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An efficient application of particle swarm optimization in model predictive control of constrained two-tank system

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ABSTRACT

Despite all the model predictive control (MPC) based solution advantages such as a guarantee of stability, the main disadvantage such as an exponential growth of the number of the polyhedral region by increasing the prediction horizon exists. This causes the increment in computation complexity of control law. In this paper, we present the efficiency of particle swarm optimization (PSO) in optimal control of a two-tank system modeled as piecewise affine. The solution of the constrained final time-optimal control problem (CFTOC) is derived, and then the PSO algorithm is used to reduce the computational complexity of the control law and set the physical parameters of the system to improve performance simultaneously. On the other hand, a new combined algorithm based on PSO is going to be used to reduce the complexity of explicit MPC-based solution CFTOC of the two-tank system; consequently, that the number of polyhedral is minimized, and system performance is more desirable simultaneously. The proposed algorithm is applied in simulation and our desired subjects are reached. The number of control law polyhedral reduces from 42 to 10 and the liquid height in both tanks reaches the desired certain value in 189 seconds. Search time and apply control law in 25 seconds.

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1. INTRODUCTION

According to the system modeling, it is possible to introduce piecewise affine (PWA) systems, a particular hybrid system class. It is described by partitioning the vast input-state space into Polyhedron regions and assigning a PWA equation to each of these regions. Discrete-time PWA systems are a very effective tool for modeling most hybrid systems [1]. These systems have established themselves as a powerful class for identification and approximation of generic nonlinear systems by multi-linearization at equilibrium [2]. A practical method in designing controllers of nonlinear systems is optimal control concepts in constrained and non-constrained processes by linear discrete-time models in the form of state space. Study [3] presents a constrained discrete model predictive control strategy for an inside greenhouse temperature. Given what has been said about the modeling advantages of most systems based on the PWA class, recently, there has been strong interest in computing the optimal form-package controller for PWA systems. These problems became known as the constrained final time-optimal control (CFTOC) [4], [5]. The most important methods of analysis of this problem are multi-parameter programming, receding horizon control (RHC), or model predictive control (MPC). MPC is an effective way to deal with constrained control problems and has found many applications and advances in industry and academic research. This method requires that the next position of the variables be predicted based on the current position, the controller input, and the process model. In other words, the sequence of control inputs that optimize the objective function is computed and applied to the process. This control concept is called the MPC [6].

Boiler turbine process control was performed with three manipulated variables and three controlled variables using MPC technique [7]. Optimal control sequence allows recalculation and feedback performance in MPC whenever a new measurement arrives; this is known as MPC. The stability of the control system and the fulfillment of conditions, constraints, and requirements during operation are ensured by the proper formulation of the objective function. RHC has the complexity and high volume of online computing related to optimizing and reducing system robustness due to the difference between the actual and MPC processes. Optimal methods based on the use of multi-parametric linear programming (MP-LP) and multi-parametric quadratic programming (MP-QP) were provided [8], [9].

The offline calculation of the optimal control rule for constrained discrete-time linear systems was performed using these methods. The resulting rules were made available as a PWA function on the polyhedrons. At present, explicit MPC techniques enable a standard method in controller design for nonlinear processes that are modeled in the PWA form, creating a substitute for intelligent controller design methods such as fuzzy logic and neural networks in high-performance applications [10]. Unfortunately, one of the main problems is the increasing complexity of the control rule obtained by increasing the prediction horizon and its effect on system performance. On the other hand, it is necessary to increase the prediction horizon for the system's optimal performance. For linear systems with parametric uncertainty by the Lyapunov function, the PWA controller is designed with low complexity [11]. Effective representation and approximation are provided by in-depth learning to MPC of linear time invariant (LTI) systems. Theoretically, at least neurons and hidden layers are considered [12]. A nonlinear robust MPC with input-dependent perturbations and states and uncertainty is presented [13]. The MPC algorithm with PWA control rules is presented for discrete-time linear systems in the presence of finite perturbation [14]. The online burden of computational of the linear MPC can be transferred to offline by using multi-parameter programming, which is named explicit MPC [15]. A flexibility algorithm is proposed to reduce the calculation volume in [15] that the designer can balance time and storage complexities.

This is done by hash tables and the associated hash functions. Two modified controllers instead of the standard MP-QP are used [16] to reduce the complexity of the multi-parameter programming of MPC. The problem of reducing the complexity of explicit MPC for linear systems is considered by PWA employing separating functions [17]. A semi-continuous PWA model based on the optimal control method for the nonlinear system is proposed [18]. First, the nonlinear system is approximated by multi-linear subsystems, and then the subsystems are incorporated into a PWA system and formulated as an optimal control problem. A computational method for optimizing and controlling a two-tank system with three control valves is presented [19]. Easy implementation and optimizability of complex objective functions, with many local minimums are the main advantages of particle swarm optimization (PSO). Furthermore, PSO can be used to search significantly wider range of candidate solutions. The dynamics of the tank system are nonlinear. The linear model is considered, and the parameters are adjusted so that the difference between the actual system and the model is minimized by solving the optimal control problem. PSO has been used to solve the problem of constrained optimization [20]. Self-adaptive of particle swarm optimization (SAPSO) is recommended to increase PSO performance. Theoretically, the convergence of the method has been investigated. Considerable interest has recently been generated to use PSO in optimization and engineering problems [21]. A new algorithm that combines model predictive control with the PSO is proposed for optimal control of constrained direct current (DC-DC) power system modeled as piecewise affine [22]. A modern strategy based on model reference command shaping (MCRS) for an overhead crane, with dual pendulum component impacts. The existing MRCS calculation has been making strides with the PSO calculation to diminish plan complexity and guarantee that synchronous alteration can be done with the feedback controller [23]. The proposed hybrid algorithm incorporates social interaction and elitism mechanisms from PSO into manta ray foraging optimization (MRFO) strategy [24]. A control strategy based on the self-tuning method and synchronous reference frame with proportional integral (PI) regulator is proposed to achieve optimal power quality in an independent microgrid. Particle swarm optimization is used to adjust the parameters of the PI controller, which ensures flexible performance and superior power quality [25]. The tuning method used for proportional integral derivative (PID) with derivative filter controller for liquid slosh system by implementing PSO algorithm [26]. PSO algorithm is applied to optimize the surface grinding process parameter in both rough and final grinding conditions [27]. This paper is organized as follows; first, the CFTOC of PWA system is briefly described. Having introduced the two-tank optimal control in section 3, in section 4, the application of PSO for the solution of the expressed problem is discussed, and finally, the simulation results and conclusion are presented.

2. CONSTRAINED FINAL TIME-OPTIMAL CONTROL (CFTOC) PROBLEM AND SOLUTION

We will do with the constrained PWA systems as (1) [4], [10]:

$$x(t+1) = f_{PWA}\left(x(t), u(t)\right) \coloneqq A_i x(t) + B_i u(t) + f_i \quad \begin{pmatrix} x(t) \\ u(t) \end{pmatrix} \in D_i \tag{1}$$

where t≥0, the domain $D \coloneqq \bigcup_{i=1}^{N_D} D_i$ of $f_{pwa}(.,.)$ is a non-empty compact set in $R^{n_x+n_u}$, $N_D<\infty$ is the number of system dynamics and $D \coloneqq \bigcup_{i=1}^{N_D} D_i$ denotes a polyhedral partition of the domain D. i.e., the closure of D_i is $\bar{D_i} \coloneqq \left\{\begin{pmatrix} x \\ u \end{pmatrix} \in R^{n_x+n_u} \middle| {D_i}^x x + {D_i}^u u \le {D_i}^0\right\}$ and $\mathrm{int}(D_i) \cap \mathrm{int}(D_j) = \emptyset\ \forall i \neq j$. The CFTOC problem for piecewise affine system (1) in the form (2a), (2b) and (2c) [4], [22]:

$$J_T^*\left(x(0)\right) = \min_{U_T} J_T(x(0), U_T) \tag{2a}$$

$$\text{subject to } \begin{cases} x(t+1) = f_{PWA}(x(0), u(t) \\ x(t) \in \chi^f \end{cases} \tag{2b}$$

$$J_T(x(0), U_T) = l_T\left(x(T)\right) + \sum_{t=0}^{T-1} l(x(t), u(t)) \tag{2c}$$

where $J_T(.,.)$ is the cost function, l(.,.) the stage cost, $l_T(.)$ the final penalty, U_T optimization variable described as the input sequence $U_T = \{u(t)\}_{t=0}^{T-1}$, T<∞ receding horizon and χf is a compact terminal target set in R^{n_x}. If the solution of the CFTOC problem is not unique, $u_T^*\left(x(0)\right) = \{u^*(t)\}_{t=0}^{T-1}$ decides one realization from the set of the conceivable optimizers.

The CFTOC problem determine a set of initial state and feasible inputs as $\chi_T \subset R^{n_x}(x(0) \in \chi_T)$, $U_{T-t} \subset R^{n_u}(u(t) \in U_{T-t}, t = 0, \dots, T-1)$ respectively. The explicit closed form solution can be expressed as $u^*(t)\colon \chi_T \to U_{T-t}$, t=0, …, T-1. The considered system is PWA (1), and the cost is based on 1, ∞ norm [22].

$$l\left(x(t), u(t)\right) \coloneqq \|Qx(t)\|_p + \|Ru(t)\|_p \tag{3a}$$

$$l\left(x(T)\right) \coloneqq \|Px(T)\|_p \tag{3b}$$

where $\|.\|_p$ with p={1,∞} speak to the standard vector standard 1,∞. The arrangement of (2) with aforementioned limitations is a time varying PWA work of the starting state x(0)∈ $\mathcal{P}_i$.

$$u^*(t) = \mu_{PWA}(x(0), t) = K_{T-t,i}x(0) + L_{T-t,i} \tag{4}$$

where $t = 0, \dots, T, \{\mathcal{P}_i\}_{i=1}^{N_p}$ is the polyhedral partition of a set of feasible state x(0), $\chi_T = \bigcup_{i=1}^{N_p} \mathcal{P}_i$, with the closure of $\mathcal{P}_i$ stated as $\bar{\mathcal{P}}_i = \{x \in R^{n_x} | \mathcal{P}_i^x x \le \mathcal{P}_i^o\}$ [2].

If a receding horizon control strategy is used for closed-loop, the control rule is expressed as time-variant PWA state-feedback of the form [4]:

$$\mu_{RH}\left(x(t)\right) \coloneqq K_{T,i}x(t) + L_{T,i}\ \ if\ x(t) \in \mathcal{P}_i \tag{5}$$

where i=1,…, Np and for t≥0, $u^*(t) = \mu_{RH}\left(x(t)\right)$. The CFTOC problem can be presented and solved for any selection of P, Q, R, albeit here it is assumed that the parameters T, Q, R, P and χ^f are selected by the following assumptions [3]. To avoid additional control actions in steering states to the origin (equilibrium point), matrices R, Q are required to have a full column rank.

3. OPTIMAL CONTROL OF TWO-TANK SYSTEM

The two-tank [28] shown in Figure 1 is a basic benchmark model to investigate and analyze the control issues for PWA system. The tanks are filled by pump acting on tank 1, continuously manipulated from 0 up to a maximum flow Q1. A switching valve V_{12} controls the flow between the tanks. This valve is assumed to be either completely opened or closed (V_{12}=0 or 1 respectively). The V_{N2} manual valve controls the nominal outflow of the second tank. It is assumed in the simulations that the manual valves, V_{N1} is always closed and that V_{N2} is open. The liquid levels to be controlled are denoted by h_1, h_2 for each tank respectively.

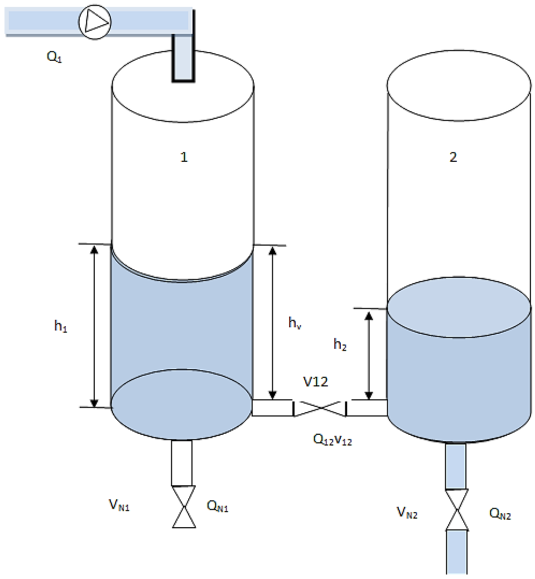

Figure 1. Two-tank system schematic

It consists of two tanks that connected to each other. We assumed that: i) the valves behavior is linear, ii) the initial volume of liquid in tanks is zero, and iii) the inflow of liquid to the first tank is constant and has the maximum value. The liquid volume of tank 1 is defined by a time varying equation as (6a) and (6b):

$$\mathrm{V1(t) = V0{,}1 + (Q1 - Q1{,}2) \times t} \tag{6a}$$

$$V1(t) = A1 \times h1(t) \tag{6b}$$

where $V_{0,1}$ is initial volume of liquid in tank 1and Q_1, $Q_{1,2}$ are inflow and outflow liquid of tank1 ($Q_{1,2}$ can be defined as inflow of liquid to tank 2), A_1 and h_1 are base area and the time varying height of liquid in tank1 respectively. The (6) can be repeated for the tank 2 with similar definition as (7a) and (7b):

$$V2(t) = V0{,}2 + (Q1{,}2 - Q2) \times t \tag{7a}$$

$$V2(t) = A2 \times h2(t) \tag{7b}$$

By combining (6), (7),

$$h_1(t) = \frac{1}{A_1}\left(V_{0,1} + \left(Q_1 - Q_{1,2}\right) \times t\right) \tag{8a}$$

$$h_2(t) = \frac{1}{A_2}\left(V_{0,2} + \left(Q_{1,2} - Q_2\right) \times t\right) \tag{8b}$$

The system is expressed as a discrete time model with a sample time (T_s=10s) by (9).

$$\begin{cases} h_1(k+1) = h_1(k) + \frac{T_s}{A_1}(Q_1(k) - k_{12}V_{12}^*(h_1(k) - h_2(k)) \\ h_2(k+1) = h_2(k) + \frac{T_s}{A_2}(k_{12}V_{12}^*(h_1(k) - h_2(k)) - k_{N2}V_{N2}^*h_2(k) \end{cases} \tag{9}$$

This model can be defined as a piecewise affine system of the shape (1), with four modes, depicted as takes after [28]:
- First Mode: V_{12}^* open, $h_1 \geq h_v$
- Second Mode: V_{12}^* open, $h_1 \leq h_v$
- Third Mode: V_{12}^* closed, $h_1 \geq h_v$
- Fourth Mode: V_{12}^* closed, $h_1 \leq h_v$

For example, the first mode of the system matrices is:

$$A_1 = \begin{bmatrix} 0.9542 & -0.0393 \\ 0.0941 & 0.9670 \end{bmatrix}\ B_1 = \begin{bmatrix} 0.0699 & 0 \\ 0 & 0 \end{bmatrix}\ C_1 = [1\ 0]\ D_1 = [0]\ and\ f_1 = \begin{bmatrix} 0.0164 \\ -0.0164 \end{bmatrix}$$

The CFTOC problem of the presented PWA system is solved by MPT [29] based on MPC for the prediction horizon=3, norm =1, $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, R= $10^{-5} \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and the explicit PWA control law has 78 polyhedral regions as shown in Figure 2 and the performance of the closed-loop system from a given initial condition is presented in Figure 3.

Using PSO algorithm, the considered purposes are going to be fulfilled simultaneously:

– The number of polyhedral of explicit MPC-based control law is minimized to reduce the complexity,
– The liquid reaches a certain height in tanks in a short time and desirable manner.

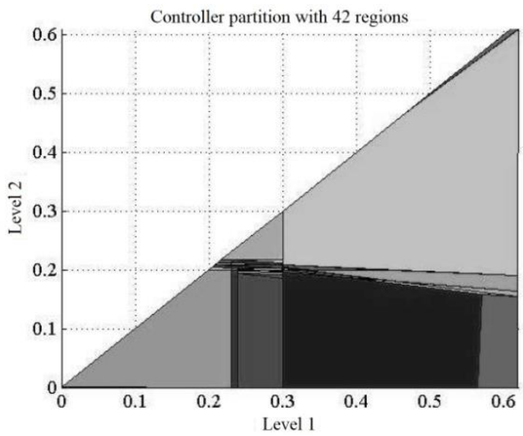

Figure 2. Controller partitions

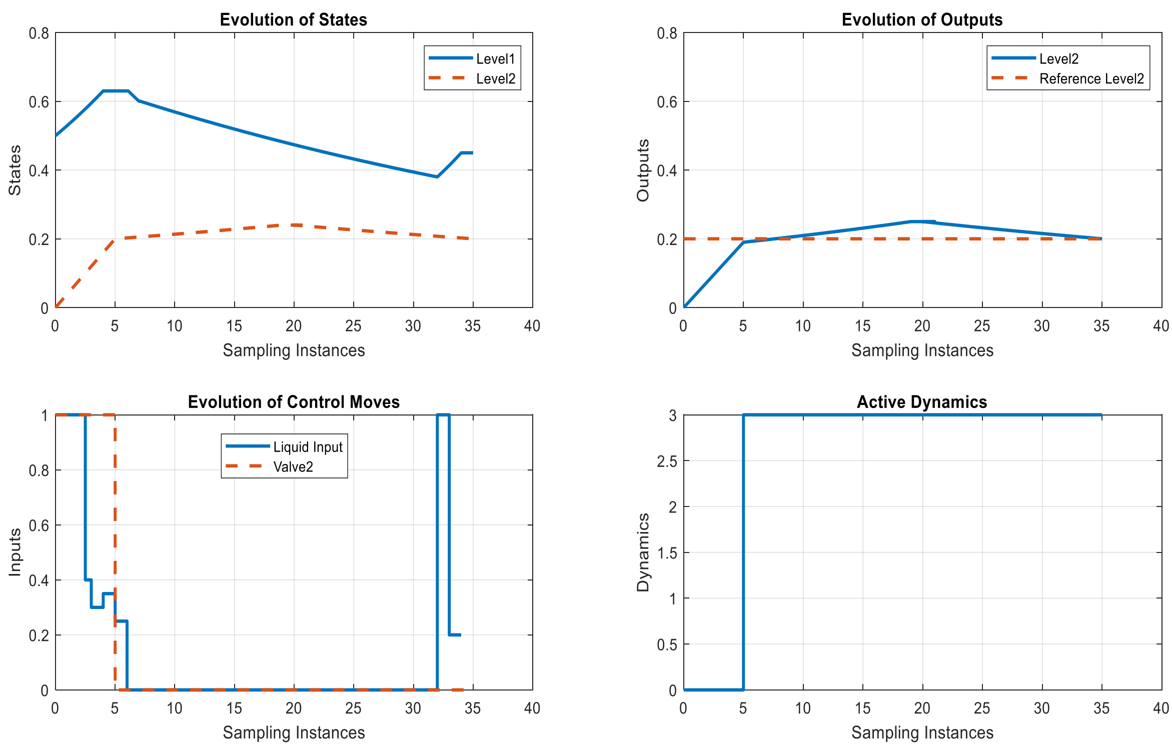

Figure 3. Closed loop system performance

4. PSO ALGORITHM APPLICATION FOR SOLVING DEFINED PROBLEM

Before long a short time later, a brief audit of PSO calculation is displayed, and after that PSO application is explored to fathom the defined problem. The PSO could be a worldwide exploratory optimization strategy, to begin with, proposed by Kennedy and Eberhart in 1995 [30]. It is made of swarm insights and is based on inquiring about the behavior of feathered creatures and the angle of development. The motion of each molecule is impacted by its best-known nearby position and is additionally coordinated to the best-known positions within the look space, which are upgraded by other particles to discover superior positions. Agreeing to Figure 4, the premise of the strategies is as follows: “Each molecule can appear with its current speed and position, the foremost ideal position of each individual, and the foremost ideal position of the environment” [22], [30]. By selecting the beginning populace X_i, V_i, the speed and position of each molecule around the look space alter concurring to correspondence (10) [30]:

$$X_i = [x_{1,i}\ x_{i,2} \cdots x_{n,i}]$$

$$V_i = [v_{1,i}\ v_{i,2} \cdots v_{n,i}]$$

$$V_{id}^{k+1} = V_{id}^{k} + c_1 \times r_1^k \times \left(V_{id}^{Lbest}\right) + c_2 \times r_2^k \times \left(V_{id}^{Gbest}\right) \tag{10a}$$

$$X_{id}^{k+1} = X_{id}^{k} + V_{id}^{k} \tag{10b}$$

$$V_{id}^{Lbest} = pbest_{id}^{k} - X_{id}^{k} \tag{10c}$$

$$V_{id}^{Gbest} = gbest_{id}^{k} - X_{id}^{k} \tag{10d}$$

where in this uniformity, V_{id}^k and X_{id}^k independently stand for the speed of the molecule *i* at its *k* times and the d-dimension amount of its position; $pbest_{id}^k$ speaks to the d-dimension amount of the person *i* at its most ideal position at its *k* times; $pbest_{id}^k$ is the amount d-dimension of the swarm at its most ideal position. To avoid a molecule from being distant from the look space, the speed of the made molecule is restricted in each course between $-v_{dmax}$ and v_{dmax} [22]. In the event that the number of v_{dmax} is as well tall, the arrangement is distant from the most excellent arrangement, something else; the ideal arrangement will be nearby. c_1 and c_2 appear the speed digit, which alters the length when flying to the biggest molecule within the entire gather and the ideal single molecule. In the event that the number of v_{dmax} is as well little, the molecule is likely distant from the target field; in case the shape is as well huge, the molecule may fly all of a sudden to the target field or past the target field.

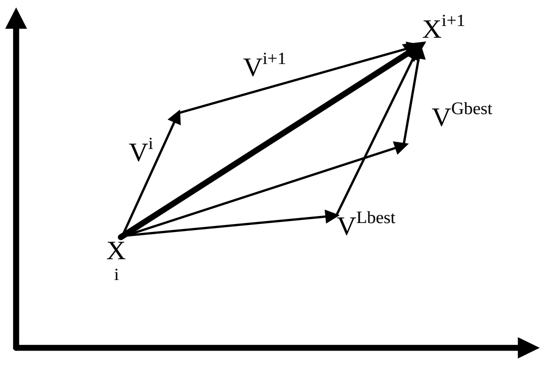

Figure 4. The basis of evolutionary PSO algorithm

Reasonable figures for c_1 and c_2 can control the molecule flight speed and will not be a halfway ideal arrangement. c_1 is more often than not rise to c_2, and they are rising to 2. r_1 and r_2 speak to an irregular story and 0-1 could be an arbitrary number. As said some time recently, our modern point is utilizing PSO for complexity lessening of unequivocal MPC-based control law by diminishment the number of its polyhedral and setting the physical parameter of framework to move forward the framework execution at the same time. In this manner, the taking after objective work has been defined: *Fitness-Function =Number of polyhedral + Output specifications*.

Where yield determinations are decided as a summation of operational determinations such as settling time, overshoot, undershoot, steady-state deviation, time consistent, and so forward. Presently, the unequivocal controller gotten in past segment as a portion of PSO ought to be considered and the taking after unused execution list is being defined:

$$\mathrm{J}_{\mathrm{new}_{\mathrm{T}}}^{*} = Min(Fitness - Function) \ = Min[number\ of\ polyhedrals\ of\ \mu_{RH}(x(0) + Output\ speciication] \tag{11a}$$

$$\mathrm{S.T}\begin{cases} \mathrm{u}^{*} = \mu_{\mathrm{RH}}\big(\mathrm{x}(\mathrm{t})\big) = \mathrm{K}_{\mathrm{T,i}}\mathrm{x}(\mathrm{t}) + \mathrm{L}_{\mathrm{T,i}} \ \text{ if } \mathrm{x}(\mathrm{t}) \in \mathcal{P}_{\mathrm{i}} \\ \text{Output} = \sum \text{desired output characteristic} \end{cases} \tag{11b}$$

It is used according to the flowchart shown in Figure 5.

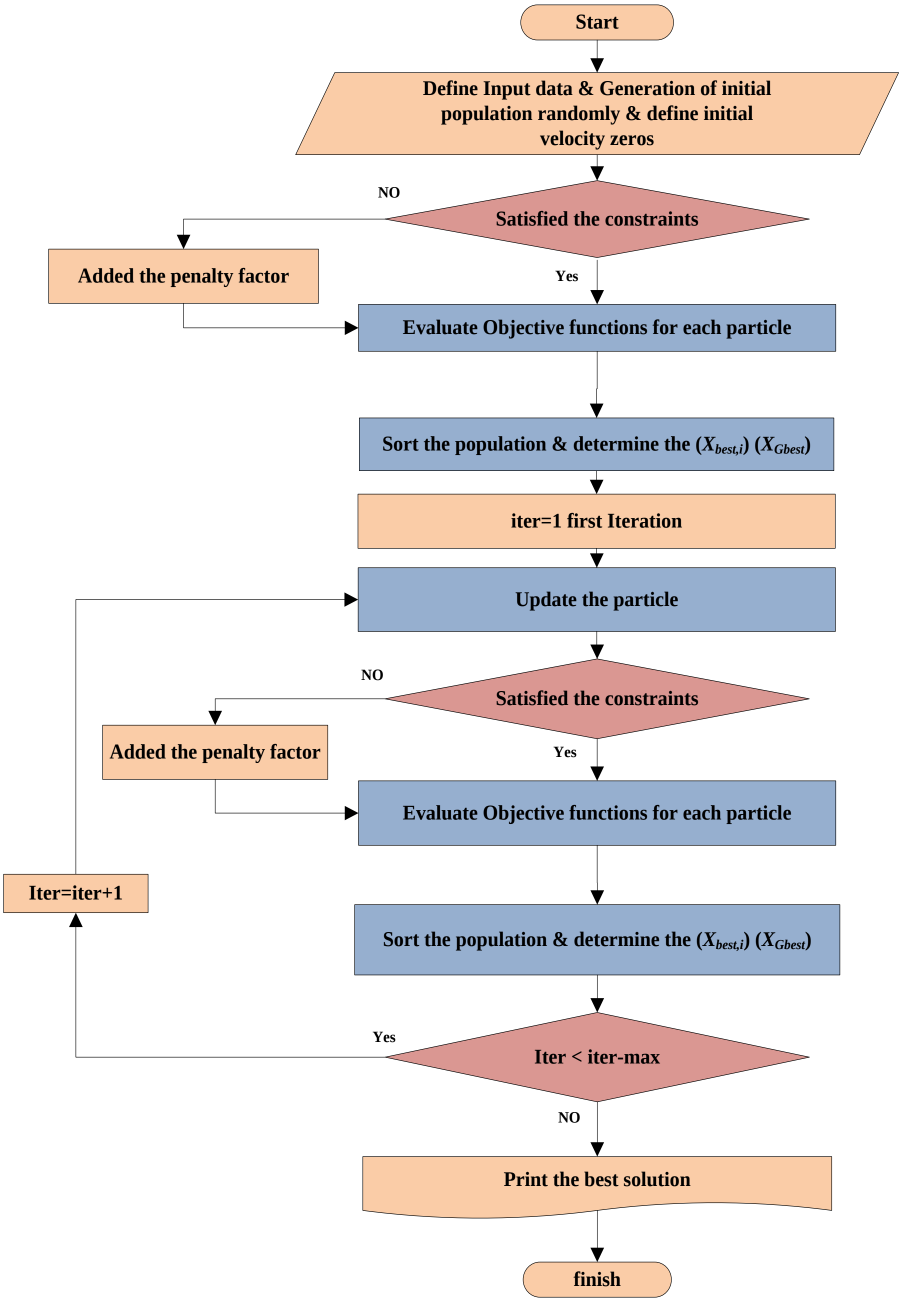

Figure 5. Flowchart of PSO algorithm application

The issue is illuminated as:
- The required information such as time step, fluid stature, and calculation parameters such as the number of population and a number of the cycles is considered.
- The initial population is made as follows [22]:

$$X_i = [\text{height of valve1, cross section of valves, maximum height, base area of each tank}]$$

$$\text{initial population} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_{N\ pop} \end{bmatrix}$$

- The yield determinations of the framework are measured based on introductory parameters, at that point, the control law (gotten by MPT) is connected, and the number of polyhedral districts is calculated. In the long-run objective work is characterized as [22]:

$$Fitness - Function = Number\ of\ polyhedral + output\ specifications.$$

where we consider yield determinations can be expected as yield details ≜ settling time + certain stature of liquid. The best arrangement among the entire population is decided, and the population is overhauled based on (10a) and (10b).
- For the predefined number of iterations, step 3 is done repetitively.

The convergence conditions are explored, and at last the leading arrangement is appeared within the output. Then number of iterations is 15, the number of populaces is 5, $c_1 = c_2 = 2$ is considered. The ideal parameters are compiled within the Table 1.

Table 1. The optimal parameters

Sampling time	Height of valve 1	Cross-section of valves	Maximum height	Area of each tank	Num P	Time(S)
10 s	0.1 m	1.00E-05	0.5	0.001 m^2	10	189

T is the required time to reach a certain height of the liquid. As presented, the number of control law polyhedral reduces from 42 to 10. In Figure 6, the output flow variation of the first tank is shown. These changes are linear. After 100 seconds, output flow increases to 10 cubic meters. In Figure 7, the liquid height in the first tank is shown. These changes are nonlinear and incremental. After 90 seconds, the liquid height of the first tank reaches 50 meters. Figure 8 shows the output flow variation of the second tank. These changes are linear. At the last moment, there was a change in the function of the output valve.

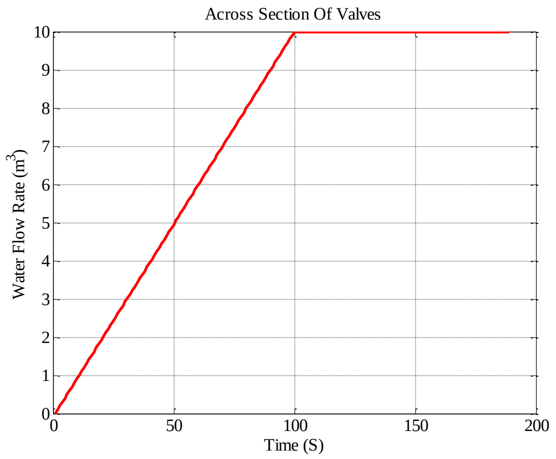

Figure 6. The output flow variation of tank 1

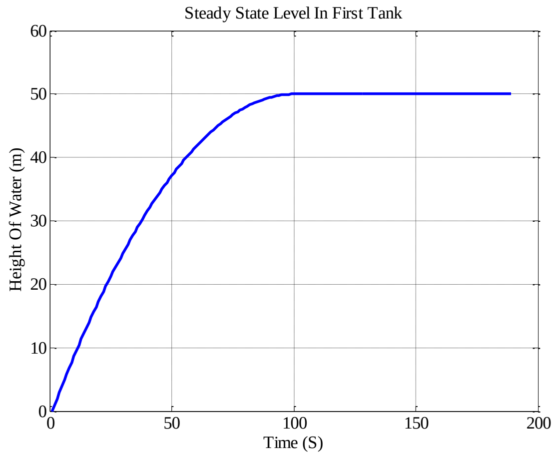

Figure 7. The liquid height in tank 1

Figure 9 shows the liquid height changes of the second tank. As you can see, this change is nonlinear and incremental. The height of the liquid in both tanks reached a certain value in 189 seconds.

Figure 10 shows the unique convergence of the objective function using the PSO algorithm. It converges after 4 iterations. Figure 11 shows the controller partition with 10 regions. Concurring to the comes about, it can be concluded that by tuning the physical parameters utilizing PSO, the number of polyhedral of MPC-based control law is minimized; in this way, the related complexity of arrangement to CFTOC will be diminished together with the change of yield details at the same time so that the fluid tallness in both tanks reach craved esteem in 189 seconds.

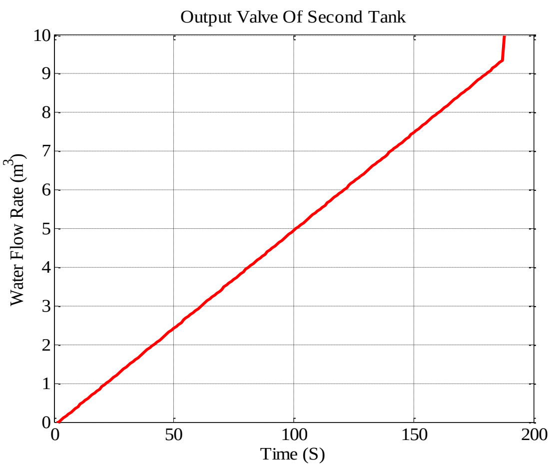

Figure 8. The output flow variation of tank 2

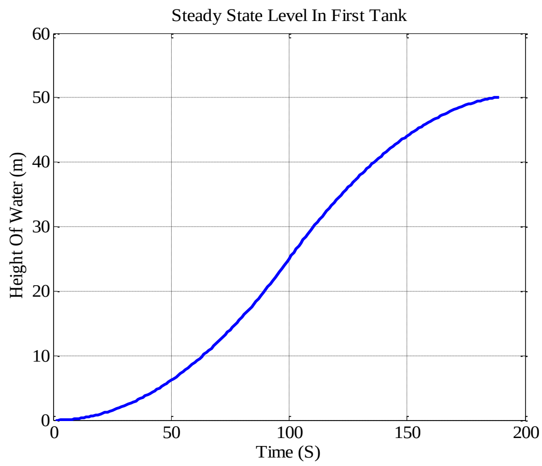

Figure 9. The liquid height in tank 2

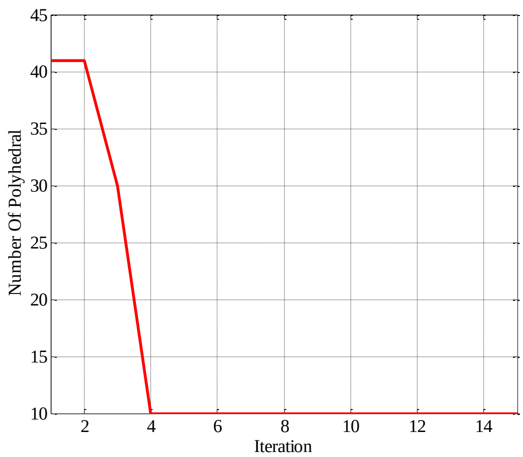

Figure 10. Convergence diagram of objective function

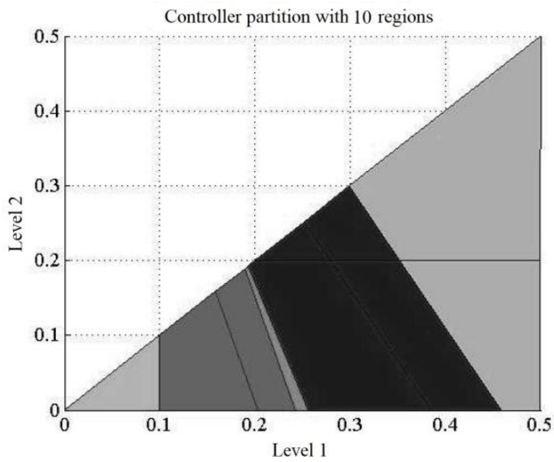

Figure 11. Controller with minimum partitions

5. CONCLUSION

Several analytical methods have been used for the CFTOC solution. Their main disadvantage is the computational complexity of the solution; therefore, the problem can be considered as NP-hard. The meta-heuristic algorithm is used to solve NP-hard optimization problems that have strategies to escape the local optimal solution and are applied to a wide range of problems. In general, the developments of meta-heuristic methods are done with the study and inspiration of optimization in nature, such as particle swarm optimization. By using PSO and an appropriate definition of the objective function, the complexity of the MPC-based solution of CFTOC was reduced and the system performance was improved simultaneously. The most massive advantage of the recommended method is that if the mentioned purposes were not in one direction, we can define a multi-objective function to fulfill aims. According to the simulation results, it is demonstrated that the number of polyhedral and the dependent complexity of CFTOC solution are reduced, the system performance such as reaching the liquid height at a certain time is desirable and the obtained steady-state error reaches zero. The number of control law polyhedral reduces from 42 to 10. The liquid height in both tanks reaches desired value in 189 seconds.

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