# 4-total edge product cordial for some star related graphs

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**ABSTRACT**

Let $G = (V(G), E(G))$ be a graph, define an edge labeling function $\psi$ from $E(G)$ to $\{0, 1, \ldots, k-1\}$ where $k$ is an integer, $2 \leq k \leq |E(G)|$, induces a vertex labeling function $\psi^*$ from $V(G)$ to $\{0, 1, \ldots, k-1\}$ such that $\psi^*(v) = \psi(e_1) \times \psi(e_2) \times \ldots \times \psi(e_n)$ mod $k$ where $e_1, e_2, \ldots, e_n$ are all edge incident to $v$. This function $\psi$ is called a $k$-total edge product cordial (or simply $k$-TEPC) labeling of $G$ if the absolute difference between number of vertices and edges labeling with $i$ and number of vertices and edges labeling with $j$ no more than 1 for all $i, j \in \{0, 1, \ldots, k-1\}$. In this paper, 4-total edge product cordial labeling for some star related graphs are determined.



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## 1. INTRODUCTION

Let us beging with a graph $G = (V(G), E(G))$ that is simple, connected, finite and undirected with order $p$ and size $q$. With regards to the standard terminology and notation, we refer to books [1]-[4]. We also provide a brief summary of definitions that are invaluable for the present study.

A graph labeling is an assignment of integers to edges or vertices or both subject to certain conditions. If the domain of the mapping is the set of edges (or vertices) then the labeling is called an edge labeling (or a vertex labeling). For a labeling function $\psi$, an edge $e$ (or a vertex $v$) is an $i$-edge (or an $i$-vertex) if $\psi(e) = i$ (or $\psi(v) = i$) where $i \in Z$. Denote the number of $i$-edges (or $i$-vertices) of $G$ under $\psi$ by $e_\psi(i)$ (or $v_\psi(i)$), respectively and let $\psi(i) = e_\psi(i) + v_\psi(i)$.

Labeling graph has applications in coding theory, especially for the design of good radar type codes, missile guidance codes, and convolution codes with optimal autocorrelation properties. Labeling graph plays important role in the study of communication network and X-ray crystallography. A detailed study of some applications of labeling graphs are given in [5]-[9].

A lot of researchers have written on cordial labeling. Cahit [10] introduced cordial labeling and there are many papers studied cordial graph of some graphs such as [11], [12]. Sundaram *et al.* [13] defined product cordial labeling and in [14]-[16] some researchers illustrate product cordial labeling of some graphs. Vadiya and Barasara [17], [18] proposed two definetions edge product cordial labeling and total edge product cordial labeling.

Azaizeh *et al.* [19] introduced $k$-total edge product cordial labeling which is:

**Definition 1.** let $\psi$ be an edge labeling function from $E(G)$ to $\{0, 1, \ldots, k-1\}$ where $k$ is an integer, $2 \leq k \leq |E(G)|$. For each vertex $v$, assign the label $\psi(e_1) \times \psi(e_2) \times \ldots \times \psi(e_n)$ (mod $k$) where $e_1, e_2, ..., e_n$ are the edges incident to vertex *v*. The function $\psi$ is called a *k*-total edge product cordial (or simply *k*-TEPC) labeling of *G* if $|(v_\psi(i) + e_\psi(i)) - (v_\psi(j) + e_\psi(j))| \leq 1$, for $i, j \in \{0, 1, \ldots, k-1\}$. A graph *G* with a *k*-total edge product cordial labeling is called *k*-total edge product cordial graph.

Some researchers studied graphs in *k*-total edge product cordial labeling [19]-[23]. Vadiya *et al.* [24] studied cordial labeling for some star related graphs and Hasni and Azaizeh [25] determined 3-total edge products cordial labeling for some star related graphs. As a continuation of these results and the results in [19], [21], the main aim of this paper is to determine the 4-total edge product cordial labeling for some star related graphs.

## 2. MAIN RESULT

Let $K_{1,n}$ denote the star with order $n+1$. We first give brief summary of definitions which are useful for our investigations.

**Definition 2.** [24] consider two stars $K_{1,n}^{(1)}$ and $K_{1,n}^{(2)}$, then $G = < K_{1,n}^{(1)}, K_{1,n}^{(2)} >$ is the graph obtained by joining apex vertices of stars to a new vertex *x*. Note that *G* has 2*n* + 3 vertices and 2*n* + 2 edges.

**Definition 3.** [24] consider *k* copies of stars namely $K_{1,n}^{(1)}, K_{1,n}^{(2)}, \ldots, K_{1,n}^{(k)}$. Then $G = < K_{1,n}^{(1)}, K_{1,n}^{(2)}, \ldots, K_{1,n}^{(k)} >$ is the graph obtained by joining apex vertices of each $K_{1,n}^{(p-1)}$ and $K_{1,n}^{(p)}$ to a new vertex $x_{p-1}$ where $2 \leq p \leq k$. Note that *G* has $k(n+2) - 1$ vertices and $k(n+2) - 2$ edges.

**Theorem 1** Graph $< K_{1,n}^{(1)}, K_{1,n}^{(2)}, \ldots, K_{1,n}^{(k)} >$ is 4-TEPC for all $n \geq 3, k \geq 1$, except when $n \equiv 1$ *or* 2 (mod 4), $k = 1$ or $n = 3$,$k = 1$.

**Proof**. Let $V(< K_{1,n}^{(1)}, K_{1,n}^{(2)}, \ldots, K_{1,n}^{(k)} >) = \{u^{(j)}, v_i^{(j)}, 1 \leq j \leq k, 1 \leq i \leq n\} \cup \{x_m, 1 \leq m \leq k-1\}$ and $E(< K_{1,n}^{(1)}, K_{1,n}^{(2)}, \ldots, K_{1,n}^{(k)} >) = \{u^{(j)} v_i^{(j)}, 1 \leq j \leq k, 1 \leq i \leq n\} \cup \{u^{(j)} x_j, u^{(j+1)} x_j, 1 \leq j \leq k-1\}$. We consider the following cases.

**Case 1:** $k \equiv 0$ (mod 4).

**Case 1.1:** $n \equiv 0$ (mod 4).

Define $\psi$ be an edge labeling function from $E(G)$ to $\{0, 1, 2, 3\}$:

$$\psi(u^{(1)} v_1^{(1)}) = 1$$
$$\psi(u^{(1)} v_i^{(1)}) = 0, 2 \leq i \leq \frac{n}{4}$$
$$\psi(u^{(j)} v_i^{(j)}) = 0, 2 \leq j \leq k, 1 \leq i \leq \frac{n}{4}$$
$$\psi(u^{(j)} v_i^{(j)}) = 1, 1 \leq j \leq k, \frac{n+4}{4} \leq i \leq \frac{n}{2}$$
$$\psi(u^{(j)} v_i^{(j)}) = 2, 1 \leq j \leq k, \frac{n+2}{2} \leq i \leq \frac{3n}{4}$$
$$\psi(u^{(j)} v_i^{(j)}) = 3, 1 \leq j \leq k, \frac{3n+4}{4} \leq i \leq n$$
$$\psi(u^{(4j-3)} x_{4j-3}) = 2, 1 \leq j \leq \frac{k}{4}$$
$$\psi(u^{(4j-2)} x_{4j-3}) = 3, 1 \leq j \leq \frac{k}{4}$$
$$\psi(u^{(4j-2)} x_{4j-2}) = 3, 1 \leq j \leq \frac{k}{4}$$
$$\psi(u^{(4j-1)} x_{4j-2}) = 1, 1 \leq j \leq \frac{k}{4}$$
$$\psi(u^{(4j-1)} x_{4j-1}) = 2, 1 \leq j \leq \frac{k}{4}$$
$$\psi(u^{(4)} x_3) = 2$$
$$\psi(u^{(4j)} x_{4j}) = 1, 1 \leq j \leq \frac{k-4}{4}$$
$$\psi(u^{(4j+1)} x_{4j}) = 1, 1 \leq j \leq \frac{k-4}{4}$$

**Case 1.2:** $n \equiv 1 \pmod 4$.
Define $\psi$ be an edge labeling function from $E(G)$ to $\{0,1,2,3\}$:

$$\psi(u^{(j)}v_i^{(j)}) = 0, 1 \le j \le k, 1 \le i \le \frac{n-1}{4}$$
$$\psi(u^{(j)}v_i^{(j)}) = 2, 1 \le j \le k, \frac{n+3}{4} \le i \le \frac{n-1}{2}$$
$$\psi(u^{(2j-1)}v_i^{(2j-1)}) = 1, 1 \le j \le \frac{k}{2}, \frac{n+1}{2} \le i \le \frac{3n-3}{4}$$
$$\psi(u^{(2j-1)}v_i^{(2j-1)}) = 3, 1 \le j \le \frac{k}{2}, \frac{3n+1}{4} \le i \le n$$
$$\psi(u^{(2j)}v_i^{(2j)}) = 3, 1 \le j \le \frac{k}{2}, \frac{n+1}{2} \le i \le \frac{3n-3}{4}$$
$$\psi(u^{(2j)}v_i^{(2j)}) = 1, 1 \le j \le \frac{k}{2}, \frac{3n+1}{4} \le i \le n$$
$$\psi(u^{(4j-3)}x_{4j-3}) = 2, 1 \le j \le \frac{k}{4}$$
$$\psi(u^{(4j-2)}x_{4j-3}) = 3, 1 \le j \le \frac{k}{4}$$
$$\psi(u^{(4j-2)}x_{4j-2}) = 2, 1 \le j \le \frac{k}{4}$$
$$\psi(u^{(4j-1)}x_{4j-2}) = 1, 1 \le j \le \frac{k}{4}$$
$$\psi(u^{(4j-1)}x_{4j-1}) = 2, 1 \le j \le \frac{k}{4}$$
$$\psi(u^{(4)}x_3) = 2$$
$$\psi(u^{(4j)}x_{4j-1}) = 1, 2 \le j \le \frac{k}{4}$$
$$\psi(u^{(4j)}x_{4j}) = 0, 1 \le j \le \frac{k-4}{4}$$
$$\psi(u^{(4j+1)}x_{4j}) = 3, 1 \le j \le \frac{k-4}{4}$$

**Case 1.3:** $n \equiv 2 \pmod 4$.
Define $\psi$ be an edge labeling function from $E(G)$ to $\{0,1,2,3\}$:

$$\psi(u^{(j)}v_i^{(j)}) = 0, 1 \le j \le k, 1 \le i \le \frac{n-2}{4}$$
$$\psi(u^{(2j-1)}v_i^{(2j-1)}) = 1, 1 \le j \le \frac{k}{2}, \frac{n+2}{4} \le i \le \frac{n}{2}$$
$$\psi(u^{(2j-1)}v_i^{(2j-1)}) = 3, 1 \le j \le \frac{k}{2}, \frac{n+2}{2} \le i \le \frac{3n+2}{4}$$
$$\psi(u^{(2j-1)}v_i^{(2j-1)}) = 2, 1 \le j \le \frac{k}{2}, \frac{3n+6}{4} \le i \le n$$
$$\psi(u^{(4j-2)}v_i^{(4j-2)}) = 1, 1 \le j \le \frac{k}{4}, \frac{n+2}{4} \le i \le \frac{n}{2}$$
$$\psi(u^{(4j-2)}v_i^{(4j-2)}) = 2, 1 \le j \le \frac{k}{4}, \frac{n+2}{2} \le i \le \frac{3n+2}{4}$$
$$\psi(u^{(4j)}v_i^{(4j)}) = 2, 1 \le j \le \frac{k}{4}, \frac{n+2}{4} \le i \le \frac{n}{2}$$
$$\psi(u^{(4j)}v_i^{(4j)}) = 3, 1 \le j \le \frac{k}{4}, \frac{n+2}{2} \le i \le \frac{3n+2}{4}$$

$$\psi(u^{(4j)}v_i^{(4j)}) = 1, 1 \leq j \leq \frac{k}{4}, \frac{3n+6}{4} \leq i \leq n$$
$$\psi(u^{(4j-3)}x_{4j-3}) = 2, 1 \leq j \leq \frac{k}{4}$$
$$\psi(u^{(4j-2)}x_{4j-3}) = 2, 1 \leq j \leq \frac{k}{4}$$
$$\psi(u^{(4j-2)}x_{4j-2}) = 0, 1 \leq j \leq \frac{k}{4}$$
$$\psi(u^{(4j-1)}x_{4j-2}) = 1, 1 \leq j \leq \frac{k}{4}$$
$$\psi(u^{(4j-1)}x_{4j-1}) = 3, 1 \leq j \leq \frac{k}{4}$$
$$\psi(u^{(4)}x_3) = 2$$
$$\psi(u^{(4j)}x_{4j-1}) = 1, 2 \leq j \leq \frac{k}{4}$$
$$\psi(u^{(4j)}x_{4j}) = 2, 1 \leq j \leq \frac{k-4}{4}$$
$$\psi(u^{(4j+1)}x_{4j}) = 2, 1 \leq j \leq \frac{k-4}{4}$$

**Case 1.4:** $n \equiv 3$ (mod 4).
Define $\psi$ be an edge labeling function from $E(G)$ to $\{0, 1, 2, 3\}$:

$$\psi(u^{(j)}v_i^{(j)}) = 0, 1 \leq j \leq k, 1 \leq i \leq \frac{n-3}{4}$$
$$\psi(u^{(j)}v_i^{(j)}) = 1, 1 \leq j \leq k, \frac{n+1}{4} \leq i \leq \frac{n-1}{2}$$
$$\psi(u^{(j)}v_i^{(j)}) = 2, 1 \leq j \leq k, \frac{n+1}{2} \leq i \leq \frac{3n-1}{4}$$
$$\psi(u^{(j)}v_i^{(j)}) = 3, 1 \leq j \leq k, \frac{3n+3}{4} \leq i \leq n$$
$$\psi(u^{(2j-1)}x_{2j-1}) = 0, 1 \leq j \leq \frac{k}{2}$$
$$\psi(u^{(2)}x_1) = 3$$
$$\psi(u^{(2j)}x_{2j-1}) = 0, 2 \leq j \leq \frac{k}{2}$$
$$\psi(u^{(4j-2)}x_{4j-2}) = 2, 1 \leq j \leq \frac{k}{4}$$
$$\psi(u^{(4j)}x_{4j}) = 3, 1 \leq j \leq \frac{k-4}{4}$$
$$\psi(u^{(2j+1)}x_{2j}) = 1, 1 \leq j \leq \frac{k-2}{2}.$$

We now have $\psi(0) = \psi(1) = \psi(3) = \frac{k}{2}(n+2) - 1, \psi(2) = \frac{k}{2}(n+2)$. Therefore, $|\psi(i) - \psi(j)| \leq 1$ for $0 \leq i < j \leq 3$. Hence $\psi$ is 4-TEPC labeling.

**Case 2:** $k \equiv 1$ (mod 4).
**Case 2.1:** $n \equiv 0$ (mod 4).
Define $\psi$ be an edge labeling function from $E(G)$ to $\{0, 1, 2, 3\}$:

$$\psi(u^{(j)}v_i^{(j)}) = 0, 1 \leq j \leq k, 1 \leq i \leq \frac{n}{4}$$

$$\psi(u^{(j)}v_i^{(j)}) = 1, 1 \le j \le k, \frac{n+4}{4} \le i \le \frac{n}{2}$$
$$\psi(u^{(j)}v_i^{(j)}) = 2, 1 \le j \le k, \frac{n+2}{2} \le i \le \frac{3n}{4}$$
$$\psi(u^{(j)}v_i^{(j)}) = 3, 1 \le j \le k, \frac{3n+4}{4} \le i \le n$$
$$\psi(u^{(2j-1)}x_{2j-1}) = 2, 1 \le j \le \frac{k-1}{2}$$
$$\psi(u^{(2j)}x_{2j}) = 3, 1 \le j \le \frac{k-1}{2}$$
$$\psi(u^{(j+1)}x_j) = 1, 1 \le j \le k-1.$$

We now have $\psi(0) = \frac{k}{2}(n+2), \psi(1) = \psi(2) = \psi(3) = \frac{k}{2}(n+2) - 1$. Therefore, $|\psi(i) - \psi(j)| \le 1$ for $0 \le i < j \le 3$. Hence $\psi$ is 4-TEPC labeling.

**Case 2.2:** $n \equiv 1 \pmod 4$, $k \ne 1$.

Define $\psi$ be an edge labeling function from $E(G)$ to $\{0, 1, 2, 3\}$:

$$\psi(u^{(j)}v_i^{(j)}) = 0, 1 \le j \le k, 1 \le i \le \frac{n-1}{4}$$
$$\psi(u^{(j)}v_i^{(j)}) = 1, 1 \le j \le k, \frac{n+3}{4} \le i \le \frac{n-1}{2}$$
$$\psi(u^{(2j-1)}v_i^{(2j-1)}) = 3, 1 \le j \le \frac{k+1}{2}, \frac{n+1}{2} \le i \le \frac{3n-3}{4}$$
$$\psi(u^{(2j-1)}v_i^{(2j-1)}) = 2, 1 \le j \le \frac{k+1}{2}, \frac{3n+1}{4} \le i \le n$$
$$\psi(u^{(2j)}v_i^{(2j)}) = 2, 1 \le j \le \frac{k-1}{2}, \frac{n+1}{2} \le i \le \frac{3n-3}{4}$$
$$\psi(u^{(2j)}v_i^{(2j)}) = 3, 1 \le j \le \frac{k-1}{2}, \frac{3n+1}{4} \le i \le n$$
$$\psi(u^{(4j-3)}x_{4j-3}) = 0, 1 \le j \le \frac{k-1}{4}$$
$$\psi(u^{(4j-2)}x_{4j-3}) = 3, 1 \le j \le \frac{k-1}{4}$$
$$\psi(u^{(4j-1)}x_{4j-1}) = 3, 1 \le j \le \frac{k-1}{4}$$
$$\psi(u^{(4)}x_3) = 3$$
$$\psi(u^{(4j)}x_{4j-1}) = 2, 2 \le j \le \frac{k-1}{4}$$
$$\psi(u^{(2j)}x_{2j}) = 1, 1 \le j \le \frac{k-1}{2}$$
$$\psi(u^{(2j+1)}x_{2j}) = 1, 1 \le j \le \frac{k-1}{2}.$$

We now have $\psi(0) = \psi(1) = \psi(3) = \frac{k(n+2)-1}{2}, \psi(2) = \frac{k(n+2)-3}{2}$. Therefore, $|\psi(i) - \psi(j)| \le 1$ for $0 \le i < j \le 3$. Hence $\psi$ is 4-TEPC labeling.

**Case 2.3:** $n \equiv 2 \pmod 4$, $k \ne 1$.

Define $\psi$ be an edge labeling function from $E(G)$ to $\{0, 1, 2, 3\}$:

$$\psi(u^{(j)}v_i^{(j)}) = 0, 1 \le j \le k, 1 \le i \le \frac{n-2}{4}$$
$$\psi(u^{(2j-1)}v_i^{(2j-1)}) = 1, 1 \le j \le \frac{k+1}{2}, \frac{n+2}{2} \le i \le \frac{3n+2}{4}$$

$$\psi(u^{(2j-1)}v_i^{(2j-1)}) = 3, 1 \le j \le \frac{k+1}{2}, \frac{3n+6}{4} \le i \le n$$
$$\psi(u^{(2j)}v_i^{(2j)}) = 3, 1 \le j \le \frac{k-1}{2}, \frac{n+2}{2} \le i \le \frac{3n+2}{4}$$
$$\psi(u^{(2j)}v_i^{(2j)}) = 1, 1 \le j \le \frac{k-1}{2}, \frac{3n+6}{4} \le i \le n$$
$$\psi(u^{(2j-1)}x_{2j-1}) = 0, 1 \le j \le \frac{k-1}{2}$$
$$\psi(u^{(2j)}x_{2j}) = 3, 1 \le j \le \frac{k-1}{2}$$
$$\psi(u^{(2)}x_1) = 3$$
$$\psi(u^{(j)}x_{j-1}) = 1, 3 \le j \le k.$$

We now have $\psi(0) = \psi(1) = \psi(3) = \frac{k}{2}(n+2) - 1, \psi(2) = \frac{k}{2}(n+2)$. Therefore, $|\psi(i) - \psi(j)| \le 1$ for $0 \le i < j \le 3$. Hence $\psi$ is 4-TEPC labeling.

**Case 2.4:** $n \equiv 3$ (mod 4).

Define $\psi$ be an edge labeling function from $E(G)$ to $\{0, 1, 2, 3\}$:

$$\psi(u^{(j)}v_i^{(j)}) = 0, 1 \le j \le k, 1 \le i \le \frac{n-3}{4}$$
$$\psi(u^{(j)}v_i^{(j)}) = 1, 1 \le j \le k, \frac{n+1}{4} \le i \le \frac{n-1}{2}$$
$$\psi(u^{(j)}v_i^{(j)}) = 2, 1 \le j \le k, \frac{n+1}{2} \le i \le \frac{3n-1}{4}$$
$$\psi(u^{(j)}v_i^{(j)}) = 3, 1 \le j \le k, \frac{3n+3}{4} \le i \le n$$
$$\psi(u^{(2j)}x_{2j}) = 0, 1 \le j \le \frac{k-1}{2}$$
$$\psi(u^{(2j+1)}x_{2j}) = 0, 1 \le j \le \frac{k-1}{2}$$
$$\psi(u^{(4j-3)}x_{4j-3}) = 2, 1 \le j \le \frac{k-1}{4}$$
$$\psi(u^{(4j-1)}x_{4j-1}) = 3, 1 \le j \le \frac{k-1}{4}$$
$$\psi(u^{(2j)}x_{2j-1}) = 1, 1 \le j \le \frac{k-1}{2}$$

We now have $\psi(0) = \frac{k(n+2)-3}{2}, \psi(1) = \psi(2) = \psi(3) = \frac{k(n+2)-1}{2}$. Therefore, $|\psi(i) - \psi(j)| \le 1$ for $0 \le i < j \le 3$. Hence $\psi$ is 4-TEPC labeling.

**Case 3:** $k \equiv 2$ (mod 4).

**Case 3.1:** $n \equiv 0$ (mod 4).

Define $\psi$ be an edge labeling function from $E(G)$ to $\{0, 1, 2, 3\}$:

$$\psi(u^{(1)}v_1^{(1)}) = 2$$
$$\psi(u^{(1)}v_i^{(1)}) = 1, 2 \le i \le \frac{n}{2}$$
$$\psi(u^{(1)}v_i^{(1)}) = 3, \frac{n+2}{2} \le n$$
$$\psi(u^{(2)}v_1^{(2)}) = 1$$
$$\psi(u^{(2)}v_2^{(2)}) = 3$$

$$\psi(u^{(2)}v_i^{(2)}) = 0, 3 \leq i \leq \frac{n+2}{2}$$
$$\psi(u^{(2)}v_i^{(2)}) = 2, \frac{n+4}{2} \leq i \leq n$$
$$\psi(u^{(j)}v_i^{(j)}) = 0, 3 \leq j \leq k, 1 \leq i \leq \frac{n}{4}$$
$$\psi(u^{(j)}v_i^{(j)}) = 1, 3 \leq j \leq k, \frac{n+4}{4} \leq i \leq \frac{n}{2}$$
$$\psi(u^{(j)}v_i^{(j)}) = 2, 3 \leq j \leq k, \frac{n+2}{2} \leq i \leq \frac{3n}{4}$$
$$\psi(u^{(j)}v_i^{(j)}) = 3, 3 \leq j \leq k, \frac{3n+4}{4}$$
$$\psi(u^{(j)}x_j) = 1, 1 \leq j \leq k-1$$
$$\psi(u^{(2)}x_1) = 0$$
$$\psi(u^{(2j)}x_{2j-1}) = 2, 2 \leq j \leq \frac{k}{2}$$
$$\psi(u^{(2j+1)}x_{2j}) = 3, 1 \leq j \leq \frac{k-2}{2}.$$

We now have $\psi(0) = \psi(1) = \psi(2) = \frac{k}{2}(n+2) - 1, \psi(3) = \frac{k}{2}(n+2)$. Therefore, $|\psi(i) - \psi(j)| \leq 1$ for $0 \leq i < j \leq 3$. Hence $\psi$ is 4-TEPC labeling.

**Case 3.2:** $n \equiv 1 \pmod 4$.

Define $\psi$ be an edge labeling function from $E(G)$ to $\{0, 1, 2, 3\}$:

$$\psi(u^{(j)}v_i^{(j)}) = 0, 1 \leq j \leq k, 1 \leq i \leq \frac{n-1}{4}$$
$$\psi(u^{(j)}v_i^{(j)}) = 1, 1 \leq j \leq k, \frac{n+3}{4} \leq i \leq \frac{n-1}{2}$$
$$\psi(u^{(2j-1)}v_i^{(2j-1)}) = 2, 1 \leq j \leq \frac{k}{2}, \frac{n+1}{2} \leq i \leq \frac{3n-3}{4}$$
$$\psi(u^{(2j-1)}v_i^{(2j-1)}) = 3, 1 \leq j \leq \frac{k}{2}, \frac{3n+1}{4} \leq i \leq n$$
$$\psi(u^{(2j)}v_i^{(2j)}) = 3, 1 \leq j \leq \frac{k}{2}, \frac{n+1}{2} \leq i \leq \frac{3n-3}{4}$$
$$\psi(u^{(2j)}v_i^{(2j)}) = 2, 1 \leq j \leq \frac{k}{2}, \frac{3n+1}{4} \leq i \leq n$$
$$\psi(u^{(j)}x_j) = 1, 1 \leq j \leq k-1$$
$$\psi(u^{(4j-2)}x_{4j-3}) = 1, 1 \leq j \leq \frac{k+2}{4}$$
$$\psi(u^{(4j-1)}x_{4j-2}) = 0, 1 \leq j \leq \frac{k-2}{4}$$
$$\psi(u^{(4j)}x_{4j-1}) = 2, 1 \leq j \leq \frac{k-2}{4}$$
$$\psi(u^{(4j+1)}x_{4j}) = 3, 1 \leq j \leq \frac{k-2}{4}.$$

We now have $\psi(0) = \psi(2) = \psi(3) = \frac{k}{2}(n+2) - 1, \psi(1) = \frac{k}{2}(n+2)$. Therefore, $|\psi(i) - \psi(j)| \leq 1$ for $0 \leq i < j \leq 3$. Hence $\psi$ is 4-TEPC labeling.

**Case 3.3:** $n \equiv 2 \pmod 4$.

Define $\psi$ be an edge labeling function from $E(G)$ to $\{0, 1, 2, 3\}$:

$$\psi(u^{(1)}v_1^{(1)}) = 2$$

$$\psi(u^{(1)}v_i^{(1)}) = 1, 2 \leq i \leq \frac{n}{2}$$
$$\psi(u^{(1)}v_i^{(1)}) = 3, \frac{n+2}{2} \leq i \leq n$$
$$\psi(u^{(2)}v_1^{(2)}) = 1$$
$$\psi(u^{(2)}v_2^{(2)}) = 3$$
$$\psi(u^{(2)}v_i^{(2)}) = 0, 3 \leq i \leq \frac{n+2}{2}$$
$$\psi(u^{(2)}v_i^{(2)}) = 2, \frac{n+4}{2} \leq i \leq n$$
$$\psi(u^{(j)}v_i^{(j)}) = 0, 3 \leq j \leq k, 1 \leq i \leq \frac{n-2}{4}$$
$$\psi(u^{(j)}v_i^{(j)}) = 1, 3 \leq j \leq k, \frac{n+2}{4} \leq i \leq \frac{n-2}{2}$$
$$\psi(u^{(2j+1)}v_i^{(2j+1)}) = 2, 1 \leq j \leq \frac{k-2}{2}, \frac{n}{2} \leq i \leq \frac{3n-2}{4}$$
$$\psi(u^{(2j+1)}v_i^{(2j+1)}) = 3, 1 \leq j \leq \frac{k-2}{2}, \frac{3n+2}{4} \leq i \leq n$$
$$\psi(u^{(2j+2)}v_i^{(2j+2)}) = 3, 1 \leq j \leq \frac{k-2}{2}, \frac{n}{2} \leq i \leq \frac{3n-2}{4}$$
$$\psi(u^{(2j+2)}v_i^{(2j+2)}) = 2, 1 \leq j \leq \frac{k-2}{2}, \frac{3n+2}{4} \leq i \leq n$$
$$\psi(u^{(j)}x_j) = 1, 1 \leq j \leq k-1$$
$$\psi(u^{(2j)}x_{2j-1}) = 0, 1 \leq j \leq \frac{k}{2}$$
$$\psi(u^{(2j+1)}x_{2j}) = 1, 1 \leq j \leq \frac{k-2}{2}.$$

We now have $\psi(0) = \psi(1) = \psi(2) = \frac{k}{2}(n+2) - 1, \psi(3) = \frac{k}{2}(n+2)$. Therefore, $|\psi(i) - \psi(j)| \leq 1$ for $0 \leq i < j \leq 3$. Hence $\psi$ is 4-TEPC labeling.

**Case 3.4:** $n \equiv 3$ (mod 4).

Define $\psi$ be an edge labeling function from $E(G)$ to $\{0, 1, 2, 3\}$:

$$\psi(u^{(j)}v_i^{(j)}) = 0, 1 \leq j \leq k, 1 \leq i \leq \frac{n-3}{4}$$
$$\psi(u^{(j)}v_i^{(j)}) = 1, 1 \leq j \leq k, \frac{n+1}{4} \leq i \leq \frac{n-1}{2}$$
$$\psi(u^{(j)}v_i^{(j)}) = 2, 1 \leq j \leq k, \frac{n+1}{2} \leq i \leq \frac{3n-1}{4}$$
$$\psi(u^{(j)}v_i^{(j)}) = 3, 1 \leq j \leq k, \frac{3n+3}{4} \leq i \leq n$$
$$\psi(u^{(2j-1)}x_{2j-1}) = 0, 1 \leq j \leq \frac{k}{2}$$
$$\psi(u^{(2j)}x_{2j-1}) = 0, 1 \leq j \leq \frac{k}{2}$$
$$\psi(u^{(4j-2)}x_{4j-2}) = 2, 1 \leq j \leq \frac{k-2}{4}$$
$$\psi(u^{(4j)}x_{4j}) = 3, 1 \leq j \leq \frac{k-2}{4}$$
$$\psi(u^{(2j+1)}x_{2j}) = 1, 1 \leq j \leq \frac{k-2}{2}.$$

We now have $\psi(0) = \frac{k}{2}(n+2), \psi(1) = \psi(2) = \psi(3) = \frac{k}{2}(n+2) - 1$. Therefore, $|\psi(i) - \psi(j)| \leq 1$ for $0 \leq i < j \leq 3$. Hence $\psi$ is 4-TEPC labeling.

**Case 4:** $k \equiv 3$ (mod 4).
**Case 4.1:** $n \equiv 0$ (mod 4).
Define $\psi$ be an edge labeling function from $E(G)$ to $\{0, 1, 2, 3\}$:

$$\psi(u^{(j)}v_i^{(j)}) = 0, 1 \leq j \leq k, 1 \leq i \leq \frac{n}{4}$$
$$\psi(u^{(j)}v_i^{(j)}) = 1, 1 \leq j \leq k, \frac{n+4}{4} \leq i \leq \frac{n}{2}$$
$$\psi(u^{(j)}v_i^{(j)}) = 2, 1 \leq j \leq k, \frac{n+2}{2} \leq i \leq \frac{3n}{4}$$
$$\psi(u^{(j)}v_i^{(j)}) = 3, 1 \leq j \leq k, \frac{3n+4}{4} \leq i \leq n$$
$$\psi(u^{(2j-1)}x_{2j-1}) = 2, 1 \leq j \leq \frac{k-1}{2}$$
$$\psi(u^{(2j)}x_{2j}) = 3, 1 \leq j \leq \frac{k-1}{2}$$
$$\psi(u^{(j+1)}x_j) = 1, 1 \leq j \leq k-1.$$

We now have $\psi(0) = \frac{k}{2}(n+2), \psi(1) = \psi(2) = \psi(3) = \frac{k}{2}(n+2) - 1$. Therefore, $|\psi(i) - \psi(j)| \leq 1$ for $0 \leq i < j \leq 3$. Hence $\psi$ is 4-TEPC labeling.
**Case 4.2:** $n \equiv 1$ (mod 4).
Define $\psi$ be an edge labeling function from $E(G)$ to $\{0, 1, 2, 3\}$:

$$\psi(u^{(j)}v_i^{(j)}) = 0, 1 \leq j \leq k, 1 \leq i \leq \frac{n-1}{4}$$
$$\psi(u^{(j)}v_i^{(j)}) = 1, 1 \leq j \leq k, \frac{n+3}{4} \leq i \leq \frac{n-1}{2}$$
$$\psi(u^{(2j-1)}v_i^{(2j-1)}) = 2, 1 \leq j \leq \frac{k+1}{2}, \frac{n+1}{2} \leq i \leq \frac{3n+1}{4}$$
$$\psi(u^{(2j-1)}v_i^{(2j-1)}) = 3, 1 \leq j \leq \frac{k+1}{2}, \frac{3n+5}{4} \leq i \leq n$$
$$\psi(u^{(2j)}v_i^{(2j)}) = 3, 1 \leq j \leq \frac{k-1}{2}, \frac{n+1}{2} \leq i \leq \frac{3n+1}{4}$$
$$\psi(u^{(2j)}v_i^{(2j)}) = 2, 1 \leq j \leq \frac{k-1}{2}, \frac{3n+5}{4} \leq i \leq n$$
$$\psi(u^{(j)}x_j) = 1, 1 \leq j \leq k-1$$
$$\psi(u^{(4j-2)}x_{4j-3}) = 1, 1 \leq j \leq \frac{k+1}{4}$$
$$\psi(u^{(4j-1)}x_{4j-2}) = 3, 1 \leq j \leq \frac{k+1}{4}$$
$$\psi(u^{(4j)}x_{4j-1}) = 0, 1 \leq j \leq \frac{k-3}{4}$$
$$\psi(u^{(4j+1)}x_{4j}) = 2, 1 \leq j \leq \frac{k-3}{4}.$$

We now have $\psi(0) = \frac{k(n+2)-3}{2}, \psi(1) = \psi(2) = \psi(3) = \frac{k(n+2)-1}{2}$. Therefore, $|\psi(i) - \psi(j)| \leq 1$ for $0 \leq i < j \leq 3$. Hence $\psi$ is 4-TEPC labeling.
**Case 4.3:** $n \equiv 2$ (mod 4).
Define $\psi$ be an edge labeling function from $E(G)$ to $\{0, 1, 2, 3\}$:

$$\psi(u^{(j)}v_i^{(j)}) = 0, 1 \leq j \leq k, 1 \leq i \leq \frac{n-2}{4}$$
$$\psi(u^{(j)}v_i^{(j)}) = 1, 1 \leq j \leq k, \frac{n+2}{4} \leq i \leq \frac{n-2}{2}$$
$$\psi(u^{(1)}v_{\frac{n}{2}}^{(1)}) = 1$$
$$\psi(u^{(1)}v_i^{(1)}) = 2, \frac{n+2}{2} \leq i \leq \frac{3n-2}{4}$$
$$\psi(u^{(j)}v_i^{(j)}) = 2, 2 \leq j \leq k, \frac{n}{2} \leq i \leq \frac{3n-2}{4}$$
$$\psi(u^{(j)}v_i^{(j)}) = 3, 1 \leq j \leq k, \frac{3n+2}{4} \leq i \leq n$$
$$\psi(u^{(2j-1)}x_{2j-1}) = 0, 1 \leq j \leq \frac{k-1}{2}$$
$$\psi(u^{(2j)}x_{2j}) = 1, 1 \leq j \leq \frac{k-1}{2}$$
$$\psi(u^{(2)}x_1) = 2$$
$$\psi(u^{(j)}x_{j-1}) = 1, 3 \leq j \leq k.$$

We now have $\psi(0) = \psi(1) = \psi(2) = \frac{k}{2}(n+2) - 1, \psi(3) = \frac{k}{2}(n+2)$. Therefore, $|\psi(i) - \psi(j)| \leq 1$ for $0 \leq i < j \leq 3$. Hence $\psi$ is 4-TEPC labeling.

**Case 4.4:** $n \equiv 3$ (mod 4).

Define $\psi$ be an edge labeling function from $E(G)$ to $\{0, 1, 2, 3\}$:

$$\psi(u^{(j)}v_i^{(j)}) = 0, 1 \leq j \leq k, 1 \leq i \leq \frac{n-3}{4}$$
$$\psi(u^{(j)}v_i^{(j)}) = 1, 1 \leq j \leq k, \frac{n+1}{4} \leq i \leq \frac{n-1}{2}$$
$$\psi(u^{(j)}v_i^{(j)}) = 2, 1 \leq j \leq k, \frac{n+1}{2} \leq i \leq \frac{3n-1}{4}$$
$$\psi(u^{(j)}v_i^{(j)}) = 3, 1 \leq j \leq k, \frac{3n+3}{4} \leq i \leq n$$
$$\psi(u^{(4j-3)}x_{4j-3}) = 0, 1 \leq j \leq \frac{k+1}{4}$$
$$\psi(u^{(2j)}x_{2j}) = 0, 1 \leq j \leq \frac{k-1}{2}$$
$$\psi(u^{(4j-1)}x_{4j-1}) = 3, 1 \leq j \leq \frac{k-3}{4}$$
$$\psi(u^{(2j)}x_{2j-1}) = 1, 1 \leq j \leq \frac{k-1}{2}$$
$$\psi(u^{(2j+1)}x_{2j}) = 2, 1 \leq j \leq \frac{k-1}{2}.$$

We now have $\psi(0) = \psi(1) = \psi(2) = \frac{k(n+2)-1}{2}, \psi(3) = \frac{k(n+2)-3}{2}$. Therefore, $|\psi(i) - \psi(j)| \leq 1$ for $0 \leq i < j \leq 3$. Hence $\psi$ is 4-TEPC labeling. This completes the proof:

Graphs in Figures 1 to 4 show examples for case 1 , case 2, case 3, and case 4 respectively.

**Theorem 2** The graph $K_{1,n}$ is not 4-TEPC for $n \equiv 1$ (mod 4) or $n \equiv 2$ (mod 4).

**Proof**. Let $V(K_{1,n}) = \{u, v_i, 1 \leq i \leq n\}$ and $E(K_{1,n}) = \{uv_i, 1 \leq i \leq n\}$. Assume $n \equiv 1$ (mod 4), $n = 4t + 1$. If $K_{1,4t+1}$ is 4-TEPC, then we must have $\psi(i) = 2t + 1$ for three numbers from $\{0, 1, 2, 3\}$and fourth one $\psi(i) = 2t$. But this is impossible since each possible 4-TEPC labeling, $\psi(uv_i) = \psi(v_i)$ and $\psi(u) = 0$. Therefore just $\psi(0)$ has odd number and $\psi(1), \psi(2)$ and $\psi(3)$ must be even number, this is a contradiction. The proof for $n \equiv 2$ (mod 4) is similar as above. Hence, this completes the proof.

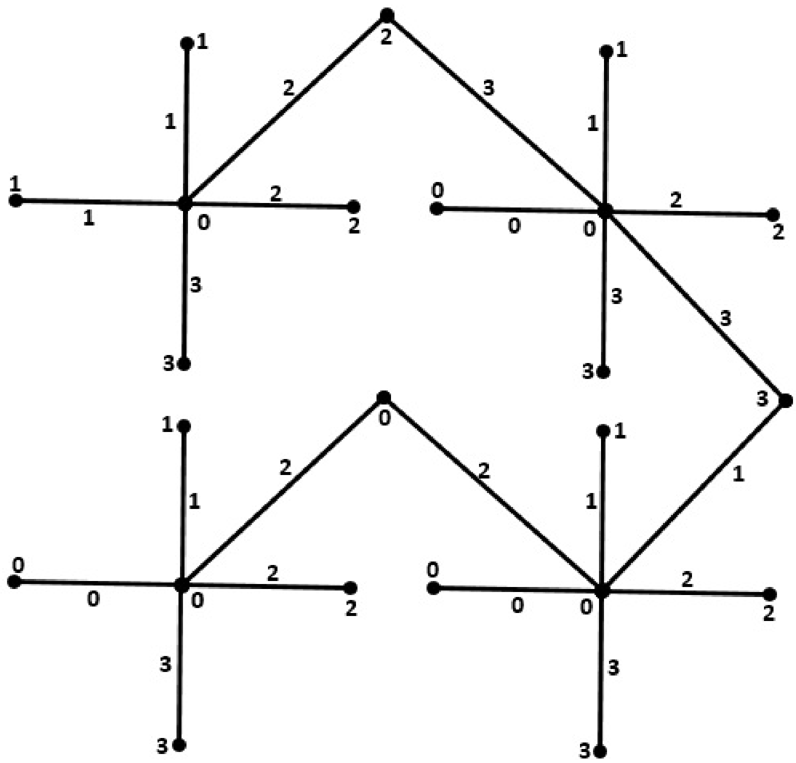

Figure 1. The graph $< K_{1,4}^{(1)}, K_{1,4}^{(2)}, K_{1,4}^{(3)}, K_{1,4}^{(4)} >$ and its 4-TEPC labeling

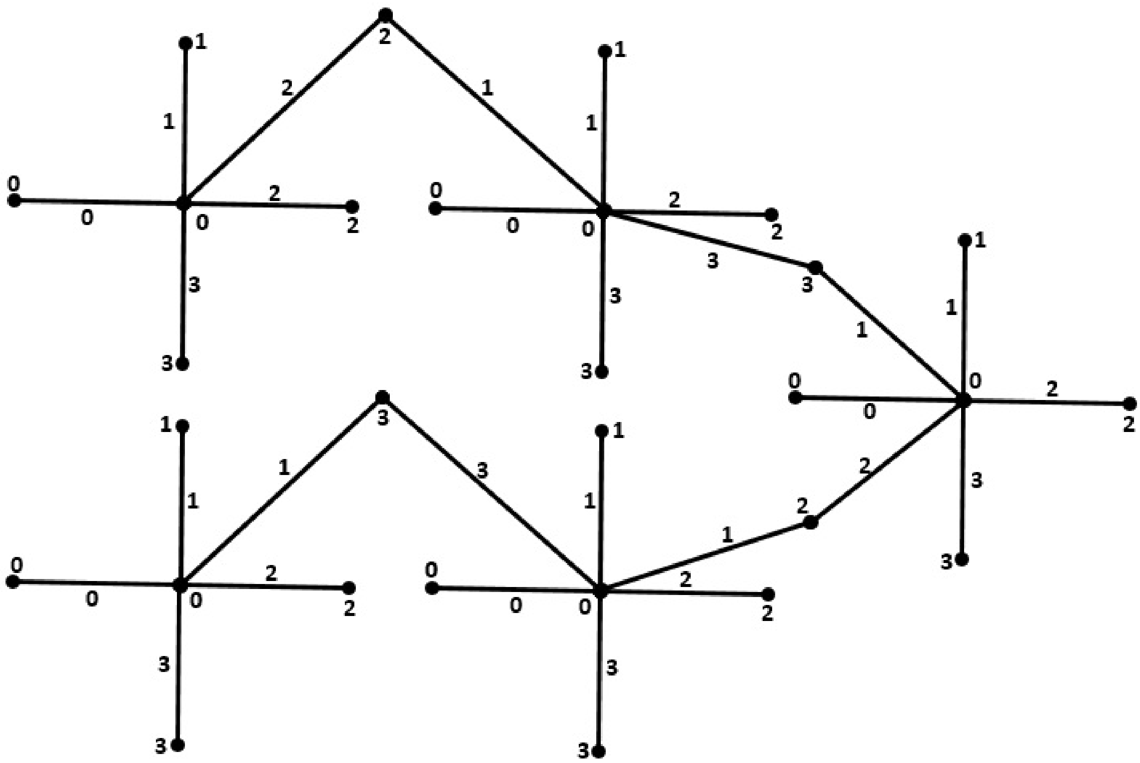

Figure 2. The graph $< K_{1,4}^{(1)}, K_{1,4}^{(2)}, K_{1,4}^{(3)}, K_{1,4}^{(4)}, K_{1,4}^{(5)} >$ and its 4-TEPC labeling

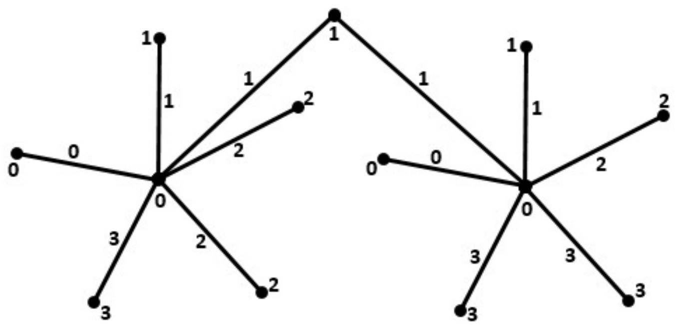

Figure 3. The graph $< K_{1,5}^{(1)}, K_{1,5}^{(2)} >$ and its 4-TEPC labeling

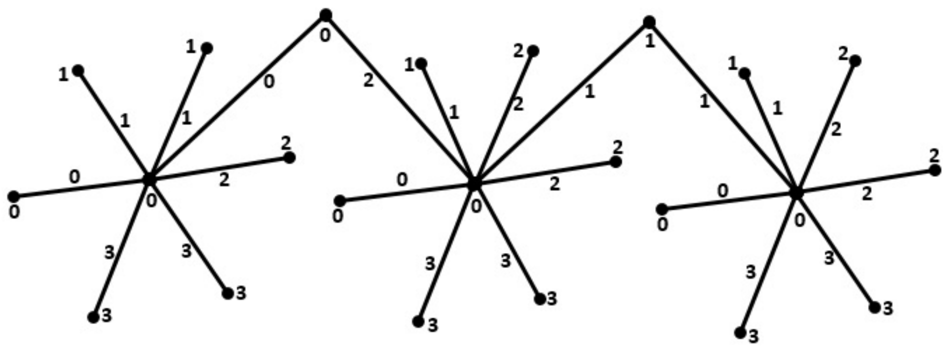

Figure 4. The graph $< K_{1,6}^{(1)}, K_{1,6}^{(2)}, K_{1,6}^{(3)} >$ and its 4-TEPC labeling

## 3. CONCLUSION

We completely determined the 4-total edge product cordial labeling for some star related graphs. The labeling pattern is elucidated using illustrations. Investigate 4-total edge product cordial labeling graph of other standard graphs such complete graph, wheel related graphs (friendship, helm, closed helm, fan, double fan, web, flower, and others), and study the 5-, 6- and in general, k-total edge product cordial labeling of some star related graphs are an open area of research.

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