

A novel 4 dimensional hyperchaotic system with its control, synchronization and implementation

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Article Info

Article history:

Received Aug 27, 2020

Revised Oct 8, 2020

Accepted Dec 5, 2020

Keywords:

Adaptive controller

Chaotic

Hyperchaotic

Lyapunov exponents

Synchronization

ABSTRACT

This paper presents a new hyperchaotic system which shows some interesting features, the system is 4-dimensional with 4 nonlinearities. An extensive numerical analysis has showed that the system has some interesting features and strange behaviors. The numerical analysis includes studying the effect of system parameters and initial conditions. Some of the important properties of the system with parameter set, in which the system is hyperchaotic, such as Lyapunov exponents and Lyapunov dimension, dissipation and symmetry are found and discussed. In the next part of our work, a tracking controller for the proposed system is designed and then a synchronization control system for two identical systems is designed. The design procedure uses combination of a simple synergetic control with adaptive updating laws to identify the unknown parameters derived basing on Lyapunov theorem. Hardware implementation based on microcontroller unit (MCU) board is proposed and tested and used to experimentally validate the designed control and synchronization systems. As an application, the designed synchronization system is used as a secure analogue communication system. Using MATLAB, Simulation study for the control and synchronization systems is presented. The simulation and experimental study have been showed excellent results.

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1. INTRODUCTION

The dynamical system which highly effected by initial conditions is known as chaotic system. Chaotic systems are characterized by their strange attractors. Due to their attraction, theoretical importance and possible scientific and engineering applications, chaotic systems received great attention in the literature and scientific research [1]. Analytically, there are several methods to investigate the existence of chaotic behavior in dynamical systems, the most important of these methods are Lyapunov exponents and bifurcation diagrams [2]. Dynamical systems having positive maximum Lyapunov exponent are chaotic [3]. The largest maximum positive exponent system is the more chaos one. Hyperchaotic system is a chaotic system with more than one positive Lyapunov exponents [4, 5]. Hyperchaotic systems show more dynamical complexity comparing with that systems which have one Lyapunov exponent or simply ordinary chaotic systems [6]. Chaotic systems get great attention in last decades due to their theoretical and practical applications and in various fields such as in secure communication, neural networks, laser, nonlinear circuits, mobile robots, oscillators, artificial neural networks, chemical reactors, finance systems, circuits and others [7-11].

According to some researchers, chaos can be regarded as the third largest scientific revolution in the past and current century [12, 13].

Since the first arise of chaotic systems by Lorenz [14], several chaotic and hyperchaotic systems with ordinary differential equations, complex and fractional order have been introduced and extensively studied [4, 7, 9, 15-17]. Despite of this fact “there was a lot of new chaotic and hyperchaotic systems proposed along the last years,” but it is still advantageous for the field of chaotic in theoretical and practical areas to find and analyses new chaotic systems [17, 18]. Due to their dynamical complexity over chaotic systems, hyperchaotic systems admits more benefits in some applications, especially in the secure communication where it has been shown that encryption with hyperchaotic system gives more efficient secure communication system [19].

In classical chaotic theory, the excitation of strange attractors of chaotic systems comes from their unstable equilibria, so they are referred to as self-excited chaotic systems [8, 9]. More recently, another type of chaotic systems has been appeared, where no equilibria or just a stable equilibrium point system with chaotic behavior [20, 21]. These types of systems have been known as hidden attractors chaos and the reason for this name is very clear. Hidden attractor chaotic systems became now well known as the second source for chaotic systems [22]. Synchronization of two or more chaotic systems received considerable attention due to its theoretical and practical importance in several fields such as secure communication [23, 24]. Sliding mode control theory, adaptive control, active control and other approaches have been used for solving the challenges of the problem of chaotic systems synchronization [25].

Motivated by the previous discussion, we present in this work, a new hyperchaotic system with some interesting features. An extensive study including the important dynamical properties of the proposed system is presented. A tracking and synchronization controllers are designed. Hardware MCU based implementation is proposed and finally an application of the synchronization system is proposed and tested by simulation and experimental study.

The rest of the paper is organized as follows: In section 2, the new system is introduced with a brief detail. The dynamical system analysis and its properties are presented in section 3. A tracking control system design for the system is presented in section 4. In section 5, a synchronization control system is designed. In section 6, a description of the proposed hardware implementation is given. In section 7, simulation and experimental study is presented and used to validate the effectiveness of the control and synchronization system and to use the designed synchronization system to build and test a secure communication system. Finally, in section 7, paper conclusion has been drawn.

2. THE PROPOSED HYPERCHAOTIC SYSTEM

The following system of equations which have three quadratic and one quartic nonlinearity terms represents the new system:

$$\begin{aligned}\dot{x}_1 &= ax_2 - 12x_1 \\ \dot{x}_2 &= bx_1x_4 \\ \dot{x}_3 &= 10x_1^2 - 12x_3^2 \\ \dot{x}_4 &= 20 - 0.5x_2^4\end{aligned}\tag{1}$$

In this system, a and b are positive real parameters.

System (1) shows chaotic behavior for a certain value of parameters set and for certain initial conditions. The system characterized by its high independent on initial conditions. It has been found that for a certain set of parameters, the system may exhibit chaotic, hyperchaotic, periodic and quasi periodic motions depending on the initial conditions.

3. SYSTEM ANALYSIS

In this section, the system is analyzed. The analysis will mainly base on numerical simulation. Equilibria analysis, Phase portrait of the system, bifurcation diagrams and Lyapunov exponents are the main tools used in analyzing the system.

3.1. Equilibrium points

The equilibrium points play significant role in the dynamical behavior of the nonlinear systems. Finding these points and investigating their types is essential in analyzing nonlinear systems. The only restriction in investigating systems stability of nonlinear systems by linearization technique is the case of zero real part of the Eigen values of one or more of the equilibria points. To obtain the equilibrium points of a

system, the left side of the system equations should be equated to zero. Then, solving for the states of the system (the $X=\{x_1, x_2, x_3, x_4\}$ vector). The equilibrium points have been found as a function of system parameters. The system has 4 equilibria and as listed:

$$\begin{aligned} x_1 &= \left\{ a(2f)^{\frac{1}{4}}, (2f)^{\frac{1}{4}}, \frac{a(2f)^{\frac{1}{4}}}{12}, 0 \right\} \\ x_2 &= \left\{ -a(2f)^{\frac{1}{4}}, -(2f)^{\frac{1}{4}}, \frac{a(2f)^{\frac{1}{4}}}{12}, 0 \right\} \\ x_3 &= \left\{ a(2f)^{\frac{1}{4}}, (2f)^{\frac{1}{4}}, -\frac{a(2f)^{\frac{1}{4}}}{12}, 0 \right\} \\ x_4 &= \left\{ -a(2f)^{\frac{1}{4}}, -(2f)^{\frac{1}{4}}, -\frac{a(2f)^{\frac{1}{4}}}{12}, 0 \right\} \end{aligned} \quad (2)$$

where $(C_{N\Box})_{cone}$ is the normal-force-curve slope of pointed cone with β/λ'_N and η'_{Cyl} , and $(C_{N\Box})_{sphere}$ is the normal-force-curve slope of hemispherical nose.

To determine the types of these points, the spectral or Eigen values of these points are found and as listed in (3).

$$\begin{aligned} eg(x_1) &= \left\{ -12, 0, -2a(2f)^{\frac{1}{4}}, -2(2f)^{\frac{3}{4}} \right\} \\ eg(x_2) &= \left\{ -12, 0, -2a(2f)^{\frac{1}{4}}, 2(2f)^{\frac{3}{4}} \right\} \\ eg(x_3) &= \left\{ -12, 0, 2a(2f)^{\frac{1}{4}}, -2(2f)^{\frac{3}{4}} \right\} \\ eg(x_4) &= \left\{ -12, 0, 2a(2f)^{\frac{1}{4}}, 2(2f)^{\frac{3}{4}} \right\} \end{aligned} \quad (3)$$

Then, all of the Eigen values of the fixed points have zero value, which means that these points are non-hyperbolic. Then, linearization cannot be useful or enough tool to determine the stability and behavior of our system in neighborhoods of the equilibrium points and the overall system stability. The non-hyperbolic points are center points but their stability cannot be ensured by linearization.

To get some indication about the stability of the equilibrium points, the system has been simulated using initial conditions which are close to the equilibrium points. Choosing $a, b = \{17, 20\}$, getting then the equilibrium points of the system at these parameters. The numerical simulation shows that the system admits periodic motion when simulated with initial conditions close to x_1 and x_2 , while it is unstable when using close initial conditions to x_3 and x_4 , where x_1, x_2, x_3 and x_4 are the first to fourth equilibria points. This shows that the system may exhibit chaos behavior with self-excitation or by hidden attractors.

3.2. Effects of the system parameters

To study the effect of the system parameters on the system behavior, bifurcation diagrams have been used. First, the parameter b has been fixed to 7 and a has been changed from 0.1 to 20 with 0.01 as a step size. The system was simulated at each step for 500 second with $\{1, 1, 1, 1\}$ as an initial condition, the maxima of x_2 have been found. We ignored the first 100 second of flow to ensure destroying transient flow. Figure 1 shows the resultant diagram. Then, the same process has been repeated for b with $a=7$, Figure 2 shows the bifurcation diagram in this case.

It is known from literature that there are three main routes to chaos. The first one is period doubling in which the period of periodic motion of the system is doubled at specific point of bifurcation parameter and then and at a specific value of bifurcation parameter the motion becomes chaotic. The second route is the intermittency chaos in which periodic motion is replaced by chaotic at a certain value of bifurcation parameter. The last route is quasi periodic to chaos in which the system motion changes from fixed to periodic and quasi periodic and lastly chaotic motion appear. Analyzing Figures 1 and 2, shows that the system rout to chaos is the intermittency chaos. From this figures, it can also be noted that periodic motion appears interchangeably with chaos motion in some region of bifurcation diagram.

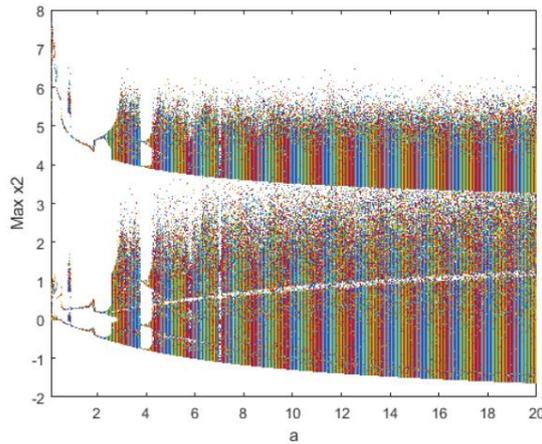


Figure 1. Bifurcation diagram for a parameter

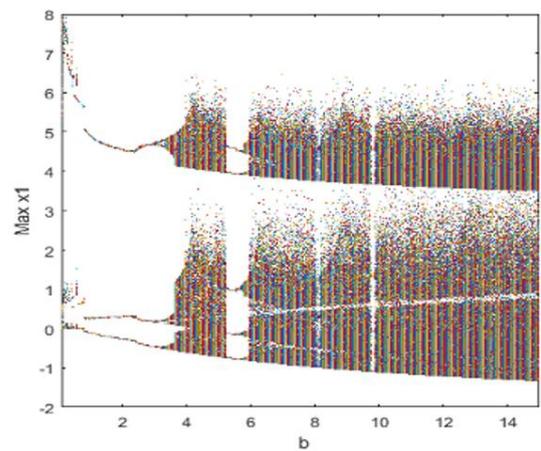


Figure 2. Bifurcation diagram for b parameter

From the bifurcation diagrams, the regions in which the system is chaotic is clear and from these regions we selected $a=17$ and $b=20$. In the rest of this paper the system parameters will be fixed at these values. At these values of system parameters, the system is hyperchaotic. To prove the hyperchaotic nature of the system, we used Lyapunov exponents. The Lyapunov exponents have been found by using Alan wolf algorithm [26]. Figure 3 shows the dynamics of Lyapunov exponents for these values of a and b and for 1000 seconds. The settling values of the exponents are:

$$L_1 = 2.56, L_2 = 0.023, L_3 = -14.6, L_4 = -40.24$$

Then, the first and the second exponents are positive which means that the system is hyperchaotic. Figure 4 shows the 2 dimensional hyperchaotic attractor of the system for x_1, x_2 phase plane and Figure 5 shows phase portrait of the system for x_1, x_3 plane.

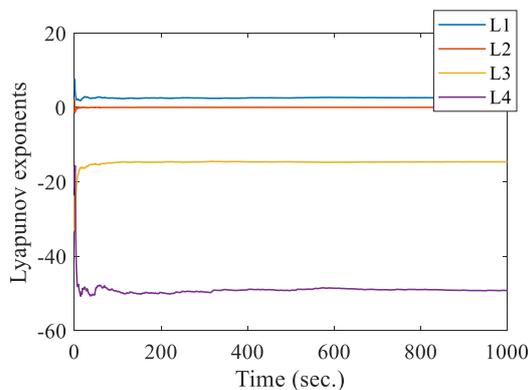


Figure 3. Lyapunov exponents

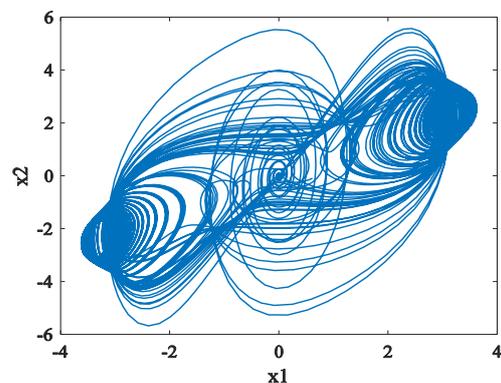


Figure 4. 2 D phase portrait of x_1, x_2

3.3. The effects of the initial conditions

In this subsection, the initial condition effects on the system behavior are studied. As mentioned previously, the system parameters are fixed at $a=17$ and $b=20$.

3.3.1. The sensitivity to the initial conditions

The system shows very high sensitivity to small changes in initial condition. The high sensitivity to initial condition of the system is tested by simulating the system for initial condition $X_0=\{1,1,1,1\}$ and for 100 seconds, then a small perturbation to this initial condition is entered where X_0 becomes $\{1.00001,1,1,1\}$ and the system has been simulated again for the same period of time. Figure 6 shows the motion of x_1 from 10 to 20 seconds. It is clear that the system motion has changed significantly for this small change of IC.

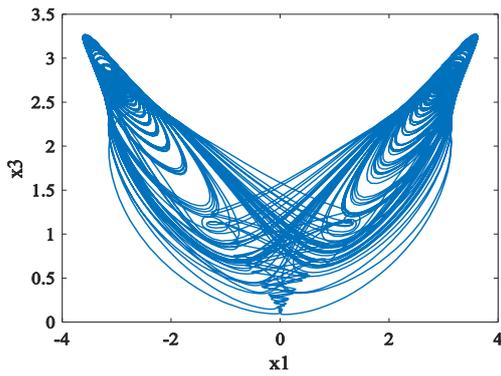


Figure 5. 2 D phase portrait of x_1, x_3

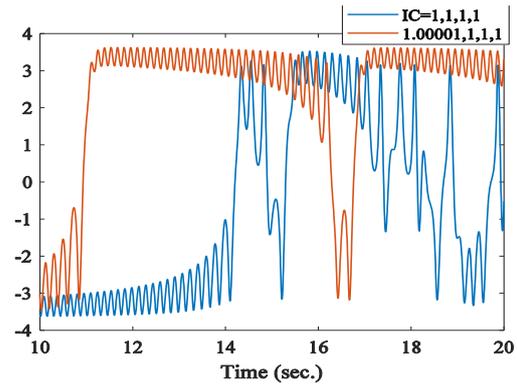


Figure 6. The response of x_1 for 2 close ICs

3.3.2. The effects of the initial conditions on the system behavior

For the selected parameter values, the system is hyperchaotic for most initial conditions as shown previously. Even though the system shows different behavior for some initial conditions. It shows periodic and quasi periodic motions for some ICs. Also, it shows coexisting limit cycle attractors for some initial conditions. It also shows a strange behavior for some initial conditions, where the system motion is settle down to a periodic motion for a long time (more than 400 second), but after that this motion is replaced by a quasi-periodic motion. Then, the system for these ICs shows 2 transient motion, the first is fast and destroyed after a few seconds and the second which is a periodic motion takes a long time and the settling motion is quasi periodic. Table 1 lists some examples of these initial condition’s effects and Figure 7 to 9 show the corresponding responses.

Table 1. The ICs and the corresponding motion description

The initial conditions	The behavior
{1.1,2.5,2.1,0}, {-1.1,-2.5,2.1,0}	2 coexisting limit cycle
{1.1,2.5,2.1,0},{1.11,2.51,2.11,0.01}, {1.12,2.52,2.12,0.02}, {1.12,2.52,2.12,0.02}	4 coexisting limit cycle
{13.4,1,1,1}	Transient, periodic, quasi periodic

3.4. Some dynamical features

In this subsection, some important dynamical properties of the proposed system are found and discussed.

3.4.1. Symmetry

It can easily be noted that the system is invariant under the transformation:

$$(x_1, x_2, x_3, x_4) \rightarrow (-x_1, -x_2, x_3, x_4)$$

which means that if x_1, x_3, x_1, x_3 are a solution for the system equations, then $-x_1, -x_3, x_1, x_3$ is also a solution for it,

3.4.2. Lyapunov dimension

The Lyapunov dimension is a measure of system chaotic behavior degree, it can be defined according to Kaplan-York [27] conjecture and as (5):

$$D_k = J + \frac{\sum_{i=1}^J L_i}{|L_{J+1}|} \tag{5}$$

where J should be selected such that $\sum_{i=1}^J L_i > 0$ and $\sum_{i=1}^{J+1} L_i < 0$.

Then, for our system:

$$D_k = 3 + \frac{\sum_{i=1}^3 L_i}{|L_4|} = 3 + \frac{L_1+L_2+L_3}{|L_4|} = 2.76$$

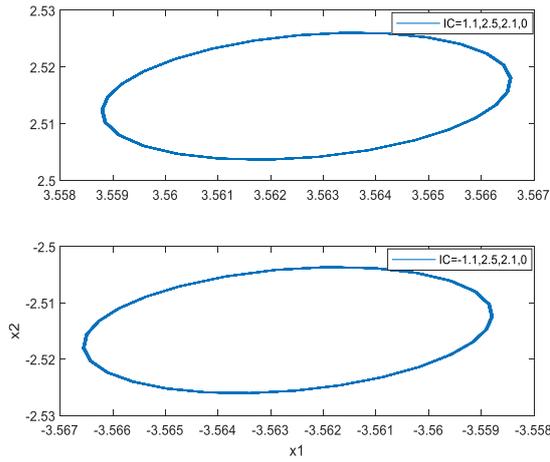


Figure 7. 2 coexisting limit cycles attractors

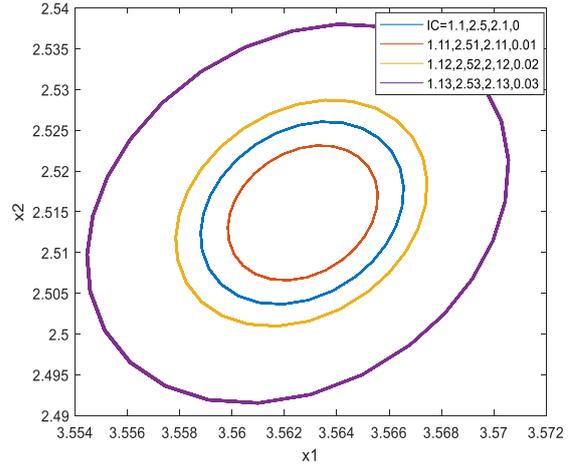


Figure 8. 4 coexisting limit cycles attractors

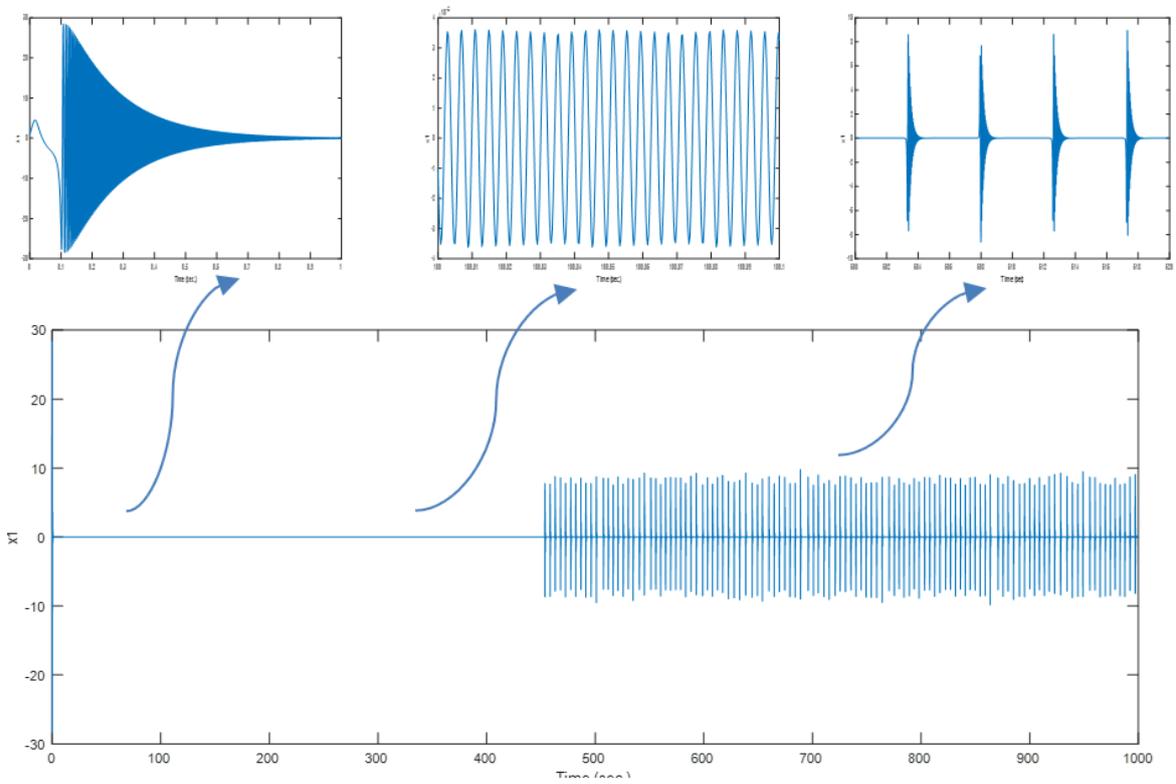


Figure 9. Response of system for IC={13.4,1,1,1}

4. TRACKING CONTROL DESIGN

A controller to track a specific reference signals for the states of the system is designed in this section. To this end, we suppose that a vector of control input signals $U = \{u_1, u_2, u_3, u_4\}$ is added to the system, then the system equations now read:

$$\begin{aligned}
 \dot{x}_1 &= ax_2 - 12x_1 + u_1 \\
 \dot{x}_2 &= cx_1x_3 + u_2 \\
 \dot{x}_3 &= 10x_1^2 - 12x_3^2 + u_3 \\
 \dot{x}_4 &= 20 - 0.5x_2^4 + u_4
 \end{aligned}
 \tag{6}$$

In the controller design, we assumed that the system parameters a and b are unknown, so they need to be estimated. Simple synergetic control design with adaptive laws for unknown parameters estimation are used to design the controller. The design procedure relay on selecting the control inputs such that the following error dynamics of the system is satisfied:

$$\dot{e}_i + k_i e_i = 0, \quad i=1,2,3,4 \quad (7)$$

In this equation, $e_i = r_i - x_i$, r_i is the desired output of the system states x_i and k_i are design parameters which are strictly positive parameters for stable system and their values should be selected such that we get the desired response for error system. Substituting (6) into (7) and solving for U , the following is obtained:

$$\begin{aligned} u_1 &= \dot{r}_1 + k_1 e_1 - a_n x_2 + 12x_1 \\ u_2 &= \dot{r}_2 + k_2 e_2 - b_n x_1 x_3 \\ u_3 &= \dot{r}_3 + k_3 e_3 - 10x_1^2 + 12x_3^2 \\ u_4 &= \dot{r}_4 + k_4 e_4 - 20 + 0.5x_2^4 \end{aligned} \quad (8)$$

where, a_n and b_n are the estimates of a and b . Substituting (8) into (6), the controlled system is obtained:

$$\begin{aligned} \dot{x}_1 &= \dot{r}_1 + k_1 e_1 + (a - a_n)x_2 \\ \dot{x}_2 &= \dot{r}_2 + k_2 e_2 + (b - b_n)x_1 x_3 \\ \dot{x}_3 &= \dot{r}_3 + k_3 e_3 \\ \dot{x}_4 &= \dot{r}_4 + k_4 e_4 \end{aligned} \quad (9)$$

using $x_i = r_i - e_i$ and $\dot{x}_i = \dot{r}_i - \dot{e}_i$ in 9 and after some manipulation, the following error dynamics is obtained:

$$\begin{aligned} \dot{e}_1 &= a_e e_2 - k_1 e_1 - a_e r_2 \\ \dot{e}_2 &= -k_2 e_2 + b_e (r_1 e_3 + e_1 r_3 - r_1 r_3 - e_1 e_3) \\ \dot{e}_3 &= -k_3 e_3 \\ \dot{e}_4 &= -k_4 e_4 \end{aligned} \quad (10)$$

In 10, a_e and b_e are the errors between real and estimated values of a and b , i.e. $a_e = a - a_n$ and $b_e = b - b_n$. Now, to stabilize the dynamics in (10), the estimated values should converge to the unknown parameter values, a Lyapunov theory is used to find update laws for a_n and b_n to converge to a and b and obtaining stable dynamics. The following positive definite function is selected as a Lyapunov function candidate:

$$V(t) = 1/2(e_1^2 + e_2^2 + e_3^2 + e_4^2 + a_e^2 + b_e^2) \quad (11)$$

Differentiating $V(t)$:

$$\dot{V}(t) = e_1 \dot{e}_1 + e_2 \dot{e}_2 + e_3 \dot{e}_3 + e_4 \dot{e}_4 - \dot{a}_n a_e - \dot{b}_n b_e \quad (12)$$

Substituting \dot{e}_i from 10 and rearranging, the following result is obtained:

$$\begin{aligned} \dot{V}(t) &= -k_1 e_1 - k_2 e_2 - k_3 e_3 - k_4 e_4 + a_e (e_1 e_2 - r_2 e_1 - \dot{a}_n) \\ &\quad + b_e (r_1 e_2 e_3 + r_3 e_1 e_2 - e_2 r_1 r_3 - e_1 e_2 e_3 - \dot{b}_n) \end{aligned} \quad (13)$$

If we update a_n and b_n as in (14):

$$\begin{aligned} \dot{a}_n &= e_1 e_2 - r_2 e_1 \\ \dot{b}_n &= r_1 e_2 e_3 + r_3 e_1 e_2 - e_2 r_1 r_3 - e_1 e_2 e_3 \end{aligned} \quad (14)$$

Then

$$\dot{V}(t) = -k_1 e_1 - k_2 e_2 - k_3 e_3 - k_4 e_4 \quad (15)$$

And this is a semidefinite function, ensuring then system stability.

5. SYNCHRONIZATION CONTROLLER DESIGN

In this section, a controller to synchronize two identical of the proposed system is designed. The system (1) is uncontrolled system or the master and its states output should be tracked or synchronized by the second system which is often called the slave in the literature. The Slave system is the same system (1) added to it the control signals $U = \{u_1, u_2, u_3, u_4\}$. Master system is represented by system 1 with a and b assumed to be unknown, while the slave system is represented by the following system:

$$\begin{aligned}\dot{y}_1 &= ay_2 - 12y_1 + u_1 \\ \dot{y}_2 &= cy_1y_3 + u_2 \\ \dot{y}_3 &= 10y_1^2 - 12y_3^2 + u_3 \\ \dot{y}_4 &= 20 - 0.5y_2^4 + u_4\end{aligned}\quad (16)$$

The synchronization error is defined as:

$$e_i = y_i - x_i, i = 1,2,3,4 \quad (17)$$

using this relation with (1), (16) and $\dot{e}_i = \dot{y}_i - \dot{x}_i, i = 1,2,3,4$, the following system of equations is obtained:

$$\begin{aligned}\dot{e}_1 &= ae_2 - 12e_1 + u_1 \\ \dot{e}_2 &= b(y_1y_4 - x_1x_4) + u_2 \\ \dot{e}_3 &= 10(y_1^2 - x_1^2) - 12(y_3^2 - x_3^2) + u_3 \\ \dot{e}_4 &= 0.5(x_2^4 - y_2^4) + u_4\end{aligned}\quad (18)$$

Using similar design procedure used in tracking controller design, our target here also is to satisfy the following error dynamics:

$$\dot{e}_i + k_i e_i = 0, \quad i=1,2,3,4 \quad (19)$$

Combining 19 and 18 and solving for U, we obtain:

$$\begin{aligned}u_1 &= -k_1 e_1 - a_n e_2 + 12e_1 \\ u_2 &= -k_2 e_2 - b_n (y_1 y_4 - x_1 x_4) \\ u_3 &= -k_3 e_3 - 10(y_1^2 - x_1^2) + 12(y_3^2 - x_3^2) \\ u_4 &= -k_4 e_4 - 0.5(x_2^4 - y_2^4)\end{aligned}\quad (20)$$

where a_n and b_n are the estimates of a and b.

Substituting (20) into 18, yields:

$$\begin{aligned}\dot{e}_1 &= -k_1 e_1 + a_e e_2 \\ \dot{e}_2 &= -k_2 e_2 + b_e (y_1 y_4 - x_1 x_4) \\ \dot{e}_3 &= -k_3 e_3 \\ \dot{e}_4 &= -k_4 e_4\end{aligned}\quad (21)$$

where a_e and b_e are the error between the real parameter values of a and b and the estimate values a_n and b_n .

The parameter estimation will be designed by Lyapunov theory. The following function is selected as a Lyapunov function candidate:

$$V(t) = 1/2(e_1^2 + e_2^2 + e_3^2 + e_4^2 + a_e^2 + b_e^2) \quad (22)$$

then,

$$\dot{V}(t) = e_1 \dot{e}_1 + e_2 \dot{e}_2 + e_3 \dot{e}_3 + e_4 \dot{e}_4 - \dot{a}_n a_e - \dot{b}_n b_e \quad (23)$$

and,

$$\dot{V}(t) = -k_1 e_1 - k_2 e_2 - k_3 e_3 - k_4 e_4 + a_e (e_1 e_2 - \dot{a}_n) - b_e (e_2 (y_1 y_3 - x_1 x_3) + \dot{b}_n) \quad (24)$$

If we put:

$$\begin{aligned} \dot{a}_n &= e_1 e_2 \\ \dot{b}_n &= e_2 (y_1 y_4 - x_1 x_4) \end{aligned} \quad (25)$$

then,

$$\dot{V}(t) = -k_1 e_1 - k_2 e_2 - k_3 e_3 - k_4 e_4 \quad (26)$$

which is a negative semidefinite and the prove of system stability is completed.

5.1. System implementation

The proposed system has been implemented using microcontroller platform. We propose implementing of the system using the so-called Arduino MCU board, where Arduino uno board has been used to implement the system. Arduino boards are easy to use MCUs development boards. The used board is the Arduino uno which has 32 KB flash memory for storing the program and 2 KB of SRAM memory for storing program data and 1 KB EEPROM which can be used for storing static data of the program. These specifications can cover all the requirements of implementing our system. The implementation program uses fourth order Runge-Kuta method to solve the ODE system of equations of our system with step size equal to 0.001 second. For the purpose of testing the system, we have used MATLAB to plot the phase portraits of the system states plane where the MCU board and laptop have been connected via serial communication. Appendix A shows the implementation program and appendix B shows the program used to receive the data from MCU board. Figure 10 shows the phase portrait of x_1, x_2 plane received from MCU board for 30 seconds run time.

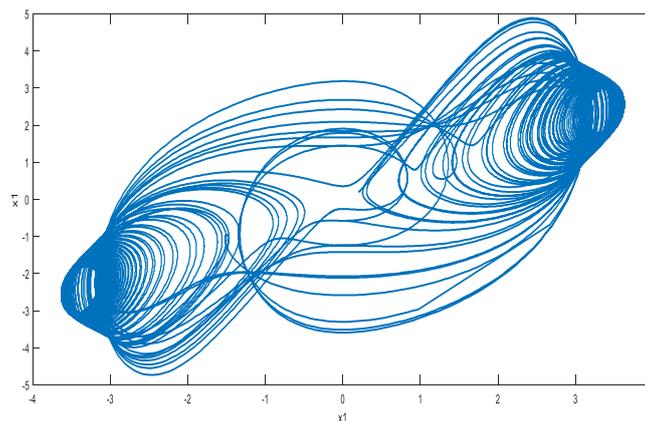


Figure 10. x_1, x_2 portrait of the implemented system

6. SIMULATION AND EXPERIMENTAL STUDY

In this section, the theoretical results of tracking control system and synchronization of the proposed hyperchaotic system are investigated by simulation and experimentally. Also, a secure communication using our synchronization system is introduced and tested. As mentioned earlier, MATLAB 2018a script is used to write the simulation programs. The experimental study relayed on the system implemented on the MCU Arduino board and as described in the previous section.

6.1. Control system

The tracking control system has been tested by simulation with two vectors of reference signals. First, we assumed that the system is to be stabilized from initial conditions, then, Reference signals= $R = \{r_1, r_2, r_3, r_4\} = \{0, 0, 0, 0\}$ The initial conditions for the states $X(0) = \{x_1(0), x_2(0), x_3(0), x_4(0)\} = \{1, -1, 3, 2\}$. The states responses of the system are shown in Figure 11. Next, sinusoidal signals are assumed as a reference signal. Using the same initial conditions, the responses are shown in Figure 12. From Figure 11 and 12, it is very clear that the designed control system performance is excellent.

$$R = \{r_1, r_2, r_3, r_4\} = \{\sin(20t), 2\sin(20t), 3\sin(20t), 4\sin(20t), \}$$

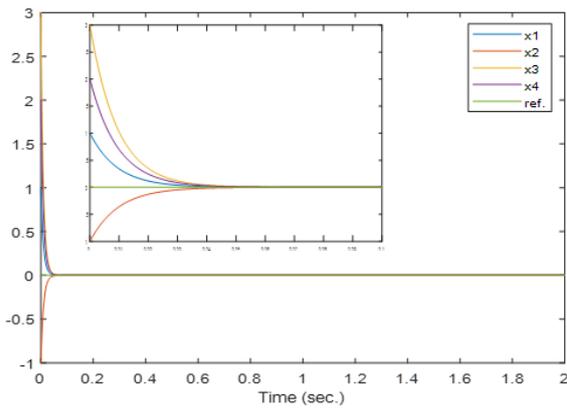


Figure 11. System stabilization from initial conditions

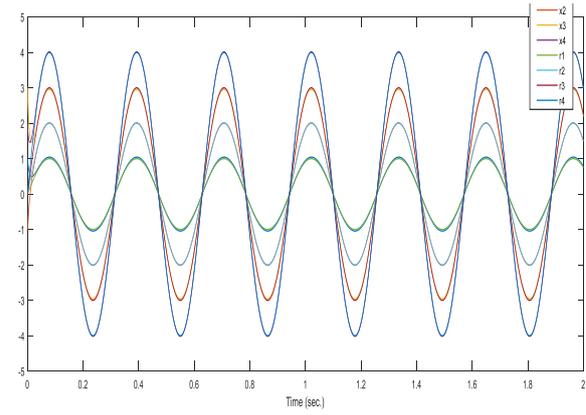


Figure 12. Tracking system responses for sinusoidal signals

6.2. Synchronization system

The designed synchronization system is tested by using the MCU based system designed in the previous section as a master and the slave system is implemented on computer side using MATLAB with the following initial conditions for master and slave respectively. The synchronization errors are shown in Figure 13. This figure shows an excellent synchronization error response.

$$X = \{x_1, x_2, x_3, x_4\} = \{1, 2, 3, 1\}$$

$$Y = \{y_1, y_2, y_3, y_4\} = \{2, 1, 1, 2\}$$

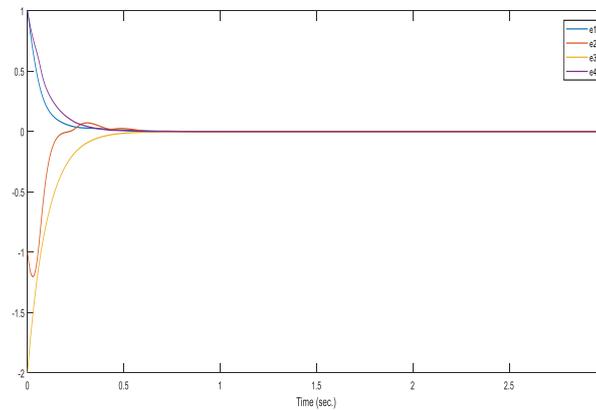


Figure 13. Errors of the synchronization system

6.3. Secure communication application

In this subsection, an application of chaotic system synchronization is presented. In analog secure communication using synchronized chaotic systems, the transmitted signal (*sig*) is added to one of the master systems states (say x_1) to form the scripted signal (*tr*) to be transmitted to the receiver side with other signal required by slave to synchronize its output with master system. At the receiver side, the scripted (*tr*) signal is subtracted from the counterpart state of the slave (y_1 in our case) to form the decrypted signal (*rs*), then:

At the transmitter:

$$tr = sig + x_1$$

And at the receiver:

$$rs = tr - y_1$$

It is clear that if full synchronization is achieved, signal is completely retrieved, i.e. $rs=sig$.

In our test, we used the implemented system on MCU based as a transmitter side and the laptop as a receiver side. A sinusoidal signal ($sig=4\sin(30t)$) is selected as a signal to be transmuted. Figure 14 shows the transmitted (sig) signal and the received (rs) signal. From this figure, it can be noted that the transmitted signal is tracked at the receiver with no error and in very short time.

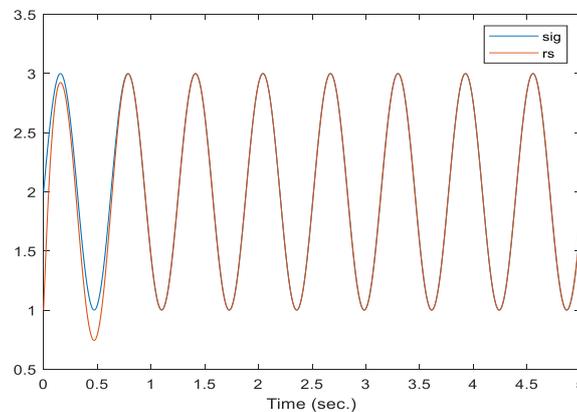


Figure 14. The transmitted and received signals and error

7. CONCLUSION

In this paper, a new 4-dimensional hyperchaotic system has been proposed. The system has been extensively analyzed and its dynamical properties have been investigated. It has been found that the system has some interesting features and strange behaviors. After that, a tracking and synchronization systems for the introduced system have been designed, assuming unknown system parameters. The design procedure depended on simple controller with Lyapunov theory for the purpose of deriving update laws for the unknown parameters. Hardware realization is proposed where MCU board is used to implement the system. It is well known that MCU implementation gives a lot of features like cost effective, reliability and flexibility. Simulation and experimental study have showed the effectiveness of the designed control and synchronization systems and also the effectiveness of the secure communication system which can be designed basing on the designed synchronization system.

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