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New approach on global optimization problems based on meta-heuristic algorithm and quasi-Newton method

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ABSTRACT

This paper presents an innovative approach in finding an optimal solution of multimodal and multivariable function for global optimization problems that involve complex and inefficient second derivatives. Artificial bees colony (ABC) algorithm possessed good exploration search, but the major weakness at its exploitation stage. The proposed algorithms improved the weakness of ABC algorithm by hybridizing with the most effective gradient based methods which are Davidon-Flecher-Powell (DFP) and Broyden-Flecher-Goldfarb-Shanno (BFGS) algorithms. Its distinguished features include maximizing the employment of possible information related to the objective function obtained at previous iterations. The proposed algorithms have been tested on a large set of benchmark global optimization problems and it has shown a satisfactory computational behaviour and it has succeeded in enhancing the algorithm to obtain the solution for global optimization problems.

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1. INTRODUCTION

The collective behaviors of self-organized, independent and decentralized swarms, for example flocks of birds, schools of fishes, branches of organic plans and social insects with self-organization and division of labor as the core components are categorized as swarm intelligence (SI). Self-organization is characterized by low-level components interactions whereas the division of labor included simultaneous tasks perform by specialized individuals in cooperation. Both of these fundamental concepts are essential and adequate properties to attain swarm intelligence behaviors [1]–[4]. Artificial bee colony (ABC) algorithm is one of the newest swarm intelligence based algorithms which was proposed by Karaboga [5]. ABC algorithm simulates the foraging behavior of honey bees. It is very direct, robust and population-based. In the ABC algorithm, the process begins with some initial solutions (population), which try to improve them toward optimal solutions by distributing the bees colony into two separate groups which are called employed and unemployed bees.

There are three essential components of forage selection namely food sources, employed and unemployed. Food sources are defined as the value function and depend on numerous factors (adjacency to the nest, aggregation of the energy, and the ease of extracting this energy). Employed bees are linked with a particular food source at which they are being occupied at. They bring with them vital information regarding particular sources such as the distance and direction from the nest, the profitability of the source, and then share this information with a certain probability. Meanwhile, the unemployed bees are continuously searching and looking out for a food source to be exploited [5]. The number of food sources is directly proportional to the number of employed bees since for every food source there is only one employed bee. The computation of fitness value is dependent on the quality of the food source and is associated with its position. The process of searching for food source by the swarm of bees reflects the process involved in finding the optimal solution [6].

There are two imperative mechanisms in the ABC algorithm which are the exploitation and exploration process. The exploitation process occurs whenever the employed bees approach and work on the food sources. Once the employed bees determine the amounts of nectar of the food sources, the onlooker bees will proceed to the value of source that has the highest probability and then calculates the amount of nectar. The end of exploitation process is indicated by exhaustion of food sources. In the meantime, exploration process started when bee scouts are deployed to search and find new food sources randomly [7], [8]. The improvement and modifications to this algorithm have been determined is numerous literature such as [9]–[19]. Nevertheless, there are some inefficiencies occurred in terms of exploitation and convergence ability associated with ABC algorithm. Based on previous studies ABC algorithm performance is superior during exploration stage however, the performance is weaker during exploitation stage. At the biggest scale level, the exploration process is related to the ability of exploring and seeking for global optimum independently while the exploitation involves the distribution of the acquired knowledge to search for better solutions. This fact is the main goal of this research, where in the exploitation phase a quasi-Newton scheme is introduced so that it can increase the exploitability of the ABC algorithm.

The rest of this paper is structured as follows. Section 2 introduces the problem formulation of the global optimization. Section 3 describes the original ABC algorithm. Section 4 presents the proposed modification of ABC algorithm through manipulation of Davidon-Flecher-Powell (DFP) and Broyden-Flecher-Goldfarb-Shanno (BFGS) schematics, numerical results and discussion are discussed in section 5, and the conclusion of the work is summarized in section 6.

2. PROBLEM FORMULATION

Considering the problem of finding a global solution of,

$$\min_{x \in D} f(x) \tag{1}$$

where $f : D \subseteq \mathbb{R}^n \to \mathbb{R}$ is a continuous function, and D is closed and bounded region. In order to obtain the global minimum point, any unconstrained algorithm should obtain a point x^* such that,

$$f(x^*) \leq f(x), \quad \forall (x \in D) \tag{2}$$

In recent literature many algorithms have been proposed to solve the problem (1), see for example [20]–[22]. However, this paper focused on tackling the particular difficult case of problem (1) in which the evaluations of second derivatives are too complex and time consuming. In addition, the initial guess point to start the algorithm involves high level of difficulty to predict. Therefore, exploring and developing a more efficient method to resolve the problem (1) is a significant and valuable scientific research area.

3. ARTIFICIAL BEES COLONY ALGORITHM

The algorithm consists of four main parts [23]. The first part initialized the population of unemployed bees for exploration search and employed bees for the food source by using (3):

$$x_{ij} = x_j^{\min} + \alpha\Big(x_j^{\max} - x_j^{\min}\Big), \; i = 1, \cdots, n, j = 1, \cdots, D \tag{3}$$

where x_{ij} represents jth dimension of the ith employed bee. $x_j^{\max}$ is the upper boundary, $x_j^{\min}$ is the lower boundary of the food source position in dimension jth respectively. α represents the uniform random number within the interval [0, 1]. The maximum number of cycles (MNC) is utilized to control the number of iterations and it also acts as a termination criterion. In the second part, the fitness value will entice the onlooker bees to execute the exploitation search. A new candidate solution will be generated for each employed bee. The initial solution in the first part is replicated to a new solution $(v_i = x_i)$. Then, a uniform randomly chosen parameter j of the solution is revised by using (4):

$$v_{ij} = x_{ij} + \phi(x_{ij} - x_{kj}). \tag{4}$$

$i, k \in \{1, \cdots, n\}$, $j \in \{1, \cdots, D\}$ and $i \neq k$, j is randomly chosen parameter. x_k is a randomly chosen solution in the neighborhood of x_i and ϕ is the uniform random number within the interval $[0, 1]$. The fitness value of the solution can be determined by (5):

$$\mathsf{fit}_i = \begin{cases} \frac{1}{1+f_i} & \text{if} \quad f_i \geq 0 \\ 1+ \mid f_i \mid & \text{if} \quad f_i \leq 0 \end{cases} \tag{5}$$

where f_i is the objective value for ith candidate solution. If the current solution is superior than previous one then the candidate solution substitutes to the current solution and the abandonment is reset to zero, otherwise it will be incremented by one. In the third part, all information related to the nectar amount and respective position of food sources will be communicated by the employed bees to the onlooker bees. The onlookers will proceed to evaluates the nectar information from employed bees and choose the food source based on the highest probability which is dependent on the nectar amount by (6).

$$p_i = \frac{\mathsf{fit}_i}{\sum_{j=1}^{n} \mathsf{fit}_j}. \tag{6}$$

In the next step, the candidate solution is updated by using (4). If the current solution is better than the previous one then candidate solution substitutes the current solution and the abandonment is reset to zero, otherwise it will be incremented by one. In the fourth part, the employed bee will turn into a scout bee if the abandonment limit reached the predefined limit and look out for a new food source. Scout bee will then check all potential solutions for a predefined limits. The scout bee will generate the solution by using (3) and set the abandonment back to zero. Then scout bee turns into the employed bee and hence prevents stagnation of the algorithm. Based on the explanation above, the original ABC algorithm is given in algorithm 1.

Algorithm 1 Basic algorithm of ABC bees colony

Data: Initilization: the population of solution x_{ij} $i = 1, \cdots, n; j = 1, \cdots, D$; $\mathsf{trial} = 0$ is the non improvemnet number of the solution $x_{i,j}$ used for abondonment, maximum cycle number, limit = A food source which could not be improved through limit trial and abandoned by its employed bee.

Evaluate the population Set cycle = 1 **repeat**
- {Produce a new food source population for employed bees} **for** *i=1 to n* **do**
  - Produce a new food source v_i for employed bees of the food source x_i by using (4) and determine its quality Apply a greedy selection process between v_i and x_i and select the better one **if** x_i *does not improve* **then**
    - $\mathsf{trial} = \mathsf{trial}_i + 1$ Otherwise $\mathsf{trial}_i + 1 = 0$
  - **end**
- **end**
- Calculate the probability value p_i by using (6) for the solutions using fitness values by using (5) {Produce a new food source population for onlooker bees} Set $t = 0, i = 0$ **repeat**
  - **if** $random \leq p_i$ **then**
    - {Produce a new v_i food source by using (4) for onlooker bees} Apply a greedy selection process between v_i and x_i and select the better one **if** *Solution* x_i *does not improve* **then**
      - $\mathsf{trial}_i = \mathsf{trial}_i + 1$ Otherwise $trial_i + 1 = 0$ $t = t + 1$
    - **end**
  - **end**
- **until** $t = n$
- {Determine scout} **if** $\max(\mathit{trial}_i) > limit$ **then**
  - Replace x_i with a new randomly produces solution by using (3)
- **end**
- Memorize the best solution achieved so far $\mathsf{cycle} = \mathsf{cycle} + 1$

until {*cycle = Maximum cycle Number*}

4. ARTIFICAL BEES COLONY ALGORITHM ENHANCEMENT

Many attempts have been made in the exploitation process to enhance the performance of the ABC algorithm. In this section, the exploitation part is enhanced by using a quasi-Newton algorithm. The algorithm does not need explicit expressions of second partial derivatives. This is normally denoted as variable metric methods [24]–[27]. As implied by the term, the basic concept of this method is the classical Newton's method, whereby the new point is calculated.

$$x_{k+1} = x_k - H_k^{-1} g_k, \tag{7}$$

Where H is the Hessian matrix and g is the gradient of the objective function, respectively. The basic principle in the quasi-Newton algorithm is the search direction which is based on an $n \times n$ direction matrix B which supports the same objective as the inverse of the Hessian matrix in Newton's method. This matrix is computed by using the first-order derivative of $f(x)$ collected from the previous iterations. Moreover, as the number of iterations increased, B progressively achieving a more precise representation of the inverse of Hessian and for convex quadratic objective functions that become identically to the inverse of Hessian in $(n+1)$ iterations. The two most important schemes of quasi-Newton are DFP and BFGS schemes. They will be described in detail how it replaces the exploitation process in the ABC algorithm.

Based on the theoretical concept of Newton's method which locally approximates the problem (1) by using the quadratic function. The minimum point of quadratic approximation is utilized as the starting point for the subsequent iterations. This brings to next Newton's method iteration as given in (7). Generally, if the initial point is not sufficiently close to the optimal solution, then this method does not has the descent property. Nevertheless, the quasi-Newton method guarantees that the algorithm processes the descent property by amending the iteration:

$$x_{k+1} = x_k - \alpha_k B_k g_k, \tag{8}$$

where

$$B_k = \begin{cases} I_k & \text{for the steepest-descent algorithm} \\ H_k^{-1} & \text{for the Newton's method} \end{cases} \tag{9}$$

and B_k is an $n \times n$ real matrix that approximate the Hessian matrix. During constructing the approximation to the inverse of Hessian, objective function and gradient will be used solely. The B_{k+1} computation is achieved by an incremental update to B_k in the following subsection.

4.1. Davidon-Flecher-Powell scheme

This scheme was developed by Davidon in 1959 and modified by Fletcher and Powell in 1963. The algorithm guarantees that B_k to be positive definite for all k. However, for the case of larger non-quadratic problems, the algorithm tends to get stuck in certain problematic situations. This phenomenon is associated with B_k becoming nearly singular [20]. The scheme of DFP is given,

$$d_k = -B_k . g_k .$$

Updating the matrix B_k using (10):

$$B_{k+1} = B_k + U_k + C_k \tag{10}$$

where U_k and C_k are calculated by using

$$U_k = \frac{s.s_k^T}{s_k . y_k}; \quad C_k = \frac{-z_k . z_k}{y_k . z_k}$$

$$s_k = \alpha d_k; \quad y_k = g_{k+1} - g_k$$

and

$$g_{k+1} = \nabla f(x_{k+1}); \quad z_k = B_k . y_k .$$

4.1.1. The quasi-Newton with DFP scheme algorithm

The quasi-Newton with DFP Scheme algorithm is recorded. The next subsection is describing the refinement of the ABC Algorithm by introducing the DFP Scheme. In this work, the DFP scheme is used in the exploitation phase of the ABC algorithm. By doing this, the convergence of the original ABC algorithm can be improved. The quasi-Newton with DFP Scheme algorithm is represented in algorithm 2.

Algorithm 2 Quasi-Newton with DFP scheme algorithm

Data: $x_0 \in \mathbb{R}^n$, and tolerance, $\epsilon \geq 0$. Setting $\lambda_k = 2$, $\beta = 0.618$ and $\sigma = 0.8$.
Result: Minimizer x_k*
while $\left(\|d_k\| \leq \epsilon\right)$ **do**
  $\lambda_k = \max\{s, s\beta, s\beta^2, \cdots\}, s > 0, \beta \in (0,1), \sigma \in (0,1)$ **while** $\left(\left(f(x_k + \lambda_k d_k) - f(x_k)\right) \leq \sigma\lambda_k g_k^T d_k\right)$ **do**
    $\lambda_k = \lambda_k \beta$
  end
  $x_{k+1} = x_k + \lambda_k d_k$ Compute B_{k+1} by using (10) $k = k+1$ $d_k = -B_k g_k$
end
x_k is a minimizer

4.2. Broyden-Flecher-Godfard-Shanno scheme

The search direction d_k at k-th iteration is provided by the solution of the analogue of (7) and given as (11):

$$B_k d_k = -g_k \tag{11}$$

where B_k will approximate the Hessian matrix. Initially, $B_k^{-1} = I$ and $g_k = \nabla f(x_k)$ are chosen. The matrix B will be updated iteratively at each iterations [28]–[34]. By letting $y_k = g_{k+1} - g_k$ and $s_k = x_{k+1} - x_k$, thus the update of B_{k+1} is given by (12):

$$B_{k+1} = B_k + \frac{y_k y_k^T}{y_k^T s_k} - \frac{B_k s_k s_k^T B_k^T}{s_k^T B_k s_k}. \tag{12}$$

The following subsection illustrated the quasi-Newton with BFGS scheme algorithm which replace the exploitation stage of ABC algorithm.

4.2.1. The quasi-Newton with BFGS scheme algorithm

The quasi-Newton with BFGS Scheme algorithm is given. The next subsections describe the refinement of the ABC algorithm by introducing the DFP and BFGS Schemes. In this work, the DFP and BFGS schemes are used in the exploitation phase of the ABC algorithm. By doing this, the convergence of the original ABC algorithm can be improved. The quasi-Newton with BFGS Scheme algorithm is given in algorithm 3.

Algorithm 3 Quasi-Newton with BFGS scheme algorithm

Data: $x_0 \in \mathbb{R}^n$, and tolerance, $\epsilon \geq 0$. Setting $\lambda_k = 2$, $\beta = 0.618$ and $\sigma = 0.8$.
Result: Minimizer x_k*
while $\left(\|d_k\| \leq \epsilon\right)$ **do**
  $\lambda_k = \max\{s, s\beta, s\beta^2, \cdots\}, s > 0, \beta \in (0,1), \sigma \in (0,1)$ **while** $\left(\left(f(x_k + \lambda_k d_k) - f(x_k)\right) \leq \sigma\lambda_k g_k^T d_k\right)$ **do**
    $\lambda_k = \lambda_k \beta$
  end
  $x_{k+1} = x_k + \lambda_k d_k$ Compute B_{k+1} by using (12) $k = k+1$ $d_k = -B_k g_k$
end
x_k is a minimizer

4.2.2. The improvement of ABC algorithm with quasi-Newton schemes

The improvement of ABC algorithm using DFP and BFGS schemes which replaces the exploitation process is explained as follows. Furthermore, the hybridization of the ABC algorithm with the DFP scheme and the hybridization of the ABC algorithm with the BFGS scheme in the exploitation stage are referred to as the ABC-DFP algorithm and the ABC-BFGS algorithm, respectively and represented in algorithm 4. The numerical results are presented and given in the following section. The results show that the proposed algorithm is capable in finding the global optimal solution effectively compared to the original ABC algorithm. Moreover, the convergence ability of the proposed algorithm is reported in the following section.

Algorithm 4 ABC Algorithm with quasi-Newton scheme

```
Data: Initilization: the population of solution x_ij i = 1, ··· , n; j = 1, ··· , D; trial = 0 is the non improvemnet number of the solution
      x_{i,j} used for abondonment, maximum cycle number, limit = A food source which could not be improved through limit trial and
      abandoned by its employed bee.
Evaluate the population  Set cycle = 1  repeat
|  {Produce a new food source population for employed bees}  for i=1 to n do
|  |  Produce a new food source v_i for employed bees of the food source x_i by using Algorithm (2)  Apply a greedy selection process
|  |    between v_i and x_i and select the better one  if x_i does not improve then
|  |  |  trial = trial_i + 1  Otherwise trial_i + 1 = 0
|  |  end
|  end
|  Calculate the probability value p_i by using (6) for the solutions using fitness values by using (5)  {Produce a new food source
|    population for onlooker bees}  Set t = 0, i = 0  repeat
|  |  if random ≤ p_i then
|  |  |  {Produce a new v_i food source by using Algorithm (2)}  Apply a greedy selection process between v_i and x_i and select the
|  |  |    better one  if Solution x_i does not improve then
|  |  |  |  trial_i = trial_i + 1  Otherwise trial_i + 1 = 0  t = t + 1
|  |  |  end
|  |  end
|  until t = n
|  {Determine scout}  if max(trial_i) > limit then
|  |  Replace x_i with a new randomly produces solution by using (3)
|  end
|  Memorize the best solution achieved so far  cycle = cycle + 1
until {cycle = Maximum cycle Number}
```

5. NUMERICAL RESULTS AND DISCUSSION

The original ABC and the enhanced ABC algorithm with the DFP and BFGS schemes are presented. The performance improvement is compared to the original ABC algorithm using eighteen selected global optimization problems taken from [4]. The numerical results comparison is depicted in Table 1 whereby the first column represents the name of the test problem while the second together with the third columns represent results obtained by using BFGS and DFP schemes in the exploitation part of ABC algorithm. Meanwhile the final column depicted the results obtained using the original ABC algorithm.

Table 1. The performance of enhanced and original ABC algorithms

Problem	Function	BFGS	DFP	ABC Algorithm
1	Griewank (5 variable)	1.313384×10^{-3}	1.630763×10^{-6}	2.934116×10^{-10}
2	Sphere	2.055125×10^{-13}	2.055125×10^{-13}	7.855848×10^{-6}
3	Rosenbrock (5 variable)	1.828925×10^{-13}	8.927977×10^{-12}	4.081273×10^{-1}
4	Rastrigin (5 variable)	6.069244	5.820506	2.448230×10^{2}
5	Rastrigin (2 variable)	4.477316×10^{-1}	3.482357×10^{-1}	8.809675×10^{-11}
6	Ackley	2.630402×10^{-6}	2.774587×10^{-6}	2.181409×10^{-7}
7	Beale	3.561118×10^{-14}	3.846104×10^{-14}	4.939879×10^{-5}
8	Goldstein–Price	3.000000	3.000000	3.018673
9	Booth	2.611529×10^{-13}	2.526855×10^{-13}	1.127488×10^{-5}
10	Matyas	4.725471×10^{-15}	5.988413×10^{-15}	1.684934×10^{-5}
11	Lévi	4.526399×10^{-12}	1.919452×10^{-11}	5.493683×10^{-3}
12	Three-hump camel	1.672050×10^{-13}	1.791896×10^{-13}	3.411217×10^{-11}
13	Easom	-1.000000	-1.000000	-1.000000
14	Adjiman	-5.002938	-4.998900	-1.001762×10^{1}
15	Bird	-1.067645×10^{2}	-1.067645×10^{2}	-1.067645×10^{2}
16	Bohachevsky 1	4.293244×10^{-12}	4.742057×10^{-12}	1.137368×10^{-13}
17	Bohachevsky 2	1.800000×10^{-1}	1.800000×10^{-1}	1.800000×10^{-1}
18	Bohachevsky 3	3.065742×10^{-13}	3.727768×10^{-13}	5.237552×10^{-6}

Results obtained proved the proposed algorithm has the ability to find the global optimum solution effectively when compared to the original ABC algorithm except for the Griewank and Rastrigin functions. Moreover, convergence ability of the proposed algorithm is still better than the original ABC algorithm. Conclusively, the proposed algorithm attained a smaller margin of error in eight out of the eighteen test problems.

The success of the new approach is because of the Armijo line search beeing used in the enhancement process. According to literature, the quasi-Newton method using Armijo line search will hold global convergent ability like the steepest descent method. Therefore, the new approach algorithms can perform as great as the steepest descent method [15].

The performance of all two variants of the new approach which ABC-DFP and ABC-BFGS algorithms have been compared to the original ABC algorithm in 10 designated global optimization problems [4]. The numerical results showed by the graph of the algorithm to obtain a global optimum solution with the number of iterations. Obviously, numerical results indicated that the new approach algorithms are able to obtain more accurate a global solution than original ABC algorithm except in Griewank function. However, in all selected problems the new approach showed their convergent ability is better than the original ABC algorithm.

6. CONCLUSION

This paper attempted to address the issue at exploitation stage for ABC algorithm. The complexities and inefficiencies in evaluating second derivatives and difficulty in predicting initial point is tackled during this research. The new proposed algorithm explored modification of the exploitation part in ABC algorithm by introducing a quasi-Newton algorithm that included DFP and BFGS schemes. All the considering algorithms are capable in obtaining the global optimal solution of the global optimization problems. Numerical results comparison proven that the new proposed algorithm is capable in getting the optimal global solution effectively in the majority of test functions. This finding is significant in terms of enhancing the algorithm to obtain the solution associated with global optimization problems. On the other hand, the ability of the original DFP and BFGS which used to find local optimum, have been improved by hybridizing with ABC and able to determine the global optimum solution in nonconvex multimodal optimization problems.

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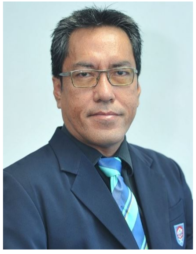

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