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Two-stage parametric identification procedure to predict satellite orbital motion

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ABSTRACT

The paper presents a new step-by-step procedure for constructing a navigation satellite motion model. At the first stage of the procedure, the parameters of the radiation pressure model are estimated using the maximum likelihood method. The statistic estimator based on the continuous-discrete adaptive unscented Kalman filter is proposed for the solar radiation model parameters estimation. Step-by-step scheme of filtering algorithm used for the software development are given. At the second stage, the parameters of the unaccounted perturbations model are estimated based on the results of residual differences measurements. The obtained results lead to significant improvement of prediction quality of the satellite trajectory.

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1. INTRODUCTION

The representation of a dynamic system by a nonlinear stochastic model makes it possible to take into account factors caused by nonlinear laws of nature and perform a better analysis. To obtain a model with good predictive properties, informative measurement data and a suitable model structure capable of accurately describing the dynamics of the process are required; therefore, when constructing models of nonlinear systems, parametric identification methods are used. Traditionally, the maximum likelihood (ML) method is used to solve the problem of parametric identification [1]–[3]. In case of using dynamic models with Gaussian noise, the corresponding identification criterion is written on the basis of the equations of the extended Kalman filter (EKF) [4], [5]. Although the EKF is widely used, this filter has some drawbacks. EKF applies the standard technique to linearize a nonlinear model. It requires the sufficient differentiability of the dynamic state and the susceptibility to biasing and to divergence of the state estimates. This method is sub-optimal and can easily lead to the divergence. The EKF achieves only the first-order accuracy and produces a good result only if the initial estimation error and disturbing noises are small.

These difficulties can be successfully overcome with such nonlinear filters as the cubature Kalman filter [6], [7] and the unscented Kalman filter (UKF) [8]–[12]. Julier *et al.* [8], [9] proposed derivative free alternative to the EKF for state estimation the unscented Kalman filter (UKF). The UKF has been developed for the case of highly nonlinear state estimation problems. The UKF performs a Gaussian approximation with a limited number of points (sigma points), using the unscented transform. This technique is used to linearize a function a random variable via the linear regression based on the points drawn from the prior distribution of the random variable. The UKF has the same computational complexity as the EKF has [12], but UKF does not require the Jacobians computing and can achieve the second-order accuracy of the Taylor expansion.

When solving practical problems, statistical parameters of noise are set inaccurately or they are completely unknown. The presence of outliers in the measurement data makes the further determination of such characteristics complicated. When using the incorrect a priori information about the noise properties of the system and/or the measurements, the obtained estimates may be biased. Covariance matrices are determined according to the results of the analysis of the source data or modeling. Note that the correct determination of the covariance matrices of noise processes affects the accuracy of the estimation of the state vector.

A possible solution to the problem is seen in the use of adaptive methods of measuring data processing [13]–[17]. Adaptive algorithms allow us to jointly evaluate the state vector and covariance noise matrices. In this research, the sub-optimal Sage-Husa estimator [13], [16], [17] is combined with the UKF algorithm in order to estimate and improve the statistical properties of the process noise. Such improvement reduces the model error, suppresses the filtering divergence and improves the filtering accuracy. For the more accurate construction of a mathematical model, it is proposed to additionally evaluate the disturbances from the measurements of the residual differences, using a two-stage parametric identification procedure.

This paper has the following structure. Section 2 provides a mathematical description of the motion model of the navigation satellite. A two-step parametric identification procedure is described in section 3. The results of applying the two-stage parametric identification procedure in constructing a satellite motion model are given in section 4. The found parameters of the solar radiation model are given. A comparison of the accuracy of the constructed motion models of navigation satellite based on the ML method and the two-stage identification procedure is also given in section 4. The conclusion is provided in section 5.

2. MOTION MODEL OF THE NAVIGATION SATELLITE

The quality of the ephemeris-temporal support for Global Navigation Satellite System (GNSS) technologies depends on adequacy of the applied mathematical models describing the orbital motion of navigation satellites. Consider the following stochastic nonlinear continuous-discrete model of orbital motion of the navigation satellite [18], [19] is (1) and (2).

$$\frac{d}{dt}\big(r(t)\big) = \underbrace{-\frac{\mu \cdot M_E}{\|r(t)\|^3} r(t)}_{f(R(t))} + g_1\big(r(t)\big) + g_2\big(r(t)\big) + g_3(r(t), \dot{r}(t), \theta) + w(t), t \in [t_0, t_N] \quad (1)$$

$$s(t_{k+1}) = \underbrace{r(t_{k+1})}_{h(R(t_{k+1}))} + v(t_{k+1}), k = 0,1, \dots, N-1 \quad (2)$$

Where, $R(t) = \begin{pmatrix} r(t) \\ r(t) \end{pmatrix}, r(t) = \big(x(t), y(t), z(t)\big)^T$ is the coordinate vector of the navigation satellite in an inertial coordinate system, $r(t) = \big(V_x(t), V_y(t), V_z(t)\big)^T$ is the velocity vector of the navigation satellite in an inertial coordinate system; $f\big(R(t)\big), h\big(R(t_{k+1})\big)$ are functions, where μ is the gravitational constant, M_E is the mass of the Earth; $\|r(t)\| = \sqrt{x^2(t) + y^2(t) + z^2(t)}$ is the radius of the orbit, $g_1\big(r(t)\big)$ is the perturbations, caused by the non-sphericity of the Earth's geopotential, $g_2\big(r(t)\big)$ is the perturbations, caused by the gravitational influence of the Moon, the Sun and/or the other planets, $g_3(r(t), r(t), \theta)$ is perturbations from the solar radiation (SR); $\theta \in \Omega_\theta$ is the vector of unknown parameters; $s(t_{k+1})$ is the measurement vector (for example, pseudo range, query range, satellite laser ranging (SLR) from ground points to the navigation spacecraft). In a particular case, a posteriori ephemeris of navigation spacecraft obtained by various processing centers can act as measurements (i.e. $h\big(R(t_{k+1})\big) = r(t_{k+1})$. In (1) and (2):

– The random vectors $w(t)$ and $v(t_{k+1})$. form white Gaussian noises with unknown noises covariance matrices

$$E[w(t)] = 0, E[w(t)w^T(\tau)] = Q_w(t)\delta(t-\tau)$$
$$E[\,v(t_{k+1})] = 0, E[\,v(t_{k+1})\,v^T(t_{i+1})] = \,Q_v(t_{k+1})\delta_{k,1}$$
$$E[v(t_{k+1})w^T(\tau)] = 0, k, i = 0,1, \dots, N-1, \tau \in [t_0, t_N]$$

– The state vector $R(t)$in moment $t_0$defined by

$$E[R(t_0)] = \bar{R}(t_0), E\left[\big(R(t_0) - \bar{R}(t_0)\big)\big(R(t_0) - \bar{R}(t_0)\big)^T\right] = P(t_0)$$

and has no correlation with $w(t)$, $v(t_{k+1})$.

A description of each of the forces affecting on a satellite can be found in, for example, [19], [20]. It is important to note that some of these force models include parameters which numerical values are only partially known. In the formation of model (1) and (2) it remains problematic to take into account perturbations from solar radiation pressure on the satellite [21]–[25]. To compute $g_3(r(t), r(t), \theta)$ in an inertial coordinate system, the following SR model has been used [19], [23]–[25]:

$$g_3(r(t),\dot{r}(t),\theta) = \Lambda(r(t)) \cdot \rho^{-2}(r(t)) \cdot \big[i_1 \cdot \big(\theta_1 + \theta_2 \cos\sigma(r(t),\dot{r}(t)) + \theta_3 \sin\sigma(r(t),\dot{r}(t))\big) + \\ + i_2.\big(\theta_4 + \theta_5 \cos\sigma(r(t),\dot{r}(t)) + \theta_6 \sin\sigma(r(t),\dot{r}(t))\big) \\ + i_3 \cdot \big(\theta_7 + \theta_8 \cos\sigma(r(t),\dot{r}(t)) + \theta_9 \sin\sigma(r(t),\dot{r}(t))\big)\big] \quad (3)$$

Here $\Lambda(r(t))$ is the eclipse factor, $\rho(r(t))$ is the distance between the satellite and the Sun, $\sigma(r(t), r(t))$ is the argument of the latitude for the navigation satellite, $i_1 = \frac{r_s(t)-r(t)}{\|r_s(t)-r(t)\|}$ is ort in the direction of solar radiation, $\|\cdot\|$ is the Euclidean vector norm, $i_2 = \frac{i_1 \times r(t)}{\|i_1 \times r(t)\|}$ is ort normal to the Sun-satellite-Earth, $i_3 = i_1 \times i_2$ is ort that complements the system to the right triple of vectors.

3. RESEARCH METHOD

Informative measurement data and correctly defined model parameters affect the predictive properties of the model and the ability to describe the dynamics of the process. Usually, the construction of the model (1) consists in finding estimates of the unknown parameters of the SR model. Estimation of unknown parameters of model (1) and (2) is carried out according to measurement data and the selected identification method. The a priori assumptions allow using the ML method for the parameters estimation. ML estimates have practically important properties, as asymptotic efficiency, asymptotic unbiasedness, asymptotic normality and consistency.

We offer a two-step procedure for parametric identification of the model (1) and (2). At the first stage of the procedure, the parameters of the radiation pressure model are estimated using the ML method based on the adaptive unscented Kalman filter. At the second stage, the parameters of the unaccounted perturbations model are estimated based on the results of measurements of residual differences.

Stage 1. Building the matching model (parameter identification)

1. Solve the problem of parametric identification based on the ML method

$$\hat{\theta} = arg \min_{\theta \in \Omega_\theta} \tfrac{1}{2}\sum_{k=0}^{N-1} lndet\, P_Y(t_{k+1}) + \tfrac{1}{2}\sum_{k=0}^{N-1} \varepsilon(t_{k+1})^T P_Y^{-1}(t_{k+1})\,\varepsilon(t_{k+1}), \quad (4)$$

where $\varepsilon(t_{k+1})$ and $P_Y(t_{k+1})$ are defined based on the corresponding equations of the following adaptive UKF [13], [16], [17]:

Initialization:

– Set the values

$$\xi = 0.001, \eta = 2, \varphi = \kappa = 0, \rho = 0.998$$

– Define the initial values

$$\hat{R}(t_0|t_0) = \bar{R}(t_0), P(t_0|t_0) = P(t_0), Q_w(t_0), Q_v(t_1)$$

– Calculate

$$l = \xi^2(n+\varphi) - n\big(n = 6 \text{ is the size of } R(t)\big), \alpha_0 = \frac{l}{n+l}$$
$$\beta_0 = \frac{1}{(n+1)+(1-\xi^2+\eta)}, \alpha_i = \frac{1}{2(n+1)} = \beta_i, i = 1, \dots, 2n$$
$$a = [\alpha_0, \alpha_1, \dots, \alpha_{2n}]^T$$
$$A = \Big(I - \underbrace{[a| \dots |a]}_{2n+1}\Big) diag(\beta_0, \beta_1, \dots, \beta_{2n}) \Big(I - \underbrace{[a| \dots |a]}_{2n+1}\Big)^T$$

For $k = \overline{0, N-1}$

Prediction:

– Define, $\hat{R}(t_{k+1}|t_k), P(t_{k+1}|t_k)$ by solving of differential (5) and (6) integration

$$\frac{d}{dt}\hat{R}(t|t_k) = R_f(t|t_k)a, t_k \le t \le t_{k+1}, \tag{5}$$

$$\frac{d}{dt}P(t|t_k) = R_S(t|t_k)AR_f^T(t|t_k) + R_f(t|t_k)AR_S^T(t|t_k)$$
$$+Q_w(t),\ Q_w(t) = Q_w(t_{k+1}),\ t_k \le t \le t_{k+1}, \tag{6}$$

where the transformed set of vectors is identified as:

$$R_f(t|t_k) = [f(R_0^s(t|t_k))|f(R_1^s(t|t_k))|\ldots|f(R_{2n}^s(t|t_k))]_{n\times(2n+1)},$$

sigma points $R_i^s(t|t_k), i = \overline{1,n}$ are computed in accordance with the formula (7).

$$R_i^S(t|t_k) = \begin{cases} \hat{R}(t|t_k),\ i = 0, \\ \hat{R}(t|t_k) + \sqrt{n+l}D_i(t|t_k),\ i = \overline{1,n}, \\ \hat{R}(t|t_k) - \sqrt{n+l}D_{i-n}(t|t_k),\ i = \overline{n+1,2n}, \end{cases} \tag{7}$$

$$R_s(t|t_k) = [R_0^S(t|t_k)|R_1^S(t|t_k)|\ldots|R_{2n}^S(t|t_k)]_{n\times(2n+1)}.$$

D_i is the $i - th\ row$ of the lower triangular matrix obtained by the Cholesky decomposition of $P(t|t_k)$.
Updating:
- Find $R_S(t_{k+1}|t_k)$ using (7) with the substitution $t = t_{k+1}$.
- Calculate

$$S_h(t_{k+1}|t_k) = \left[h\left(R_0^S(t_{k+1}|t_k)\right)\middle|h\left(R_1^S(t_{k+1}|t_k)\right)\middle|\ldots\middle|h\left(R_{2n}^S(t_{k+1}|t_k)\right)\right]_{m\times(2n+1)},$$
$$\varepsilon(t_{k+1}) = s(t_{k+1}) - S_h(t_{k+1}|t_k)a,$$
$$\tau_k = \frac{1-\rho}{1-\rho^{k+1}}$$
$$\hat{Q}_v(t_{k+1}) = (1-\tau_k)\hat{Q}_v(t_k) + \tau_k[\varepsilon(t_{k+1})\varepsilon^T(t_{k+1}) -$$
$$-\sum_{i=0}^{2n}\beta_i\left(h\left(R_i^S(t_{k+1}|t_k)\right) - S_h(t_{k+1}|t_k)a\right)\left(h\left(R_i^S(t_{k+1}|t_k)\right) - S_h(t_{k+1}|t_k)a\right)^T\Big],$$
$$P_S(t_{k+1}) = S_h(t_{k+1}|t_k)AS_h^T(t_{k+1}|t_k) + Q_v(t_{k+1}),$$
$$P_{RS}(t_{k+1}) = R_S(t_{k+1}|t_k)AS_h^T(t_{k+1}|t_k).$$

- State $\hat{R}(t_{k+1}|t_{k+1})$ and covariance estimates $P(t_{k+1}|t_{k+1})$ are computed according to the equations

$$K(t_{k+1}) = P_{RS}(t_{k+1})P_S^{-1}(t_{k+1})$$
$$\hat{R}(t_{k+1}|t_{k+1}) = \hat{R}(t_{k+1}|t_k) + K(t_{k+1})\varepsilon(t_{k+1})$$
$$P(t_{k+1}|t_{k+1}) = P(t_{k+1}|t_k) - K(t_{k+1})P_S(t_{k+1})K^T(t_{k+1})$$
$$\hat{Q}_w(t_{k+1}) = (1-\tau_k)\hat{Q}_w(t_k) + \Bigg\{\tau_k\Bigg[K(t_{k+1})\varepsilon(t_{k+1})\varepsilon^T(t_{k+1})K^T(t_{k+1}) + P(t_{k+1}|t_{k+1}) -$$
$$\sum_{i=0}^{2n}\beta_i\left(f\left(R_i^S(t_{k+1}|t_k)\right) - \hat{R}(t_{k+1}|t_k)\right)\left(f\left(R_i^S(t_{k+1}|t_k)\right) - \hat{R}(t_{k+1}|t_k)\right)^T\Bigg]\Bigg\}$$

End.

Note that the cost function in (4) is known to have many local optima. Often to solve the optimization problem (4) are used Newton’s method and various quasi-Newton methods, which are the local ones. These methods are sensitive to the setting of initial values and the accuracy of determining the gradient. In general, it is impossible to determine what influenced the low accuracy of the model, the inaccurate definition of the model structure or the found local minimum. A reasonable approach to optimization problem is to use global optimization methods. In the paper for the solution (4) is used global optimization approach based on the sequential quadratic programming method.
Stage 2. Specifying the matching model (identification of unaccounted disturbances from measurements of residual differences)
- Calculate the residual differences $\Delta(t_{k+1})$ based on the estimates $\hat{\theta}$ obtained in step 1.

$$\Delta(t_{k+1}) = s(t_{k+1}) - \hat{s}_1(t_{k+1})$$

where

$$\hat{s}_1(t_{k+1}) = h\left(\hat{R}(t_{k+1}|t_{k+1})\right) \tag{8}$$

$\hat{R}(t_{k+1}|t_{k+1})$- the estimate of the state vector obtained at the first stage.
– Choose the following model

$$\Delta^i(t_{k+1}) = a_0^i + a_1^i t_{k+1} + a_2^i t_{k+1}^2 + b_1^i \cos\left(\frac{2\pi t_{k+1}}{T}\right) + b_2^i \sin\left(\frac{2\pi t_{k+1}}{T}\right) + c_1^i \cos\left(\frac{4\pi t_{k+1}}{T}\right) + c_2^i \sin\left(\frac{4\pi t_{k+1}}{T}\right) \tag{9}$$

$i = 1, \dots, m$, m - the size of $s(t_{k+1})$ (here m=3), T-defined value, and estimate of the unknown parameters $\tilde{\theta} = \left(a_0^i, a_1^i, a_2^i, b_1^i, b_2^i, c_1^i, c_2^i\right)^T, i = 1,2,3,$ using least squares method [26]:

$$\hat{\hat{\theta}} = (B^T B)^{-1} B^T \Delta(t_{k+1}),$$

where $B = \begin{bmatrix} 1 & t_1 & t_1^2 \cos\left(\frac{2\pi t_1}{T}\right) & \sin\left(\frac{2\pi t_1}{T}\right) & \cos\left(\frac{4\pi t_1}{T}\right) \sin\left(\frac{4\pi t_1}{T}\right) \\ \cdots & \cdots & \cdots \quad \cdots & \cdots & \cdots \quad \cdots \\ 1 & t_N & t_N^2 \cos\left(\frac{2\pi t_N}{T}\right) & \sin\left(\frac{2\pi t_N}{T}\right) & \cos\left(\frac{4\pi t_N}{T}\right) \sin\left(\frac{4\pi t_M}{T}\right) \end{bmatrix}$

– Calculate $\hat{\Delta}(t_{k+1})$ taking into account the estimates $\hat{\hat{\theta}}$ found by the (9).
– Calculate

$$\hat{s}_2(t_{k+1}) = \hat{s}_1(t_{k+1}) + \hat{\Delta}(t_{k+1}), k = 0, \dots, N-1 \tag{10}$$

using $\hat{\Delta}(t_{k+1})$ and $\hat{s}_1(t_{k+1})$ found by the (8).

4. RESULTS AND DISCUSSION

As the measurement data we have taken the rapid ephemeris of the GPS from 07/14/2016, obtained by the international GNSS service. In this case, the satellite makes more than one revolution around the Earth (passes through the different light zones). At the initial time, we compute the velocity of the satellite on the basis of rapid ephemeris using Everett interpolation. Estimation of the SR parameters (3) can be carried out using the ML method according to the trajectory observations in areas of total illumination and penumbra zones [27]. The quality of the parameters estimates found was determined by the accuracy of predicting the navigation satellite motion trajectory:

$$RMSE_1^i = \sqrt{\frac{1}{N}\sum_{k=0}^{N-1} \|s(t_{k+1}) - \hat{s}_i(t_{k+1})\|^2}$$

$$RMSE_2^i = \sqrt{\frac{1}{N}\sum_{k=0}^{N-1} \left\|s^*(t_{k+1}) - \hat{s}_i^*(t_{k+1})\right\|^2}$$

Here $\{s(t_{k+1}), k = 0,1, \dots, N-1\}$ is final ephemeris received on July 14, 2016, $\{\hat{s}_1^*(t_{k+1}), k = 0,1, \dots, N-1\}$ is the predicted trajectory on 06/14/2016 for the filter equation at $\hat{\theta}$ and $\{\hat{s}_2^*(t_{k+1}), k = 0,1, \dots, N-1\}$ is the obtained trajectory on 06/14/2016 based on the found parameters $\hat{\hat{\theta}}$, $s^*(t_{k+1})$ is final ephemeris received on 06/15/2016, $\{\hat{s}_1^*(t_{k+1}), k = 0,1, \dots, N-1\}$ is the obtained trajectory on 06/15/2016 for the filter equation at $\hat{\theta}$ and $\{\hat{s}_2^*(t_{k+1}), k = 0,1, \dots, N-1\}$is the obtained trajectory on 06/15/2016 based on the found parameters $\hat{\hat{\theta}}$. The obtained results are presented in Table 1.

Thus, the result of the sunlight SR parameters specification is that it is possible to increase the accuracy of the satellite trajectory prediction. The results obtained show that the model based on the two-stage parametric identification procedure is more accurate than the model obtained using the one-stage parametric identification procedure.

Table 1. Results of estimating parameters of solar radiation model

	One-stage parametric identification procedure	Two-stage parametric identification procedure
Parameters estimates	$\hat{\theta} = \begin{bmatrix} 1.06906362 \\ 0.06858372 \\ 0.04729392 \\ 0.11131100 \\ 0.06708731 \\ 0.09272575 \\ 0.09483054 \\ 0.13523427 \\ 0.10924206 \end{bmatrix}$	$\hat{\hat{\theta}} = 10^{-7} \times \begin{bmatrix} 0.0955 & 0.0668 & -0.0143 \\ -0.0010 & -0.0051 & 0.0007 \\ 0.0000 & 0.0001 & 0.0000 \\ -0.2458 & -0.1025 & 0.1188 \\ 0.1703 & -0.1729 & 0.1879 \\ -0.0009 & -0.0051 & 0.0061 \\ -0.0186 & 0.0114 & -0.0141 \end{bmatrix}$
Evaluation of the quality of building a mathematical model (km)	$RMSE_1^1 = 3.2701 \times 10^{-8}$	$RMSE_1^2 = 8.1142 \times 10^{-9}$
Evaluation of the prediction quality (km)	$RMSE_2^1 = 3.3985 \times 10^{-8}$	$RMSE_2^2 = 1.9186 \times 10^{-8}$

5. CONCLUSION

In this paper the statistic estimator based on the adaptive modification UKF with noise is used for the SR model parameters estimation. This approach improves the robustness of the traditional UKF with the respect to the variable noise distribution. The results show that in case of time-varying or uncertain noise characteristics the adaptive UKF is more efficient than the conventional UKF in terms of the fast convergence and the state estimation accuracy.

Offered approach to the construction of a satellite motion model and the prediction of ephemeris is based on a complex application of the maximum likelihood method, a nonlinear filtering algorithm and the identification of a complex component of the satellite motion model. The use of a two-stage parametric identification procedure that combines the estimation of the parameters of the matching SR model from ephemerides with the refinement of the residual accelerations made it possible to increase the accuracy of determining the unknown parameters of the satellite motion model. In the future, to improve the accuracy of the model construction, a two-stage gradient identification procedure based on a combination of several adaptive filters will be developed.

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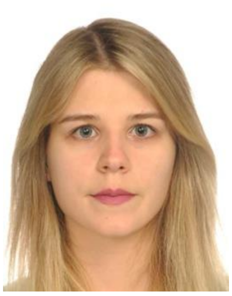

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