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Chaos and bifurcation in time delayed third order phase-locked loop

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ABSTRACT

In this paper, the modern nonlinear theory is applied to a third order phase locked loop (PLL) with a feedback time delay. Due to this delay, different behaviors that are not accounted for in a conventional PLL model are identified, namely, oscillatory instability, periodic doubling and chaos. Firstly, a Pade approximation is used to model the time delay where it is utilized in deriving the state space representation of the PLL under investigation. The PLL under consideration is simulated with and without time delay. It is shown that for certain loop gain (control parameter) and time delay values, the system changes its stability and becomes chaotic. Simulations show that the PLL with time delay becomes chaotic for control parameter value less than the one without time delay, i.e., the stable region becomes narrower. Moreover, the chaotic region becomes wider as time delay increases.

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1. INTRODUCTION

A phase-locked loop (PLL) is a versatile device used mainly in carrier synchronization, frequency synthesis, clock recovery, wireless communications and phase inverters [1-4]. When the PLL operates in the phase-locked state, the dynamic behavior of the loop is studied using linear theory. Unfortunately, the PLL may operate in the out-of-lock and in this case the dynamics behavior of the loop follows the nonlinear theory and analyzing this behavior becomes tedious [5-12]. Chaos and complex bifurcations are inherent to nonlinear systems due to dynamical instabilities. Chaos induced in phase locked loop was investigated by many researchers. Endo and Chua [13] proved the existence of horseshoe chaos in second order PLL using Melnikov's method. Later, Bradley and Straub [14] showed that chaotic PLL circuits sometimes can be useful. In fact, they utilized chaotic PLL to broaden the capture range of the PLL. Harb and Harb [15] applied modern nonlinear theory to analyze the chaotic behavior observed in a third order PLL with sinusoidal phase detector characteristics. Sarkar and Chakraborty [16] studied self-oscillations of a third order PLL in periodic and chaotic mode. Fortuna, *et al*. [17] used chaotic pulse position modulation to improve the efficiency of sonar sensors.

In recent years, many researchers studied the dynamic instabilities induced in feedback systems due to the time delay of signals [18-20]. This delay effect causes an oscillatory behavior which has been reported in nonlinear systems especially in radio engineering. Later, numerous experimental and theoretical studies have demonstrated that many nonlinear delay systems experienced a chaotic behavior as a result of dynamic instabilities. Such instabilities include period-doubling route to chaos, quasi periodicity and intermittency [21-24]. Moreover, studies have shown that the dimension of the resulted chaotic attractor is directly proportional to the time delay induced in the system independent of the form of the system. In this case, one can obtain high-dimensional chaotic attractors by increasing the time delay in the system [25-27]. This method should be performed with caution since the state space representation of a nonlinear delay system constitutes a finite-dimensional space, whereas, the dynamics span an infinite-dimensional space.

Delay effect in phase locked loops was firstly investigated by Schanz and Pelster [28] where they proved the existence of a hopf bifurcation in first order PLL with time delay using the method of multiple scale. Buckwalter and York [29] studied time delay in high-frequency phase-locked loop. Grant *et al.* [30] investigated the performance of optical phase-locked loops in the presence of non negligible loop propagation delay.

In this paper chaos and bifurcation theory will be applied to a third order phase locked loop considering a feedback time delay. Pade approximation will be used to derive the state space representation of a third order PLL. The chaotic behavior of the third order PLL with and without delay will be compared, and delay will be used as a control parameter. Unlike first and second order PLLs, third order PLL exhibit a chaotic behavior in the absence of delay [15] since the order condition for chaotic behavior in nonlinear system is valid. This paper is organized as follows: Section 2 contains the mathematical model of the PLL under consideration without delay, where the main results from previous work are presented. Also, the mathematical model and the derivation of the nonlinear differential equation describing the dynamics of the PLL under consideration with time delay is presented in this section. Simulation and discussion of the results is presented in section 3 and section 4 contains the conclusions and future work.

2. RESEARCH METHOD

2.1. Mathematical model of third order PLL without delay

The classical model of a third order phase locked loop is shown in Figure 1. It consists of a sinusoidal phase detector, a second order loop filter and a voltage controlled oscillator (VCO).

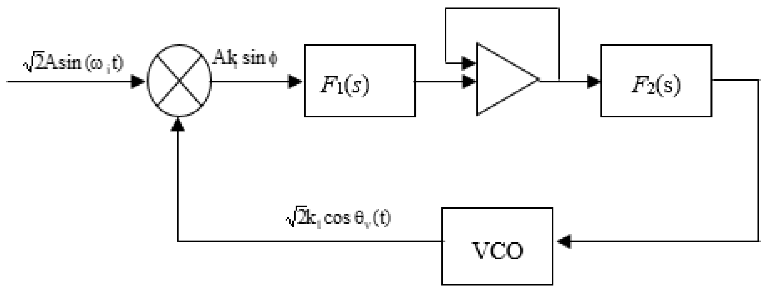

Figure 1. Block diagram of a phase-locked loop without time delay

The differential equation that describes the closed loop phase error in the PLL under consideration is given by [15]:

$$\frac{d\phi}{dt} = \frac{d\theta}{dt} - k\left[\frac{1+\tau_{z1}\, d/dt}{1+\tau_{p1}\, d/dt}\cdot\frac{1+\tau_{z2}\, d/dt}{1+\tau_{p2}\, d/dt}\right]\sin(\phi(t)) \tag{1}$$

A simplified form of (1) can be written as:

$$\frac{d^3\phi}{dt^3}+\frac{d^2\phi}{dt^2}\left(\frac{\tau_{p1}+\tau_{p2}}{\tau_{p1}\tau_{p2}}+\frac{k\tau_{z1}\tau_{z2}\cos\phi}{\tau_{p1}\tau_{p2}}\right)+\frac{d\phi}{dt}\left(\frac{1}{\tau_{p1}\tau_{p2}}+\frac{k(\tau_{z1}+\tau_{z2})\cos\phi}{\tau_{p1}\tau_{p2}}\right)-\frac{k\tau_{z1}\tau_{z2}\sin\phi}{\tau_{p1}\tau_{p2}}\left(\frac{d\phi}{dt}\right)^2$$
$$+\frac{k\sin\phi}{\tau_{p1}\tau_{p2}}=\frac{d^2\theta_1}{dt^2}+\frac{\tau_{p1}+\tau_{p2}}{\tau_{p1}\tau_{p2}}\frac{d^2\theta_1}{dt^2}+\frac{1}{\tau_{p1}\tau_{p2}}\frac{d\theta_1}{dt} \tag{2}$$

where τ_{p1}, τ_{p2}, $\tau_{z1,}$ and τ_{z2} are the loop filters time constants, k is the overall closed loop gain and ϕ is the closed loop phase error. Assume the input frequency is constant and normalize the time variable using $t'=(k/\tau_{p1}\tau_{p2})^{1/3}\, t$, the above equation becomes;

$$\dddot{\phi} + a\ddot{\phi} + b\cos(\phi)\,\ddot{\phi} + c\dot{\phi} + d\cos(\phi)\,\dot{\phi} - e\sin(\phi)\,\dot{\phi}^2 + \sin(\phi) = \delta \tag{3}$$

where $\cdot = d/dt'$, $\nu = (k/\tau_{p1}\tau_{p2})^{1/3}$, $a = (\tau_{p1}+\tau_{p2})\nu^2/k$, $b = \tau_{z1}\tau_{z}\nu^2$

$c = \nu/k$, $d = (\tau_{z1} + \tau_{z2})\nu$, $e = \tau_{z1}\tau_{z2}\nu^2$, $\delta = \omega_{os}/k$ and $\omega_{os} = \omega_{i} - \omega_{o}$

with $x_1 = \phi$, $x_2 = \dot{\phi}$, (3) becomes:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= -ax_3 - bx_3\cos(x_1) - cx_2 - dx_2\cos(x_1) + ex_2^{\,2}\sin(x_1) - \sin(x_1) + \delta \end{aligned} \tag{4}$$

Harb and Harb [15] showed that the system had a chaotic oscillation at normalized gain value of $k = 76300$ as shown in Figure 2. The system remains in chaotic region for gain value up to $k = 100000$.

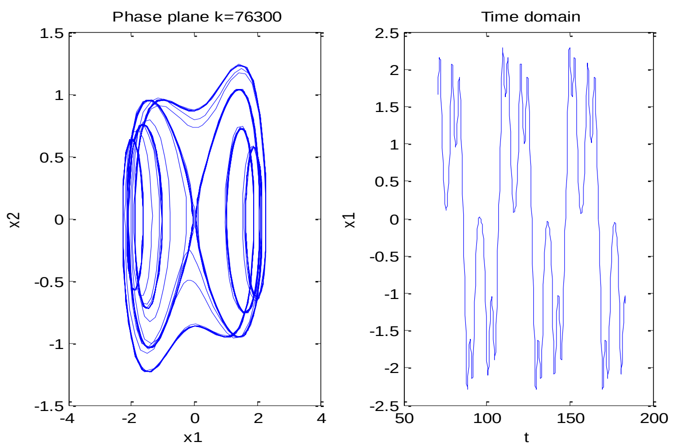

Figure 2. Chaotic behavior at normalized gain k=76300

2.2. Mathematical model of third order pll with time delay

Due to the present of the delay element, the differential equation that describes the closed loop phase error becomes:

$$\frac{d\phi}{dt} = \frac{d\theta}{dt} - k\left[\frac{1+\tau_{z1}\,{}^{d}\!/_{dt}}{1+\tau_{p1}\,{}^{d}\!/_{dt}} \cdot \frac{1+\tau_{z2}\,{}^{d}\!/_{dt}}{1+\tau_{p2}\,{}^{d}\!/_{dt}}\right]\sin(\phi(t-\tau)) \tag{5}$$

and after simplifications, the nonlinear ordinary differential equation becomes:

$$\begin{aligned} &\frac{d^3\phi}{dt^3} + \left(\frac{\tau_{p1}+\tau_{p2}}{\tau_{p1}\tau_{p2}}\right)\frac{d^2\phi}{dt^2} + \frac{1}{\tau_{p1}\tau_{p2}}\frac{d\phi}{dt} + k\frac{\tau_{z1}\tau_{z2}}{\tau_{p1}\tau_{p2}}\cos(\phi(t-\tau))\frac{d^2\phi}{dt^2} + \frac{k}{\tau_{p1}\tau_{p2}}\sin(\phi(t-\tau)) + \\ &\frac{k(\tau_{z1}\tau_{z2})}{\tau_{p1}\tau_{p2}}\frac{d}{dt}(\phi(t-\tau))\cos(\phi(t-\tau)) - k\frac{\tau_{z1}\tau_{z2}}{\tau_{p1}\tau_{p2}}\left(\frac{d}{dt}(\phi(t-\tau))\right)^2\sin(\phi(t-\tau)) = \frac{\omega_{os}}{\tau_{p1}\tau_{p2}} \end{aligned} \tag{6}$$

Let $A_1 = \dfrac{\tau_{p1}+\tau_{p2}}{\tau_{p1}\tau_{p2}}$, $A_2 = \dfrac{1}{\tau_{p1}\tau_{p2}}$, $A_3 = \dfrac{\tau_{z1}\tau_{z2}}{A_2}$, and $A_4 = \dfrac{\tau_{z1}+\tau_{z2}}{A_2}$, then (6) becomes:

$$\dddot{\phi} + A_1\ddot{\phi} + A_2\dot{\phi} + kA_3\cos(\phi(t-\tau))\ddot{\phi}(t-\tau) + kA_2\sin(\phi(t-\tau)) + kA_4\dot{\phi}(t-\tau)\cos(\phi(t-\tau)) - kA_3(\dot{\phi}(t-\tau))^2\sin(\phi(t-\tau)) = \frac{\omega_{os}}{A_2} \quad (7)$$

Define the state variables $x_1 = \phi,\ x_2 = \dot{\phi},\ x_3 = \ddot{\phi},\ x_4 = \phi(t-\tau),\ x_5 = \dot{\phi}(t-\tau),\ x_6 = \ddot{\phi}(t-\tau)$

The first five state equations are directly derived from (7) and the state variables defined above. The last state equation will be derived using Pade approximation. Using the state variables defined above, (7) becomes:

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= -A_1x_3 - A_2x_2 - kA_3\cos(x_4)x_6 - kA_2\sin(x_4) - kA_4x_5\cos(x_4) + kA_3x_5^2\sin(x_4) + \frac{\omega_{os}}{A_2} \\
\dot{x}_4 &= x_5 \\
\dot{x}_5 &= x_6
\end{aligned} \quad (8)$$

To derive the six[th] state space equation, we use the first order Pade approximation for the delay element which is given by $e^{-s\tau} = \frac{1-s\frac{\tau}{2}}{1+s\frac{\tau}{2}}$. Starting with $x_4(t)=\phi(t-\tau)$ and taking the Laplace transform of both sides yield

$$X4(s) = \Phi(s)e^{-s\tau} = X_1(s)\frac{1 - s\frac{\tau}{2}}{1 + s\frac{\tau}{2}} \quad (9)$$

In time domain, this equivalent to:

$$x_1 - \frac{\tau}{2}\dot{x}_1 = x_4 + \frac{\tau}{2}\dot{x}_4 \quad (10)$$

By differentiating (10), and using (8) we get:

$$x_2 - \frac{\tau}{2}x_3 = x_5 + \frac{\tau}{2}x_6 \quad (11)$$

Then we differentiate (11) to get:

$$\dot{x}_6 = -\frac{2}{\tau}x_6 - \frac{2}{\tau}x_3 - A_1x_3 + A_2x_2 + kA_3\cos(x_4)x_6 + kA_2\sin(x_4) + kA_4x_5\cos(x_4) - \frac{\omega_{os}}{A_2} - kA_3x_5^2\sin(x_4) \quad (12)$$

By combining (8) and (12), the third order PLL with delay is transformed into a system of sixth order ordinary differential equations with a state space representation given by:

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= -A_1x_3 - A_2x_2 - kA_3\cos(x_4)x_6 - kA_2\sin(x_4) - kA_4x_5\cos(x_4) + kA_3x_5^2\sin(x_4) + \frac{\omega_{os}}{A_2} \\
\dot{x}_4 &= x_5 \\
\dot{x}_5 &= x_6
\end{aligned}$$

$$\dot{x}_6 = -\frac{2}{\tau}x_6 - \frac{2}{\tau}x_3 - A_1x_3 + A_2x_2 + kA_3\cos(x_4)x_6 + kA_2\sin(x_4)$$
$$+kA_4x_5\cos(x_4) - \frac{\omega_{os}}{A_2} - kA_3x_5^2\sin(x_4) \tag{13}$$

3. RESULTS AND DISCUSSIONS

In this section, the equilibrium and dynamic solutions of the system are obtained. Firstly, by setting the right hand side of (13) to zero, the equilibrium solution $\phi(t)$ is obtained. The dynamic solution is found by varying the control parameters (k is used here) and using the continuation scheme method. The stability of the solutions will be studied using the Jabcobian matrix. The eigenvalues of the Jacobian matrix evaluated at the equilibrium point (as a function of k) determine the stability of the solution and the type of bifuractions occur as the controlling parameter is varied. In this paper, we wrote our own program for calculating the equilibrium points and the type of bifurcations occurred as the controlling parameter varied.

Simulation is prformed with time delay of 0.15 μsec and different values of normalized control paprameter, k. for $k = 2.5$, the equlibrium solution (constant solution) is obtained as shown in Figure 3(a) below. As k increases, the system will lose its stability via a Hopf bifurcation point H at $k_o = 3.28$, and a periodic solution is born as shown in Figure 3(b). This Hopf bifurcation point is found to be a supercritical point based on the eigenvalus of the Jacobiam matrix. For k>3.28, a sequence of deformed (asymetric) periodic solutions are observed, as shown in Figures 3(c) and (d) leading to chaos at $k = 23$ as shown in Figure 4. Table 1 shows the instability of phase error for different values of the open loop gain at constant time delay of 0.15 μ sec.

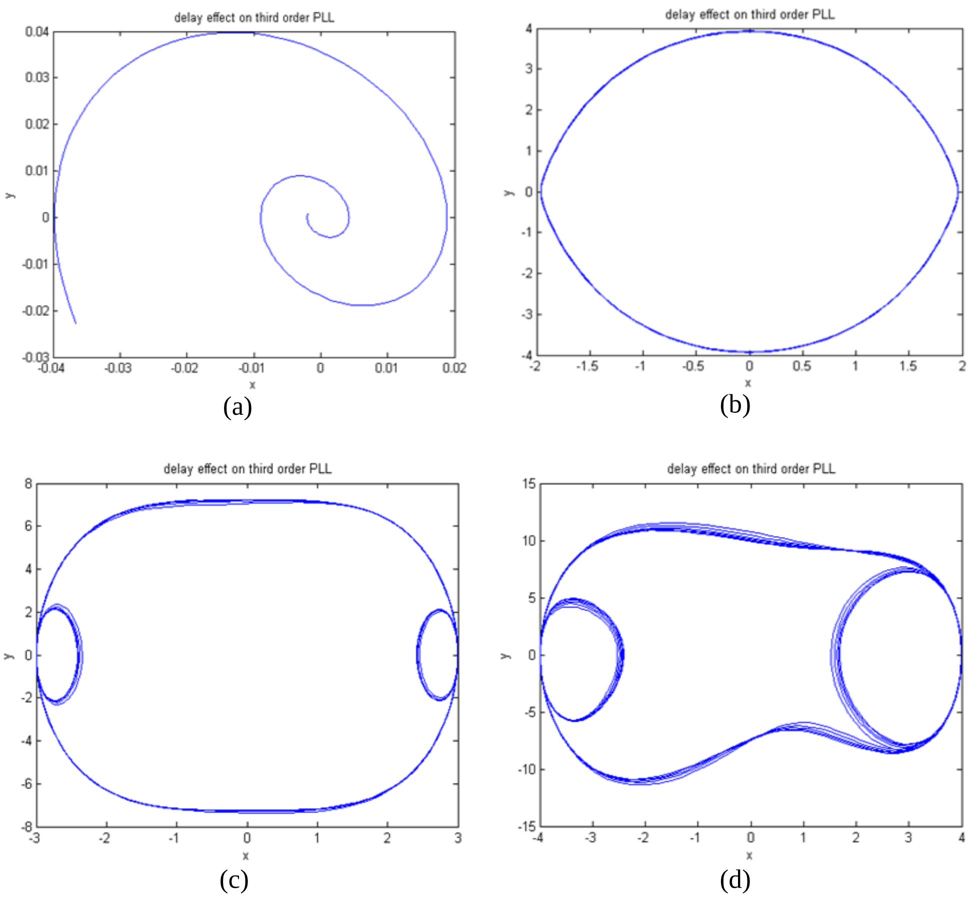

Figure 3. Phase plane plots of time delay third order PLL for different normalized loop gain (k) with $\tau = 0.15$ μsec, $x = x1, y = x2$, (a) Stable solution for $k = 2.5$, (b) Oscillatory solution for $k = 3.28$, (c) Period -2 bifurcation for $k = 6$, (d) Period -4 bifurcation for $k = 9.6$

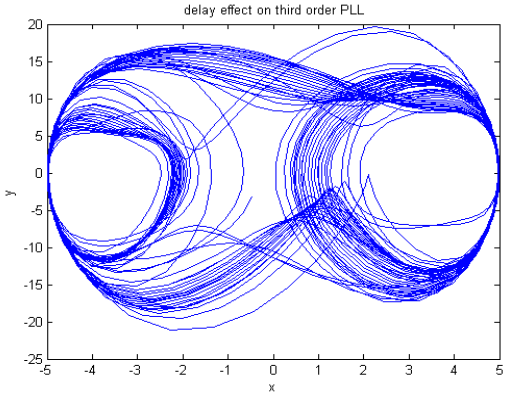

Figure 4. Phase plane plot for chaotic behavior at normalized gain $k = 23$, $\tau = 0.15$ µsec with $x = x1$ and $y = x2$

Table 1. Instability of phase error for different values of the open loop gain for third order PLL with constant time delay (0.15 µsec)

Normalized Open Loop Gain(k)	Instability
<3.28	Dc output
3.28	Oscillatory instability
6	Period-2 bifurcation
9.6	Period-4 bifurcation
13.73	Period-8 bifurcation
23	Chaos

Tables 2 shows the effect of time delay on the instabilities of the solution and its effect on the chaotic region. Without delay, the oscillatory behavior of the system starts at open loop gain of 7.341 and the chaotic behavior begins to appear at open loop gain equal of 76.3 and remains at this for values up to $\mathrm{k} = 100$. On the other hand, for time delay $= 0.15\ \mu sec$, the oscillatory behavior starts at k=3.28 and chaos starts at 23 and remains in this state for values of k up to 118.2. It is clear that the stable region becomes narrower and the chaotic region becomes wider as time delay increases.

Table 2. Instability of phase error for different values of the open loop gain for third order PLL with different time delay

Open loop gain (k) delay (1.3 µs)	Open loop gain (k) delay (0.5 µs)	Open loop gain (k) delay (0.15 µs)	Open loop gain (k) (no delay)	Instability
<0.57	<1.13	<3.28	<7.341	Dc output
0.57	1.13	3.28	7.341	Oscillatory instability
1.8	3.78	6	23.1	Period-2 bifurcation
3.5	6.2	9.6	40	Period-4 bifurcation
6	8.9	13.73	68.30	Period-8 bifurcation
9.22	13.1	23	76.300	Chaos
9.22-156	13.1-142	23-118.2	76.3-100	Chaotic region

4. CONCLUSION

In this paper, new results on nonlinear analysis of third order phase locked loop (PLL) with feedback time delay are reported. We used the modern nonlinear theory to study the effect of time delay on the stability of the solution and chaotic behavior of the PLL under investigation A first order Pade approximation was used to derive the state space representation of third order PLL. Different behavior were identified for this class of PLL's, namely, oscillatory instability, periodic doubling and chaos. It was shown that for different values of gain and time delay, the system changes its stability that leads to chaos. The study showed that as time delay increases, the PLL loses its stability faster and hence drives the PLL into chaos and broaden the chaotic region. Finally, one concluded that the effect of the time delay is really bad on the stability of the third order PLL. This study could be extended to show the effect of time delay on the capture range, pull-in range and other design parameters of third order phase locked loop with a higher order of Pade approximation.

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Ahmad M. Harb (M'98-SM'06) was born in Irbid, Jordan, in 1963. He received the Ph.D. degree from Virginia Polytechnic Institute and State University, Blacksburg, VA, in 1996, the M.S. degree from the Jordan University of Science and Technology, Irbid, Jordan, in 1990, and the B.S. degree from Yarmouk University, Irbid, Jordan, in 1987, all in electrical engineering.,Currently, Ahmad is a Professor at German Jordanian University, was a Visiting Professor at Virginia Tech., Blacksburg, VA, in 1997 and at Oakland University, Rochester, MI, in 2001. Ahmad has published more than 55 papers in various international journals and conferences. Ahmad research interests include power system analysis and control, modern of nonlinear theory (bifurcation and chaos), linear systems, power system planning, electric machines, optimal control, and power electronics. Prof. Harb is the Editor-in-Chief for the International Journal of Modern Nonlinear Theory and Applications (IJMNTA), USA. Prof. Harb received National Science Foundation (NSF) funded projects jointly with the University of Central Florida, Orlando.