International Journal of Electrical and Computer Engineering (IJECE)
Vol. 13, No. 1, February 2023, pp. 116~124
ISSN: 2088-8708, DOI: 10.11591/ijece.v13i1.pp116-124

Analysis of fractional order systems using newton iteration-based approximation technique

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Article Info

Article history:

Received Oct 31, 2021
Revised Aug 6, 2022
Accepted Aug 19, 2022

Keywords:

Fractional order plant
Integer order approximation
Nitisha-Pragya-Carlson technique
Oustaloup technique

ABSTRACT

Fractional differential equations play a major role in expressing mathematically the real-world problems as they help attain good fit to the experimental data. It is also known that fractional order controllers are more flexible than integer order controllers. But when it comes to the numerical approximation of fractional order functions inaccuracies arise if the conversion technique is not chosen properly. So, when a fractional order plant model is approximated to an integer order system, it is required that the approximated model be accurate, as the overall system performance is based on the estimated integer order model. Nitisha-Pragya-Carlson (NPC) is a recent approximation technique proposed in 2018 to derive the rational approximation of fractional order differ-integrators. In this paper, three fractional order plant models having fractional powers 3.1, 1.25 and 1.3 is analyzed in frequency domain in terms of magnitude and phase response. The performance of approximated third and second order NPC based integer model is studied and compared with the integer models developed using other existing technique. The approximation error is calculated by comparing the frequency response of the developed models with the ideal response. It has been found that in all the three examples NPC based models are very much close to the ideal values. Hence proving the efficacy of NPC technique in approximation of fractional order systems.

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1. INTRODUCTION

It is well known fact that fractional calculus plays an important role in two major areas, one is to frame a precise mathematical expression of any physical processes and the second one is designing a controller for processes industries. This is because with the help of fractional calculus an exact adaptation of the physical system can be modelled for simulation and analysis purposes and an accurate controller can be designed. Worldwide researchers have been able to explore fractional behavior in almost all branches of science, engineering, business and management [1]–[7]. The fractional order mathematical models have derivatives and integrals with fractional powers and it is not easy to implement such a model. Therefore, conversion to integer order system is done using different approximation techniques which are based on continued fraction expansion, least squares, alternate placement of poles and zeros, Newton iterations, optimization algorithms and so on [8]–[15]. The resulting integer order transfer functions depends on the technique used. It is usually seen that to achieve desired accuracy the highest power of approximated overall integer order system is very large. Such a large order system is not realizable practically. So, to reduce the order of the system reduction techniques are applied [16]–[20], [21]–[26]. In this paper the authors have presented a detailed analysis of approximating fractional order plant using the most recent Nitisha-Pragya-Carlson (NPC) technique developed in 2018 [15]. Three plants having fractional powers 3.1, 1.25 and 1.3 is analyzed in frequency domain. The NPC technique is a modified version of the Carlson technique which is based on Newton iterations [14]. The advantage of NPC technique is that it can be applied in any desired frequency range which was actually a drawback in Carlson method.

The paper consists of four sections: introduction is covered in section 1. The NPC formula and the different order reduction technique used in this paper is briefed section 2. The frequency domain analysis of three fractional order plants is detailed in section 3. Section 4 concludes the paper.

2. METHOD

The iterative formula for solving fractional order function $F(x) = a^{1/n}$ was given by Carlson and Halijak [14] as (1):

$$F(x) = \mathrm{a}^{\frac{1}{n}} = x\left[\frac{(n-1)x^n+(n+1)a}{(n+1)x^n+(n-1)a}\right] \tag{1}$$

where a is real variable and $n \in N$. The fractional order integrator was obtained after replacing $'a'$ by $\frac{1}{s}$ in (1). In this method, it is not possible to choose the frequency range in which the approximation is to be developed. But on observation it is found that the frequency responses of the approximation are constructed with centre frequency 1 rad/s.

A more generalized approximation for fractional order function proposed by NPC is an algorithm given by the recursive formula for $F(x) = a^{\pm 1/n}$ as (2):

$$F(x) = x_i = x_{(i-1)}\left[\frac{(\pm n-1)x_{(i-1)}^{\pm n}+(\pm n+1)a}{(\pm n+1)x_{(i-1)}^{\pm n}+(\pm n-1)a}\right] \tag{2}$$

where $i = 1,2,3$ ... denotes iterations. The first value x_0 for the frequency range $[R_1 R_2]$ rad/s, is given as (3):

$$x_0 = R_c^{\pm\frac{1}{n}} \tag{3}$$

where $R_c = \sqrt{R_1 R_2}$. R_1 is the lower frequency and R_2 is the upper frequency. Replacing the real variable *'a'* by the complex variable *'s'* the fractional order function becomes:

$$F(s) = s^{\pm\frac{1}{n}} \tag{4}$$

in the specified frequency range.

The approximation for fractional order differs-integrator is:

a. Fractional order differentiator $s^{1/n}$
   - First iterate:

$$x_1 = R_c^{\frac{1}{n}}\left[\frac{(n-1)R_c+(n+1)s}{(n+1)R_c+(n-1)s}\right] \tag{5}$$

   - Second iterate:

$$x_2 = x_1\left[\frac{(n-1)x_1^n+(n+1)s}{(n+1)x_1^n+(n-1)s}\right] \tag{6}$$

b. Fractional order integrator $\frac{1}{s^{1/n}}$
   - First iterate:

$$x_1 = \left(\frac{1}{R_c}\right)^{\frac{1}{n}}\left[\frac{(-n-1)\frac{1}{R_c}+(-n+1)s}{(-n+1)\frac{1}{R_c}+(-n-1)s}\right] \tag{7}$$

   - Second iterate:

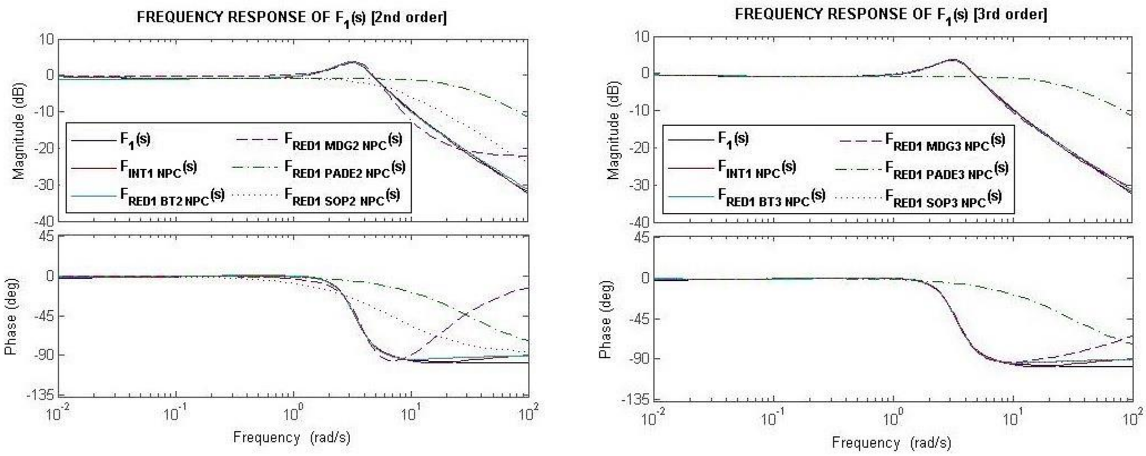
$$x_2 = x_1 \left[\frac{(-n-1)\left(\frac{1}{x_1}\right)^n + (-n+1)s}{(-n+1)\left(\frac{1}{x_1}\right)^n + (-n-1)s} \right] \quad (8)$$

3. RESULTS AND DISCUSSION

This section covers the analysis done for three fractional order plant models. For each case an equivalent approximated integer system of the fractional order plant model is developed by applying the NPC formula to all the fractional terms in the model. As the developed model is of higher order, it is reduced to second and third orders using the following approximation techniques: i) balanced truncation (BT) method, ii) matched DC gain (MDG) method, iii) moment matching (MM) (also known as Padé approximation (Pade)) method, and iv) sub-optimum (SOP) method.

A detailed description of how these reduction methods can be applied to integer approximated fractional order models is explained in [17]. All the lower order models are compared by plotting their frequency responses and calculating maximum magnitude and phase errors in the frequency range $[10^{-2}\ 10^{2}]$ rad/s. To know the maximum magnitude and phase errors it is required that the approximated models be compared with the actual response. The actual magnitude and phase values of a fractional order term for any specific frequency ω rad/s is found out as: i) actual magnitude =+/- (order of the fractional term)*20log ω dB and ii) actual phase =+/- (order of the fractional term)*90 degree.

Thus, the reference frequency response for each example is plotted and the plots of the approximated 2nd and 3rd order models are compared with the actual response. This is done to show the accuracy of approximated lower order models, as for hardware realization it is preferred that the models be of lower order. The reference plots for each of the three examples are shown as $F_1(s)$, $F_2(s)$ and $F_3(s)$ in Figures 1(a) and 1(b), Figures 2(a) and 2(b), and Figures 3(a) and 3(b) respectively.



(a) (b)

Figure 1. Frequency response of $F_1(s), F_{INT1\_NPC}(s)$ and lower order models, (a) second-order models of $F_1(s)$ and (b) third-order models of $F_1(s)$

3.1. Example 1

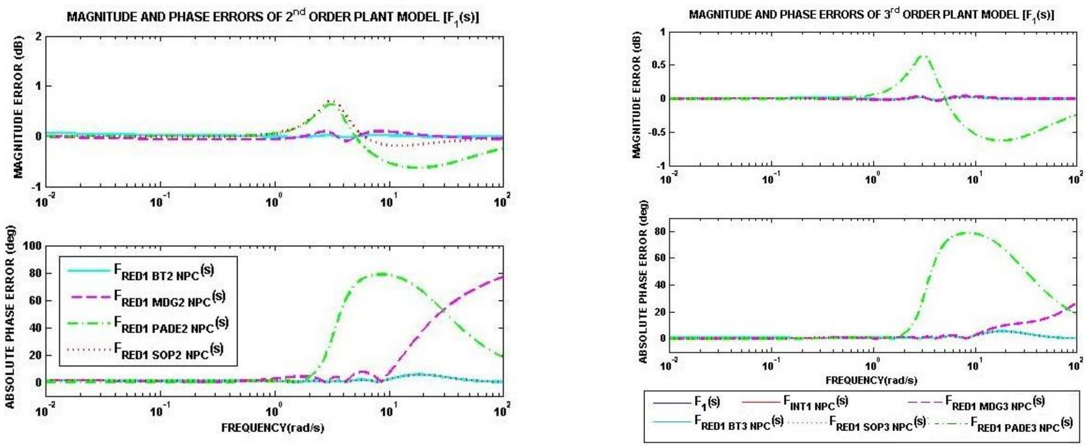
A first fractional order plant [27] considered is:

$$F_1(s) = \frac{3s^2 + 10s^{0.9} + 0.2}{0.8s^{3.1} + 3s^2 + 0.5s^{1.8} + 11s^{0.9} + 0.2} \quad (9)$$

This fractional order plant model has fractional powers 3.1, 1.8 and 0.9. To apply NPC formula the terms $s^{3.1}$ and $s^{1.8}$ are split into two as $s^{3.1} = s^3 \times s^{0.1}$ and $s^{1.8} = s \times s^{0.8}$ respectively.

The integer approximation function $\left(F_{INT1\_NPC}(s)\right)$ of $F_1(s)$ using NPC method is of order 19. The results obtained after applying further model reduction techniques are investigated. $F_{INT1\_NPC}(s)$ is reduced to second and third order models. The models reduced using BT method are termed $F_{RED1\_BT2\_NPC}(s)$ and $F_{RED1\_BT3\_NPC}(s)$ for second and third orders respectively. The models reduced using matched DC gain method are represented by $F_{RED1\_MDG2\_NPC}(s)$ and $F_{RED1\_MDG3\_NPC}(s)$ for second and third orders respectively. The models reduced using moment matching (Pade approximation) method are given as $F_{RED1\_MM2\_NPC}(s)$ and $F_{RED1\_MM3\_NPC}(s)$ for second and third orders respectively, and the models obtained using sub-optimum method are named $F_{RED1\_SOP2\_NPC}(s)$ and $F_{RED1\_SOP3\_NPC}(s)$ for second and third orders respectively.

The characteristics of lower order models obtained using NPC approximation are analyzed by plotting their frequency responses and magnitude and phase errors. The lower value of frequency is 10^{-2} rad/s and upper frequency is 10^{2} rad/s. Figures 1(a) and 1(b) show the frequency response plots of $\left(F_1(s)\right)$, its integer order approximation $\left(F_{INT1\_NPC}(s)\right)$ and second and third order models. Figures 2(a) and 2(b) shows the error plots. The Tables 1 to 4 show maximum magnitude and phase errors of the models developed using NPC and Oustaloup methods respectively. Comparing the second order models it can be seen that NPC model reduced using BT method shows better results and in third order models $F_{RED1\_BT2\_OUST}(s)$ has least maximum magnitude error where as maximum phase error is less for $F_{RED1\_SPO2\_NPC}(s)$. Thus, for implementation purposes NPC approximated second order model reduced using BT method can be used.



(a) (b)

Figure 2. Magnitude and phase error plots of lower order models of $F_1(s)$ approximated using NPC method, (a) second order models of $F_1(s)$ and (b) third order models of $F_1(s)$

Table 1. Errors of lower order models of $F_1(s)$ approximated using NPC method

Plant model	Method	Lower order models	Lower Order (3rd/2nd)	Error (decibel)	Error (degree)
$F_1(s)$	BT	$F_{RED1\_BT3\_NPC}(s)$	3	**0.0228**	5.5130
	MDG	$F_{RED1\_MDG3\_NPC}(s)$		0.0355	26.9703
	MM	$F_{RED1\_MM3\_NPC}(s)$		0.6338	78.7815
	SOP	$F_{RED1\_SOP3\_NPC}(s)$		0.0259	**5.2664**
	BT	$F_{RED1\_BT2\_NPC}(s)$	2	**0.0668**	**5.4687**
	MDG	$F_{RED1\_MDG2\_NPC}(s)$		0.0988	77.1589
	MM	$F_{RED1\_MM2\_NPC}(s)$		0.6338	78.7722
	SOP	$F_{RED1\_SOP2\_NPC}(s)$		0.7051	45.3131

Table 2. Errors of lower order models of $F_1(s)$ approximated using oustaloup method

Plant model	Method	Lower order models	Lower Order (3rd/2nd)	Error (decibel)	Error (degree)
$F_1(s)$	BT	$F_{RED1\_BT3\_OUST}(s)$	3	**0.0198**	7.1343
	MDG	$F_{RED1\_MDG3\_OUST}(s)$		0.0419	49.0691
	MM	$F_{RED1\_MM3\_OUST}(s)$		0.6342	74.3342
	SOP	$F_{RED1\_SOP3\_OUST}(s)$		0.0389	**6.8591**
	BT	$F_{RED1\_BT2\_OUST}(s)$	2	**0.0691**	**7.1273**
	MDG	$F_{RED1\_MDG2\_OUST}(s)$		0.1200	88.7147
	MM	$F_{RED1\_MM2\_OUST}(s)$		1.1232	44.7957
	SOP	$F_{RED1\_SOP2\_OUST}(s)$		0.7024	45.0358

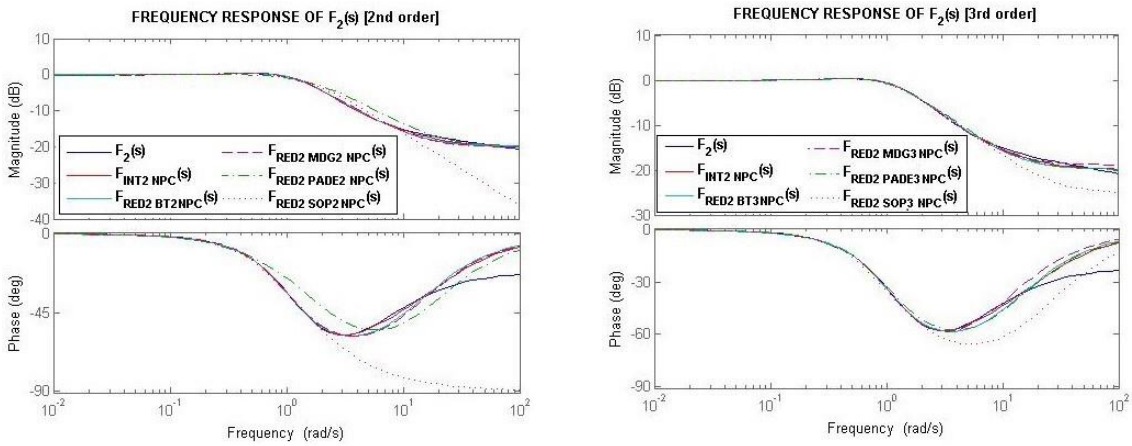
3.2. Example 2

A second fractional order plant [28] considered is:

$$F_2(s) = \frac{12.46s+64.47}{39.69s^{1.25}+12.46s+65.068} \tag{10}$$

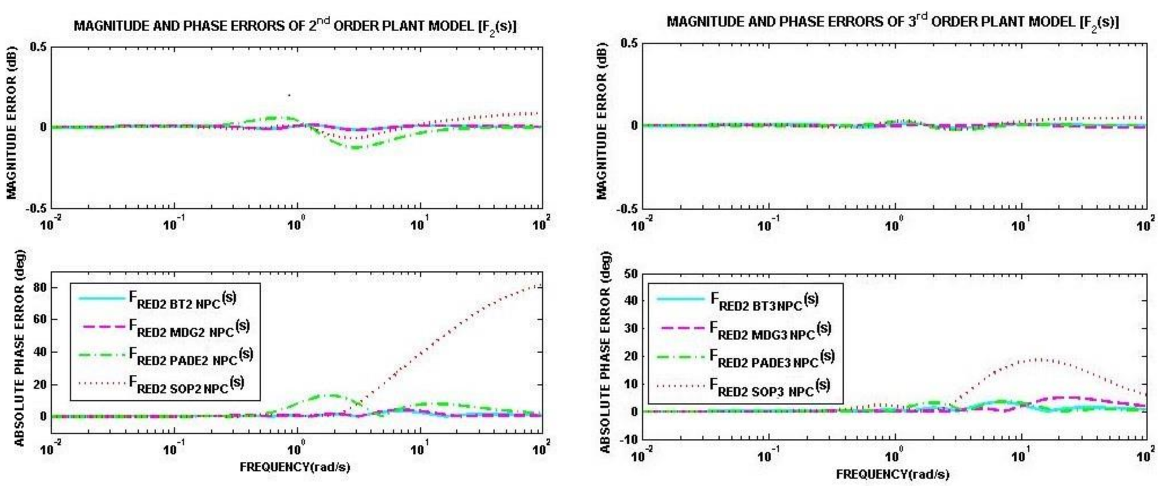
Similar analysis is also performed for $F_2(s)$. The expression of (10) has only one fractional order term $s^{0.25}$. The fractional order plant $F_2(s)$ is approximated using NPC technique. The order of the resulting integer order function, $F_{INT2\_NPC}(s)$ is 6. $F_{INT2\_NPC}(s)$ is reduced to 2nd and 3rd order models. Following terminology is used to represent the second and third order reduced models: i) BT method: $F_{RED2\_BT2\_NPC}(s)$ and $F_{RED2\_BT3\_NPC}(s)$; ii) matched DC gain method: $F_{RED2\_MDG2\_NPC}(s)$ and $F_{RED2\_MDG3\_NPC}(s)$; iii) moment matching (Pade approximation method): $F_{RED2\_MM2\_NPC}(s)$ and $F_{RED2\_MM3\_NPC}(s)$; and iv) sub-optimum method: $F_{RED2\_SOP2\_NPC}(s)$ and $F_{RED2\_SOP3\_NPC}(s)$.

The plots showing frequency versus magnitude and phase characteristics of 2nd and 3rd order models compared with $F_{INT2\_NPC}(s)$ and also with the original f-o plant $F_2(s)$ are given in Figures 3(a) and 3(b) respectively. The corresponding magnitude and phase errors for the 2nd and 3rd order reduced models are shown in Figures 4(a) and 4(b).



(a) (b)

Figure 3. Frequency response of $F_2(s), F_{INT2\_NPC}(s)$ and lower order models, (a) second-order models of $F_2(s)$ and (b) third-order models of $F_2(s)$



(a) (b)

Figure 4. Magnitude and phase error plots of lower order models of $F_2(s)$ approximated using NPC method (a) second order models of $F_2(s)$ and (b) third order models of $F_2(s)$

Table 3. Errors of lower order models of $F_2(s)$ approximated using NPC method

Plant model	Method	Lower order models	Lower Order (3rd/2nd)	Error (decibel)	Error (degree)
$F_2(s)$	BT	$F_{RED2\_BT3\_NPC}(s)$	3	0.0126	**3.3619**
	MDG	$F_{RED2\_MDG3\_NPC}(s)$		**0.0080**	5.0607
	MM	$F_{RED2\_MM3\_NPC}(s)$		0.0231	3.6180
	SOP	$F_{RED2\_SOP3\_NPC}(s)$		0.0461	18.6233
	BT	$F_{RED2\_BT2\_NPC}(s)$	2	**0.0124**	**3.3619**
	MDG	$F_{RED2\_MDG2\_NPC}(s)$		0.0146	4.1414
	MM	$F_{RED2\_MM2\_NPC}(s)$		0.0580	13.0245
	SOP	$F_{RED2\_SOP2\_NPC}(s)$		0.0871	81.8434

Table 4. Errors of lower order models of $F_2(s)$ approximated using oustaloup method

Plant model	Method	Lower order models	Lower Order (3rd/2nd)	Error (decibel)	Error (degree)
$F_2(s)$	BT	$F_{RED2\_BT3\_OUST}(s)$	3	0.0174	**4.0593**
	MDG	$F_{RED2\_MDG3\_OUST}(s)$		**0.0126**	4.6817
	MM	$F_{RED2\_MM3\_OUST}(s)$		0.0688	23.2661
	SOP	$F_{RED2\_SOP3\_OUST}(s)$		0.0160	5.1395
	BT	$F_{RED2\_BT2\_OUST}(s)$	2	0.0597	20.4803
	MDG	$F_{RED2\_MDG2\_OUST}(s)$		**0.0169**	**15.5358**
	MM	$F_{RED2\_MM2\_OUST}(s)$		0.0543	31.9633
	SOP	$F_{RED2\_SOP2\_OUST}(s)$		0.0789	68.1738

3.3. Example 3

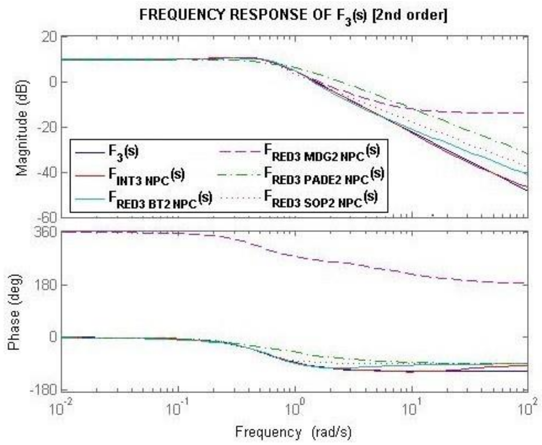
A third fractional order plant [27] considered is:

$$F_3(s) = \frac{3}{2s^{1.3}+1} \tag{11}$$

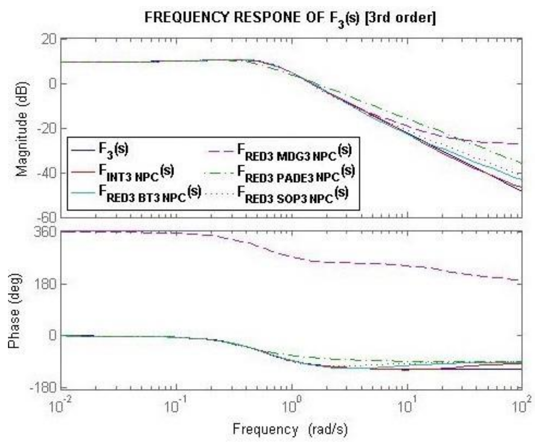
Using NPC approximation technique $F_3(s)$ is approximated to 6th order function $F_{INT3\_NPC}(s)$. $F_{INT3\_NPC}(s)$ is reduced to lower order models. The models reduced using BT method are $F_{RED3\_BT2\_NPC}(s)$ and $F_{RED3\_BT3\_NPC}(s)$; using matched DC gain method are $F_{RED3\_MDG2\_NPC}(s)$ and $F_{RED3\_MDG3\_NPC}(s)$; using moment matching method (Pade approximation method) are $F_{RED3\_MM2\_NPC}(s)$ and $F_{RED3\_MM3\_NPC}(s)$ and using sub-optimum method are $F_{RED3\_SOP2\_NPC}(s)$ and $F_{RED3\_SOP3\_NPC}(s)$ for second and third order respectively.

The performance of NPC approximated model and reduced second and third order models is analyzed by plotting their frequency responses and magnitude and phase errors in the frequency range $[10^{-2}\ 10^{2}]$ rad/s. Figures 5(a) and 5(b) show the frequency response plot of the original fractional order model $(F_3(s))$, its integer order approximation $(F_{INT3\,\mathrm{NPC}}(s))$ and second and third order models respectively and the magnitude and phase errors for the 2nd and 3rd order models are plotted in Figures 6(a) and 6(b) respectively. The Tables 5 and 6 show maximum magnitude and phase errors of the models developed using NPC and Oustaloup methods respectively. Comparing the second-order models as shown in Figure 5 (a), it is observed that the model obtained using MDG technique shows better performance in terms of magnitude response, and for the 3rd order models as shown in Figure 5(b) maximum magnitude and maximum phase error is least for the model obtained using BT technique.

The three examples considered here are fractional order plant models. The direct implementation, simulation and analysis of a model based on fractional calculus is not very easy, therefore its integer approximation is developed. The other thing which attracts our attention in the above examples is that, four different reduction techniques are used to generate 2nd and 3rd order models. This is because the parameters of the approximation algorithms are chosen in such a way that the basic characteristics of the approximated integer order model should be very close to the ideal characteristics. And it is usually found that, for accurate models the highest power of the polynomial is very high. So, it has to be reduced to lower orders. Of the four order reduction methods used in our work, BT and MDG methods are based on Hankel singular value decomposition whereas MM and SOP methods are based on Pade approximation and Powel's algorithm respectively. Now, to check the loss of accuracy on reducing the order of the model, each 2nd and 3rd order models are compared with a reference graph which is the ideal response characteristics of the fractional order plant model. The verification of the accuracy is done in the frequency domain. In all the three examples it can be seen that the 2nd and 3rd order models developed using BT and MDG methods give a good frequency response plot and have lower error values compared to models developed using MM and SOP methods.

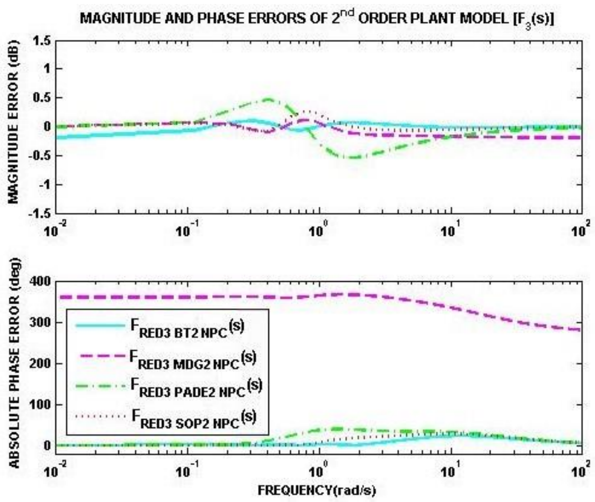


(a)

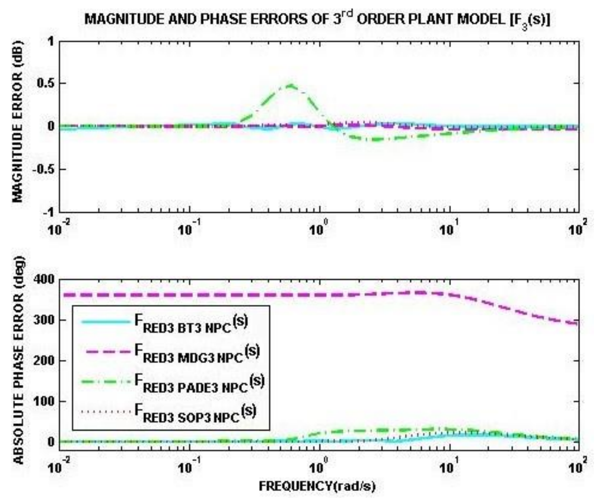


(b)

Figure 5. Frequency response of $F_3(s), F_{INT3\_NPC}(s)$ and lower order models, (a) second-order models of $F_3(s)$ and (b) third-order models of $F_3(s)$



(a)



(b)

Figure 6. Magnitude and phase error plots of lower order models of $F_3(s)$ approximated using NPC method (a) second order models of $F_3(s)$ and (b) third order models of $F_3(s)$

Table 5. Errors of lower order models of $F_3(s)$ approximated using NPC method

Plant model	Method	Lower order models	Lower Order (3rd/2nd)	Error (decibel)	Error (degree)
$F_3(s)$	BT	$F_{RED3\_BT3\_NPC}(s)$	3	**0.0125**	**22.8640**
	MDG	$F_{RED3\_MDG3\_NPC}(s)$		0.1080	366.31
	MM	$F_{RED3\_MM3\_NPC}(s)$		0.4670	39.1161
	SOP	$F_{RED3\_SOP3\_NPC}(s)$		0.2598	28.0871
	BT	$F_{RED3\_BT2\_NPC}(s)$	2	0.0319	**16.6193**
	MDG	$F_{RED3\_MDG2\_NPC}(s)$		**0.0115**	365.10
	MM	$F_{RED3\_MM2\_NPC}(s)$		0.4723	30.0869
	SOP	$F_{RED3\_SOP2\_NPC}(s)$		0.0445	22.2963

Table 6. Errors of lower order models of $F_3(s)$ approximated using Oustaloup method

Plant model	Method	Lower order models	Lower Order (3rd/2nd)	Error (decibel)	Error (degree)
$F_3(s)$	BT	$F_{RED3\_BT3\_OUST}(s)$	3	0.0332	**24.2367**
	MDG	$F_{RED3\_MDG3\_OUST}(s)$		**0.0057**	362.7609
	MM	$F_{RED3\_MM3\_OUST}(s)$		0.5724	33.8774
	SOP	$F_{RED3\_SOP3\_OUST}(s)$		0.0371	25.9575
	BT	$F_{RED3\_BT2\_OUST}(s)$	2	**0.0545**	**25.4714**
	MDG	$F_{RED3\_MDG2\_OUST}(s)$		0.1134	365.8793
	MM	$F_{RED3\_MM2\_OUST}(s)$		0.3706	58.6144
	SOP	$F_{RED3\_SOP2\_OUST}(s)$		0.2382	28.4658

4. CONCLUSION

The application of fractional order controllers and fractional filters is growing at a fast pace. This is because it provides an additional degree of freedom for fine tuning. Also, mathematical modelling of any physical process is best done using fractional differ-integrators. However, it is very important to apply a correct approximation technique to obtain an accurate integer order function of fractional systems. In this paper interger approximation of three fractional order plant models are developed using NPC technique and the results thus obtained are compared with the actual values. The error plots are shown and the maximum magnitude and phase error values for each lower order models are tabulated. The results are also compared with the integer models developed using Oustaloup technique. In all the three examples it is found that the frequency versus magnitude and phase response characteristics of the models developed using NPC method give better results and are also very much close to the ideal characteristics.

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