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A novel method for determining fixed running time in operating electric train tracking optimal speed profile

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ABSTRACT

Tracking the optimal speed profile in electric train operation has been proposed as a potential solution for reducing energy consumption in electric train operation, at no cost to improve infrastructure of existing Metro lines as well. However, the optimal speed profile needs to meet fixed running time. Therefore, this paper focuses on a new method for determining the fixed running time complied with the scheduled timetable when trains track the optimal speed profile. The novel method to ensure the fixed running time is the numerical-analytical one. Calculating accelerating time ta, coasting time tc, braking time tb via values of holding speed vh, braking speed vb of optimal speed profile with the constraint condition: the running time equal to the demand time. The other hands, vh and vb are determined by solving nonlinear equations with constraint conditions. Additionally, changing running time suit for each operation stage of metro lines or lines starting to conduct schedules by the numerical-analytical method is quite easy. Simulation results obtained for two scenarios with data collected from electrified trains of Cat Linh-Ha Dong metro line, Vietnam show that running time complied with scheduled timetables, energy saving by tracking optimal speed profile for the entire route is up to 8.7%, if the running time is one second longer than original time, energy saving is about 11.96%.

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1. INTRODUCTION

Polluted air and heavy traffic jam are knotting issues related to transport in densely populated cities. Therefore, the urban electric train is one of the useful means of transportation to address these problems. However, its major disadvantage lies in consuming a large amount of electric energy in operation [1]-[4]. Therefore, reducing energy consumption and enhancing operation effectiveness of metro lines has long been one of hot studies worldwide. Many researchers, and engineers have tried their best to obtain remarkable achievements in theory and application, which has contributed in sustainable development of urban railway transportation. These theoretical and applied achievements include: recuperating regenerative braking energy by onboard/stationary energy storage systems [5]-[12], equipping the traction substations with reversible converters or active rectifiers so as to pump back the regenerative braking energy into utility source, as a result, all regenerated energy can be recuperated [13], [14], and optimizing scheduled timetables makes the regenerated energy among trains interchange easier [4], [15], [16], lowering energy losses in the power supply system, and in on-board traction equipment [17], applying optimal theory to seek the optimal speed profile to minimize operating energy [18]-[23], using interger linear programing determining optimal position for substations in order to reduce power loss [24]. Among above approaches, tracking the optimal speed curves to decrease energy consumption becomes one of the measures which does not need to invest in equipment or infrastructure of existing metro lines, so this one is most suitable for metro lines in Vietnam to have just been installed. The researchers of South Australia University in [25]-[29] outlined an energy-saving driving strategy for a train trip on a track with uphill and downhill gradients by designing control laws to calculate location of optimal switching points, and then determining the optimal speed profile but not to mention fixed running time. Baranov *at al.* [30] proposed a solution to minimize energy consumption and consider fixed trip time by supplementing Lagrange multiplier in objective function. Calculating to find the actual time equal to demand time is not easy, and repeated. Therefore, in this paper, pontryagin's maximum principle (PMP) has been presented to determine optimal speed profile for Cat Linh-Ha Dong metro line in Vietnam, and calculating fixed-running time by numerical analytical method thanks to maple software. Simulation results are conducted in two scenarios: running time of trains tracking the optimal speed profile is equal to that of original speed profile, and the other with running time of trains is longer than the original speed profile.

2. TRAIN MODELING

The continuous-space model of urban electric train operates in three motion regimes: Accelerating, coasting, braking is shown [31]-[33].

$$\begin{cases} \dfrac{dt}{dx} = \dfrac{1}{v} \\ v\dfrac{dv}{dx} = u_{tr}f_{tr}(v) - u_{br}f_{br}(v) - w_0(v) - f_{grad}(x) \end{cases} \tag{1}$$

Where v, t, x, m represent train velocity (m / s), operation time (s), train position (m), full load mass of train $(tone)$, u_{tr}, u_{br} are defined traction and braking control variables of train, f_{tr}, f_{br}, f_{grad} are forces per unit mass; traction force applied at the wheels, braking force, mechanical force, gradient force acting on the train. where:

$$f_{tr} = \frac{F_{tr}}{m}; f_{br} = \frac{F_{br}}{m};\ f_{grad} = \frac{F_{grad}}{m}$$

In which $F_{tr}, F_{br}, W_0, F_{grad}$ are traction, braking, main resistance, gradient resistance forces, and these forces also have been differed from three motion phases for short inter-stations as shown in Figure 1.

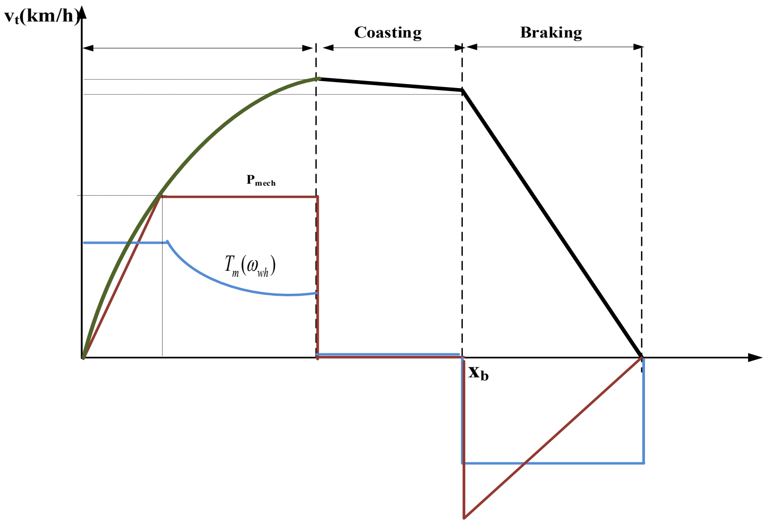

Figure 1. A typical speed profile with three motion phases for short inter-stations

Manufacturers have given traction force F_{tr}, braking force F_{br} [34]. The traction force F_{tr}, braking force F_{br} are described as show in (2):

$$F_{tr} = \begin{cases} 13.2 & (0 \leq v \leq 32) \\ -2.5 \cdot 10^{-5} v^3 + 0.007 \cdot v^2 - 0.66v + 28.35 & (32 < v \leq 80) \end{cases} \tag{2}$$

$$F_{br} = \begin{cases} 14.7 (0 \leq v \leq 65) \\ -0.254v + 31.21 (65 < v \leq 75) \\ -0.2027v + 27.36 (75 < v \leq 80) \end{cases} \tag{3}$$

Davis formula is used to calculate the basic resistance w_0 [35]

$$w_0 = \frac{W_0}{m} = a + bv + cv^2 \tag{4}$$

In which a, b, c are coefficients of train's resistance.

The gradient force F_{grad} caused by slope of road:

$$F_{grad} = mg \sin \alpha \tag{5}$$

g, α are the gravity acceleration and the rail track slope respective.

3. DETERMINATION OF FIXED TRIP TIME

Ensuring the trip time complying with scheduled timetable when train operation tracks the optimal speed profile is the main goal of this section.

3.1. Optimal speed curve determination based on PMP

The optimal speed curve is determined thanks to seek the optimal switching points between the operation regimes, and detecting these switching points based on PMP. From the state (1), boundary conditions include (6), (7).

$$v(0) = 0, v(X) = 0, t(0) = 0 \tag{6}$$

$$0 \leq v(x) \leq V(x), 0 \leq t(X) \leq T, 0 \leq x \leq X \tag{7}$$

Where $V(x)$ is the maximum allowable velocity, X is the terminal of the train operation, $v(0), v(X)$ are the velocity at the beginning, at the end of the route, T is duration of the trip is also given by the timetable.

The problem is how to lessen the train's consumption energy. The objective function is presented:

$$J = \int_0^X u_{tr} f_{tr}(v) dx \to \min \tag{8}$$

According to pontrygin's maximum principle, maximizing hamiltonian equation of the objective function J is going to find its optimal solutions. From (1) to (8), a Hamilton function is written as show in (9).

$$H = -u_{tr} f_{tr}(v) + \frac{p_1}{v} + \frac{p_2}{v}\left(u_{tr} f_{tr}(v) - u_{br} f_{br}(v) - w_0(v) - f_{grad}(x)\right) \tag{9}$$

Given, p_1, p_2 are co-state variables. Co-state variables are defined by (10):

$$\frac{dp_1}{dx} = -\frac{\partial H}{\partial t} = 0 \tag{10}$$

$$\begin{aligned}\frac{dp_2}{dx} &= -\frac{\partial H}{\partial v} = u_{tr}\frac{\partial f_{tr}}{\partial v} + p_1\frac{1}{v^2} + \frac{p_2}{v^2}\\ &\left[u_{tr}f_{tr}(v) - u_{br}f_{br}(v) - w_0(v) - f_{grad}(x)\right]\\ &-\frac{p_2}{v}\left[u_{tr}\frac{\partial f_{tr}}{\partial v} - u_{br}\frac{\partial f_{br}}{\partial v} - \frac{\partial w_0}{\partial v}\right]\end{aligned} \tag{11}$$

Define $p = \frac{p_2}{v}$, so $p \cdot v = p_2$. Therefore,

$$\frac{dp_2}{dx} = \frac{d(p \cdot v)}{dx} = p\frac{dv}{dx} + v\frac{dp}{dx} \Rightarrow \frac{dp}{dx} = \frac{1}{v}\left(\frac{dp_2}{dx} - p\frac{dv}{dx}\right) \tag{12}$$

$$\Rightarrow \frac{dp}{dx} = \frac{1}{v}\left(\frac{dp_2}{dx} - p\frac{dv}{dx}\right) \tag{13}$$

Given

$$\frac{dv}{dx} = \frac{u_{tr}f_{tr}(v) - u_{br}f_{br}(v) - w_0(v) - f_{grad}(x)}{v} \tag{14}$$

Therefore, Hamiltonian function is reformed:

$$H = (p-1)u_{tr}f_{tr} - pu_{br}f_{br} - p(w_0 + f_{grad}) + \frac{p_1}{v} \tag{15}$$

With the values of u_{tr} and u_{br} related to p

$$\begin{cases} u_{tr} = 1 & if \quad p > 1 \\ u_{tr} \in (0,1) & if \quad p = 1 \\ u_{tr} = 0 & if \quad p < 1 \end{cases} and \begin{cases} u_{br} = 0 & if \quad 0 < p < 1 \\ u_{br} \in (0,1) & if \quad p = 0 \\ u_{br} = 1 & if \quad p < 0 \end{cases} \tag{16}$$

Optimal control laws are designed to maximize Hamiltonian function:

- Full power (FP): $u_{tr} = 1, u_{br} = 0$ when $p > 1$
- Partial power (PP): $u_{tr} \in [0,1]$, $u_{br} = 0$ when $p = 1$
- Coasting (C): $u_{tr} = 0, u_{br} = 0$ when $0 < p < 1$
- Full braking (FB): $u_{tr} = 0, u_{br} = 1$ when $p < 0$
- Partial braking (PB): $u_{tr} = 0$, $u_{br} \in [0,1]$ when $p = 0$

3.2. Fixed trip time

The trip time in every operation regime needs to be calculated so that the total running time of the whole line is abided by the train scheduled timetable exactly. Calculating the total running time is divided in to three phases.

3.2.1. Accelerating phase

Equation motion of the train in optimal traction mode:

$$\frac{dv}{dt} = f_{tr}(v) - w_0(v) \tag{17}$$

Using the variable dissociation method, the running time in accelerating phase is expressed as (18).

$$\begin{aligned}&\frac{dv}{f_{tr}(v)-w_0(v)}=dt\rightarrow\int_0^{v_h}\frac{dv}{f_{tr}(v)-w_0(v)}\\&=\int_0^{t_a}dt\rightarrow\int_0^{v_h}\frac{dv}{f_{tr}(v)-w_0(v)}=t_a\end{aligned}\tag{18}$$

Where: f_{tr} is calculated as (2), from (18) the acceleration time may be employed (19):

$$\begin{aligned}t_a&=\int_0^{32}\frac{dv}{f_{tr}(v)-w_0(v)}+\int_{32}^{v_h}\frac{dv}{f_{tr}(v)-w_0(v)}\\&=\int_0^{32}\frac{dv}{\frac{13.2}{247000}-\left(a+bv+cv^2\right)}+\int_{32}^{v_h}\frac{dv}{\left(-2.5\cdot10^{-5}v^3+0.007\cdot v^2-0.66v+28.35\right)-\left(a+bv+cv^2\right)}\\&=-10^7\frac{1}{\sqrt{10^{14}ac-25\cdot10^{12}b^2-5.34\cdot10^9c}}\arctan\left(250\cdot10^{-9}\frac{2\cdot10^{13}\left(2cv+b\right)}{\sqrt{10^{14}ac-25\cdot10^{12}b^2-5.34\cdot10^9c}}\right)\Bigg|_0^{32}\\&\frac{-123.5\cdot10^6\arctan\left(\dfrac{5\cdot10^{-6}\begin{bmatrix}2.0\left(12.35\cdot10^{12}c-25.82\cdot10^9\right)v\\+12.35\cdot10^{12}b+1.52\cdot10^{12}\end{bmatrix}}{\begin{array}{l}15.25\cdot10^{15}ac-3.81\cdot10^{15}b^2-31.89\cdot10^{12}a-936.16\cdot10^{12}b\\-33.72\cdot10^{15}c+13.04\cdot10^{12}\end{array}}\right)\Bigg|_{32}^{v_h}}{\sqrt{15.25\cdot10^{15}ac-3.81\cdot10^{15}b^2-31.89\cdot10^{12}a-936.16\cdot10^{12}b-33.72\cdot10^{15}c+13.04\cdot10^{12}}}\end{aligned}\tag{19}$$

Using the MAPLE software tool, the acceleration time as a function of velocity. Acceleration distance is calculated as (20):

$$\begin{aligned}&\text{From: }\frac{dx}{dt}=v\rightarrow dx=vdt=\frac{vdv}{f_{tr}(v)-w_0(v)}\rightarrow\int_0^{x_a}dx=\int_0^{v_h}\frac{vdv}{f_{tr}(v)-w_0(v)}\\&\rightarrow x_a=\int_0^{v_h}\frac{vdv}{f_{tr}(v)-w_0(v)}\rightarrow\int_0^{x_a}dx=\int_0^{v_h}\frac{vdv}{f_{tr}(v)-w_0(v)}\\&\rightarrow x_a=\int_0^{v_h}\frac{vdv}{f_{tr}(v)-w_0(v)}\end{aligned}\tag{20}$$

3.2.2. Coasting phase

Motion equation of the train in optimal coasting mode:

$$\frac{dv}{dt}=-w_0(v)\tag{21}$$

Using the variable dissociation method, the running time in coasting phase obtains:

$$\frac{dv}{-w_0(v)}=dt\rightarrow\int_{v_h}^{v_b}\frac{dv}{-w_0(v)}=\int_{t_a}^{t_a+t_c}dt\rightarrow t_c=\int_{v_b}^{v_h}\frac{dv}{a+bv+cv^2}=2\frac{1}{\sqrt{4ac-b^2}}\arctan\left(\frac{2cv+b}{\sqrt{4ac-b^2}}\right)\tag{22}$$

In which braking velocity v_b is given as follows [35], [36].

$$\begin{aligned}&v_b=\frac{\psi(v_h)}{\varphi'(v_h)}with\varphi(v)=v\cdot w_0(v),\psi(v)=v^2\cdot w_0'(v)\\&\rightarrow v_b=\frac{v_h}{1+\dfrac{w_0(v_h)}{v_hw_0'(v_h)}}=\frac{v_h}{1+\dfrac{a+v_h(b+cv_h)}{v_h(b+2cv_h)}}\end{aligned}\tag{23}$$

Coasting distance is computed as (24):

From: $\frac{dx}{dt} = v \rightarrow dx = vdt = \frac{vdv}{-w_0(v)} \rightarrow \int_{x_a}^{x_a+x_c}$

$$dx = \int_{v_h}^{v_b} \frac{vdv}{-w_0(v)} \rightarrow x_c = \int_{v_b}^{v_h} \frac{vdv}{a + bv + cv^2} \tag{24}$$

3.2.3. Braking phase

Motion equation of the train in optimal braking mode (25):

$$\frac{dv}{dt} = -w_0(v) - f_{br}(v) \tag{25}$$

Using the variable dissociation method, the running time in braking phase is given by (26):

$$\begin{aligned} &\frac{dv}{-f_{br}(v) - w_0(v)} = dt \rightarrow \int_{v_b}^{0} \frac{dv}{-f_{br}(v) - w_0(v)} \\ &= \int_{t_a+t_c}^{t_a+t_c+t_b} dt \rightarrow \int_0^{v_b} \frac{dv}{f_{br}(v) + w_0(v)} = t_b \end{aligned} \tag{26}$$

F_{br} is calculated as shown in (3)

From (4) the braking time can be written as (27).

$$t_b = \int_0^{v_b} \frac{dv}{f_{br\max} + \left(a + bv + cv^2\right)} = 2\frac{1}{\sqrt{4ac - b^2 + 4cf_{br\max}}} \arctan\left(\frac{2cv + b}{\sqrt{4ac - b^2 + 4cf_{br\max}}}\right) \tag{27}$$

The braking distance is calculated (28).

$$\begin{aligned} &\frac{dx}{dt} = v \rightarrow dx = vdt = \frac{vdv}{-f_{br}(v) - w_0(v)} \rightarrow \int_{x_a+x_c}^{x_a+x_c+x_b} \\ &dx = \int_{v_b}^{0} \frac{vdv}{-f_{br}(v) - w_0(v)} \rightarrow x_b = \int_0^{b_b} \frac{vdv}{f_{br}(v) + w_0(v)} \end{aligned} \tag{28}$$

4. RESULTS AND DISCUSSION

The simulation parameters shown in Table 1, Table 2 collected from Cat Linh-Ha Dong metro line, Vietnam with 12 stations and 12.661 km long [34]. Simulation results carried out with two scenarios: running time complies with scheduled timetable, and running time is one second longer than scheduled timetable. The first scenario: The running time complies with scheduled timetable.

Table 1. Electric train parameters

Parameters	Unit	Value
Train grand-up	2M2T	
Mass	kg	246700
Number of traction motors		08
Max speed	km/h	80
Base speed	km/h	40
Max acceleration/braking rates	m/s^2	0.94/1

Table 2. Coefficients of basic resistance force

Parameters	Value
a	$1.19 \cdot 10^{-2}$
b	$2.56 \cdot 10^{-3}$
c	$1.54 \cdot 10^{-4}$

The second scenario: The running time is one second longer than scheduled timetable. The trip time complied with scheduled timetable indicated in Figure 2, Table 3. Figure 3 showed optimal switching points change, so do optimal accelerating, coasting, braking distances considerability. Table 3 also demonstrated that the lowest saving energy is 3.33% while the highest saving energy is to 10.15%; therefore, saving energy of the whole route is 8.7%. In the second scenario, running time is one second longer than original time indicated in Figures 4, 5, and Table 4, but saving energy of the whole route is 11.96%.

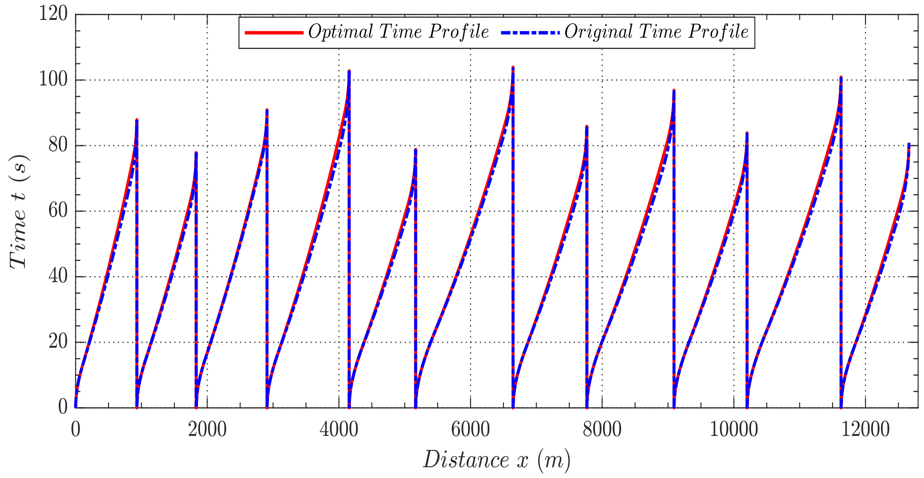

Figure 2. Responses of optimal time profile and original time profile

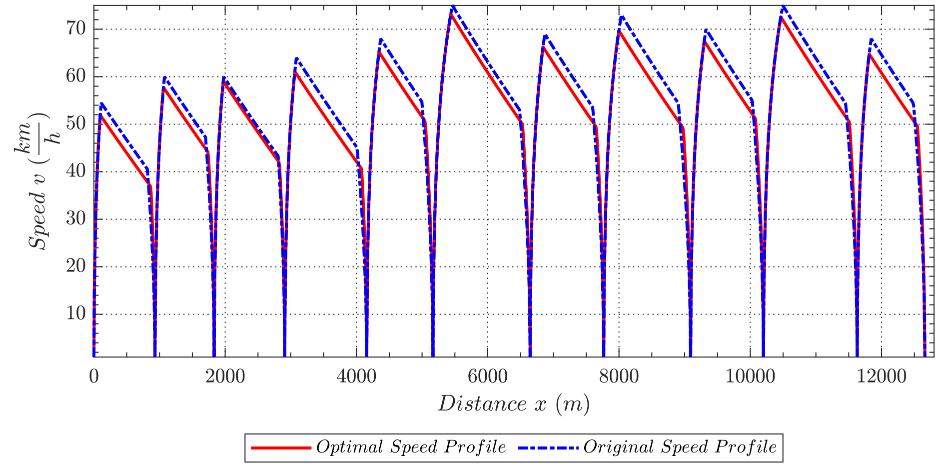

Figure 3. Responses of optimal velocity profile and original velocity profile

Table 3. Results of energy consumption with/without energy optimal strategy, and fixed trip time

Stations	Distance (m)	Trip time (s)	Actual energy consumption (kWh)	Optimal energy consumption (kWh)	Energy saving (%)
Cat Linh-La Thanh	931	88	8.31	7.50	9.75
La Thanh-Thai Ha	902	78	10.20	9.40	7.84
Thai Ha-Lang	1076	91	10.20	9.86	3.33
Lang- Thuong Dinh	1248	103	11.73	10.60	9.63
Thuong Dinh- Ring Road 3	1010	79	13.41	12.23	8.80
Ring Road 3-Phung Khoang	1480	104	16.75	15.82	5.55
Phung Khoang-Van Quan	1121	86	13.85	12.66	8.59
Van Quan - Ha Dong	1324	97	15.74	14.17	9.97
Ha Dong-La Khe	1110	84	14.30	13.18	7.83
La Khe-Van Khe	1428	101	16.75	15.53	7.28
Van Khe-Yen Nghia	1032	81	13.40	12.04	10.15
Total energy consumption			144.64	132.99	8.7

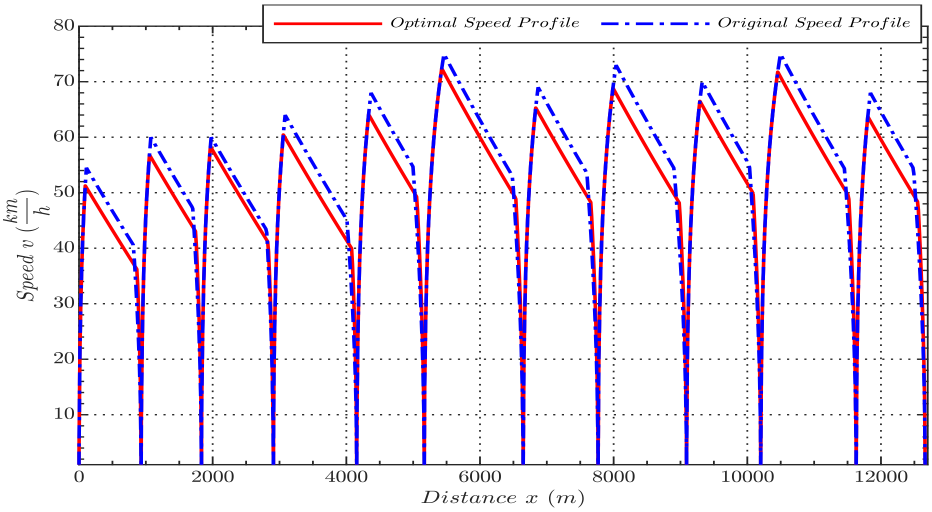

Figure 4. Responses of optimal velocity profile and original velocity profile without fixed trip time

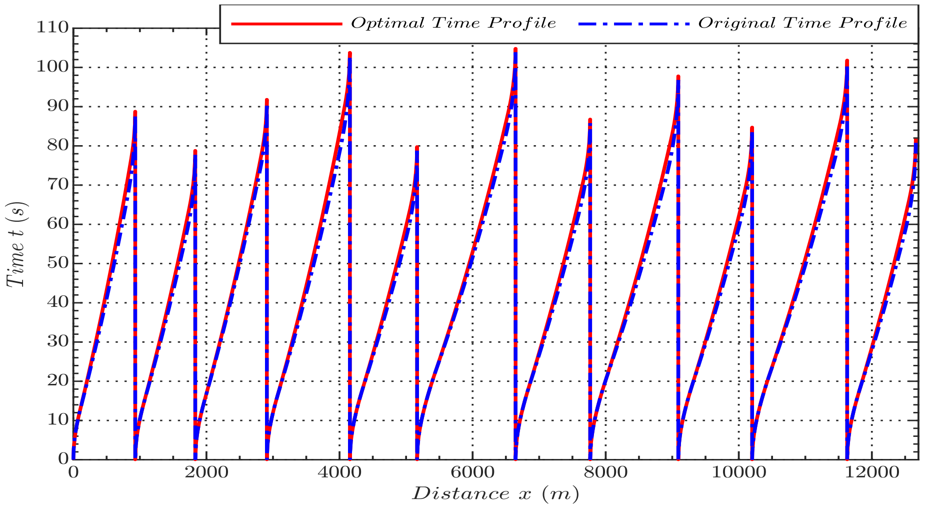

Figure 5. Responses of optimal time profile and original time profile without fixed trip time

Table 4. Results of a comparison of energy consumption with/without energy optimal strategy and non-fixed trip time

Stations	Distance (m)	Original trip time (s)	Actual energy consumption (kWh)	Optimal trip time (s)	Optimal energy consumption (kWh)	Energy saving (%)
Cat Linh-La Thanh	931	88	8.31	88.81	7.31	13.68
La Thanh-Thai Ha	902	78	10.20	78.83	9.08	12.33
Thai Ha-Lang	1076	91	10.20	91.85	9.60	6.25
Lang- Thuong Dinh	1248	103	11.73	103.82	10.37	13.11
Thuong Dinh- Ring Road 3	1010	79	13.41	79.8	11.78	13.84
Ring Road 3-Phung Khoang	1480	104	16.75	104.85	15.44	8.48
Phung Khoang-Van Quan	1121	86	13.85	86.82	12.26	12.97
Van Quan - Ha Dong	1324	97	15.74	97.83	13.80	14.06
Ha Dong-La Khe	1110	84	14.30	84.77	12.77	11.98
La Khe-Van Khe	1428	101	16.75	101.86	15.15	10.56
Van Khe-Yen Nghia	1032	81	13.40	81.82	11.62	15.32
Total energy consumption			144.64		129.18	11.96

5. CONCLUSION

Simulation results with data collected from Metro line Cat Linh-Ha Dong, Vietnam indicated that applying the numerical-analytical method to calculate running time equal to demand time is easy when using Pontryagin's maximum principle finds the optimal speed curve, and levels of energy saving (8.7%, and 11.96%). This research also has supported for designing the optimal speed profiles with the trip time suitable for operation stages of metro lines being in the technical design phase.

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