks y. This contradicts with $\mathcal{X}$, which has to be the stable extension and conflict-free.

(c) Consistency: We prove that for every extension, the conclusion set is consistent. Let $\mathcal{X}_i \in Ext(\mathcal{AF})$ be a stable extension of $\mathcal{AF}^{\succeq_{rank}} = \langle Arg_\mathcal{K}, \mathcal{R}_{\succeq_{rank}} \rangle$ Taking theorem 1, we have a preferred theory $\mathcal{P} \in Prs(\mathcal{A})$ such that $\mathcal{X}_i = Arg(\mathcal{P})$. It is easily seen that $Cons(\mathcal{X}_i) = Cl_\mathcal{T}(\mathcal{P})$. Since $\mathcal{P}$ is a preferred theory then $Cl_\mathcal{T}(\mathcal{P})$ is consistent. Thus, $Cons(\mathcal{X}_i)$ is consistent. Now, for each extension, we prove that the base of them can be consistent. In view of theorem 1, we have $\mathcal{X}_i = Arg(Base(\mathcal{X}_i))$ and $Base(\mathcal{X}_i)$ is a preferred theory due to $\mathcal{X}_i$ is a stable extension. Therefore, $\mathcal{X}_i$ is consistent.

(d) Exhaustiveness: Suppose the contrary, that $z \in Arg_\mathcal{K}$ be an argument s.t $Sup(z) \cup \{Con(z)\} \subseteq Cons(\mathcal{X})$ and $z \notin \mathcal{X}$. Since $\mathcal{X}$ is a stable extension, $\exists y, y \in \mathcal{X}$ s.t y attacks z, which means $\exists \phi, \phi \in Sup(z)$ s.t $Cl_\mathcal{T}(\{\phi, Con(y)\})$ is $\mathcal{T}$-inconsistent. We also have $Sup(z) \subseteq Cons(\mathcal{X})$ and $Con(z) \in Cons(\mathcal{X})$. By the above it follows that $Cons(\mathcal{X})$ is inconsistent, which contradicts with the Consistency postulate.

(e) Free precedence: We begin by supposing that $z \in Arg_\mathcal{K}$ is an argument where $Sup(x) \subseteq Free(\mathcal{K})$. It can be seen that with every other consistent subset of $\mathcal{A}$, $Free(\mathcal{K})$ is consistent, it follows that there is no an argument attacks z. Assume the contracdiction that there is an argument $y \in Arg_\mathcal{K}$ such that y attacks z. This means that $\exists \phi \in Sup(z)$ s.t $Cl_\mathcal{T}(\{Con(y), \phi\})$ is $\mathcal{T}$-inconsistent and $y \succeq_{rank} z$. Thus, $Cl_\mathcal{T}(Free(\mathcal{K}) \cup Sup(y))$ is $\mathcal{T}$-inconsistent. However, we know that $Sup(y)$ is consistent. This shows that $Free(\mathcal{K})$ is inconsistent with the consistent subset $Sup(y)$ of the ABox $\mathcal{A}$, a contradiction. Consequently z is unattacked by any argument, then it must be in every extension.

4. DISCUSSION

4.1. Explanation technique

In this section, we survey works on explaining query (non-)answers and entailments. As mentioned in the introduction, DL reasoning systems with explanation facilities have recently become interests in different areas of AI [10]-[12], [14]-[16]. The earliest work mainly focuses on the explanation of standard reasoning tasks and the associated types of entailments [10], [11]. The authors propose the notion of axiom pinpointing, where the idea is that we compute minimal subsets of ontological axioms, which provide a consequence. In our framework, TBox is considered to be coherent, namely, consequences of the TBox are desirable. It is clear that the works on axiom pinpointing is a first step to confirm that the errors arise from the data sources, i.e. ABoxes are inconsistent. Beside the works of computing axiom pinpointing, explanation techniques for querying in inconsistency-tolerant semantics have been recently addressed in the literature [12], [14]-[16]. Specifically, Bienvenu *et al.* [12] consider the problem of explanations in DL-Lite$_\mathcal{R}$ KBs. The authors introduced the definition of explanations for (non-) answers of the query under three semantics (brave, AR, and IAR), and the data complexity of different related problems. Their motivations are quite similar to our work. In order to explain different (brave, AR and IAR)-answers, the authors use sets of causes that minimally cover the repairs, whereas, for SPS and CPS-answers, the explanations are the sets of arguments covering the stable/preferred extensions. Having said that, their work differs from us since we explore the explanation technique for querying in prioritized DL-Lite$_\mathcal{R}$ KBs. Lukasiewicz *et al.* [15] propose explanation techniques for query answering under three inconsistency-tolerant semantics (AR, IAR, ICR semantics) in rule-based language and fulfil a complexity analysis under the combined, bounded arity-combined and fixed-program-combined complexities, besides the data complexity. In their work, a notion of minimal explanations is defined as minimal consistent subsets from sets of facts that entail the query. Note that the notion of explanations is equivalent to the concept of causes in [12]. In our paper, in contrast, we consider a different formalism expressed by DL-Lite$_\mathcal{R}$ in the context where the ABox has the preferences. While the existing approaches, such as [12], [15], have showed how to compute explanations that can provide the answers for queries holding inconsistency-tolerant semantics, our framework shows ”inconsistency of KBs” and ”why querying answers hold in the prioritized KB”. The closest related approach is proposed by Arioua *et al.* [14] who present an argumentation framework to explain query answers under the inconsistency-tolerant semantics in the presence of existential rules. The authors compute one explanation for ICR-answer by using the hitting set algorithm, applied either on the set of attacking arguments or on the sets of supporting arguments presenting in extensions (which corresponds to repairs) [16]. Contrary to our framework, their focus is on building the arguments without considering priorities in the set of facts and considering different inconsistency-tolerant semantics for Datalog$^\pm$.

4.2. Argumentation framework

In this section, we discuss our result with the related works in argumentation framework. Argumentation is a potential approach for inconsistency-tolerant reasoning over KBs. To resolve conflicting and uncertain information, several argumentation frameworks have recently been studied in different representation languages such as defeasible logics (DLs), classical Logics (CLs). Specially, GenAF presented in [22] to address reasoning for inconsistent ontologies expressed by $\mathcal{ALC}$. Garcia *et al.* [23] propose defeasible logic programming (DeLP), which is a combination of defeasible argumentation with outcomes of logic programming. These systems have several differences when comparing with our framework. First one is the way of characterizing arguments: our work constructs arguments immediately from subsets of the KB by utilizing the proof procedure, while arguments in these two systems are formed of inference trees by using two forms of rules. The other difference is that all arguments in these systems are equally strong, whereas our framework considers the preferences of arguments. In our work, we adopt the notion of preferred subtheories as the variants of inconsistency-tolerant semantics for reasoning in ontological KBs, and clarify the correspondence between diverse notions of extensions for (preference-based) AFs and preferred subtheories. Marcello et.al propose argumentative approach for reasoning under preferred subtheories in CLs [24], [25]. The authors consider the argumentative characterisations in the standard and dialectical approaches to classical logic argumentation (Cl-Arg). Moreover, they also indicate that the preferred and stable semantics of argumentation frameworks instantiated by default theories coincide. From above analyses, all approaches can be viewed as the study for characterizing preferred subtheories inference based on argumentation theory. However, all works noted so far focus on classical logics associated with priorities while our work takes into account different formalism, namely, description logics. An argument-based approach closely related to our work, i.e. an argumentation framework is built from an inconsistent ontological KB to handle inconsistency under the locally optimal, Pareto optimal, globally optimal semantics, which is proposed by Madalina and Rallou [26]. Their framework supposes that all attacks always succeed and preferences of arguments can be used to select only the best extensions. By contrast, our framework only considers preferences of arguments to formally define attack relations amongst arguments, namely, for any argument R and P, an attack relation can be a successful attack iff R attacks P and R is stronger than its attacker.

5. CONCLUSION

The main contribution of the paper is to consider the use of argumentation framework to address the problem of explaining query answer in prioritized DL-Lite$_{\mathcal{R}}$ KBs. More specifically, we proposed an prioritized argumentation framework, which corresponds to a given prioritized DL-Lite$_{\mathcal{R}}$ KB. The advantage of utilizing argumentation framework is to permits (by considering the support set) to track the provenance from data sources used to deduce query answers and to see (by considering the attack relations) which pieces of data are incompatible together. Moreover, we clarified the closed relation between the prioritized DL-Lite$_{\mathcal{R}}$ KB and the proposed argumentation framework. The significant result paves the way for applying algorithms and argument game proof theories to establish preferred subtheories inferences in the prioritized DL-Lite$_{\mathcal{R}}$ KB.

Study of the model-theoretic relations between other semantics of argumentation framework and inconsistency-tolerant semantics would be open problems for furture works. The study will have a huge impact on the knowledge representation (KR) and the argumentation theory (AT) community. It shows how KR community could receive benefits from the results of the argumentation theory and whether AT community could utilize the outcomes of KR community.

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