

Design of an adaptive state feedback controller for a magnetic levitation system

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ABSTRACT

This paper presents designing an adaptive state feedback controller (ASFC) for a magnetic levitation system (MLS), which is an unstable system and has high nonlinearity and represents a challenging control problem. First, a nonadaptive state feedback controller (SFC) is designed by linearization about a selected equilibrium point and designing a SFC by pole-placement method to achieve maximum overshoot of 1.5% and settling time of 1s (5% criterion). When the operating point changes, the designed controller can no longer achieve the design specifications, since it is designed based on a linearization about a different operating point. This gives rise to utilizing the adaptive control scheme to parameterize the state feedback controller in terms of the operating point. The results of the simulation show that the operating point has significant effect on the performance of nonadaptive SFC, and this performance may degrade as the operating point deviates from the equilibrium point, while the ASFC achieves the required design specification for any operating point and outperforms the state feedback controller from this point of view.

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1. INTRODUCTION

Magnetic levitation technology has recently become an interesting topic of study, since it is a good solution for many motion systems [1, 2]. The advantages of a MLS are its abilities to eliminate friction by eliminating the contact between moving and stationary parts [3], decreasing the cost of maintenance, and achieving precise position [4]. The MLSs has become suitable for trains, bearings, vibrating isolation systems, and levitation of wind tunnel [1, 4].

By magnetic levitation, a ferromagnetic mass is suspended in the air by an electric magnetic field [5]. The basic control aim is to precisely position the levitating object [6]. To stabilize the MLS, the magnetic field strength must be varied by changing the current of the coil [5, 7]. Since the MLS is unstable and has high nonlinearity, designing a controller for this system with adequate specifications is not a trivial task; thus, the control of this system has received considerable interest [4], and it has become a platform to test different control algorithms [1, 5].

Several control approaches were used to stabilize the MLS, such as feedback linearization [8-10], which requires an accurate model of this system; however, obtaining an accurate model represents a problem because of the high nonlinearity of this system and the variation of the gain parameter with the distance between the levitating object and the magnet. Linearization-based methods were also used, where the system is linearized about a certain equilibrium point and a controller is designed to stabilize the system, such as PID controller [1, 2, 5, 6, 7, 11], fractional order PID controller [4, 12-15], LQR [1, 2, 16, 17], lead compensator [1], H_∞ controller [18, 19], fuzzy logic controller (FLC) [16, 20, 21], and adaptive FLC [22]; however, the performance of such controllers degrade when the deviation between the operating

point and the equilibrium point (the point that the system was linearized about) increases. To handle this problem, sliding mode controller (SMC) [23-25], adaptive SMC [26], PID-notch filters [27], and linearization-gain scheduling controller PID controller [28], linearization-gain scheduling PI controller [29], and linearization-adaptive PD controller [30] were designed to provide robustness against operating point variation. This paper proposes an ASFC to stabilize the MLS, where the controller parameters become a function of the operating point, and pole placement method is used to design the controller. The rest of this paper is: section 2 presents the mathematical model of the MLS, section 3 presents the design of an ASFC for this system by pole placement, simulation results and discussions are given in section 4, and finally the conclusions that can be drawn from the obtained results are given in section 5.

2. MATHEMATICAL MODEL OF THE MLS

In a MLS, a ferromagnetic ball is levitated by a magnetic field, and the ball position is fed back to control the current of the coil [31]. The position of the ball is

$$m\ddot{y} = -k\dot{y} + mg + F(y, i) \quad (1)$$

where m and y are the mass the vertical position of the ball, k is a viscous friction coefficient, g is the gravity acceleration, $F(y, i)$ is the electromagnet force, and i is the coil current [31]. The inductance which is a function of the ball position is approximately

$$L(y) = L_1 + \frac{L_0}{1+\frac{y}{a}} \quad (2)$$

where L_1 is the electromagnetic coil inductance without the suspended ball, L_0 is the inductance due to the ball, and a is the air gap when the levitated ball is in equilibrium [32]. The inductance has its highest value $L_1 + L_0$ as the ball touches the magnet and decreases to L_1 when it is removed away from the coil. If $E(y, i) = \frac{1}{2}L(y)i^2$ is the electromagnet stored energy, the force F is [31]

$$F(y, i) = \frac{\partial E}{\partial y} = -\frac{L_0 i^2}{2a(1+\frac{y}{a})^2} \quad (3)$$

The magnetic flux linkage is

$$\phi = L(y)i \quad (4)$$

and according to Kirchoff's voltage law, the coil voltage is

$$v = \dot{\phi} + Ri \quad (5)$$

where R is the circuit resistance. Using $x_1 = y$, $x_2 = \dot{y}$, and $x_3 = i$ as state variables, $u = v$ as control input, and y as the controlled output, the state matrix equation and the output equation become:

$$\dot{\mathbf{x}} = \begin{bmatrix} x_2 \\ g - \frac{k}{m}x_2 - \frac{L_0 a x_3^2}{2m(a+x_1)^2} \\ \frac{1}{L(x_1)} \left(-Rx_3 + \frac{L_0 a x_2 x_3}{(a+x_1)^2} + u \right) \end{bmatrix} = \mathbf{f}(\mathbf{x}, u) \quad (6)$$

$$y = x_1 \quad (7)$$

The equilibrium point of system (6) can be found by setting $\dot{\mathbf{x}} = 0$. If this point is designated by $(\mathbf{x}_{ss}, u_{ss})$ where $\mathbf{x}_{ss} = [x_{1ss} \ x_{2ss} \ x_{3ss}]^T = [r, x_{2ss}, I_{ss}]^T$, and $u_{ss} = V_{ss}$, then

$$0 = x_{2ss}, \quad (8)$$

$$0 = g - \frac{k}{m}x_{2ss} - \frac{L_0 a I_{ss}^2}{2m(a+r)^2} \quad (9)$$

$$0 = \frac{1}{L(r)} \left(-RI_{ss} + \frac{L_0 a x_2 I_{ss}}{2m(a+r)^2} + V_{ss} \right) \quad (10)$$

Solving (8)-(10) for r , x_{2ss} , and I_{ss} in terms of V_{ss} yields

$$[r, x_{2ss}, I_{ss}]^T = \left[\frac{1}{R} \sqrt{\frac{L_0 a}{2mg}} V_{ss} - a, 0, \frac{V_{ss}}{R} \right]^T \quad (11)$$

The linearization of system (6) about the equilibrium point (x_{ss}, u_{ss}) is

$$\dot{x} = Ax + Bu \quad (12)$$

where $A = \left. \frac{\partial f}{\partial x} \right|_{(x_{ss}, u_{ss})}$ and $B = \left. \frac{\partial f}{\partial u} \right|_{(x_{ss}, u_{ss})}$. For any equilibrium point (x_{ss}, u_{ss}) , at least one of the three eigenvalues of matrix A has positive real part. Thus, by indirect Lyapunov's Theorem, the system is unstable. The values of the parameters of the MLS are given in Table 1.

Table 1. Parameters of the MLS

Parameter	Description	Value
m	Mass of the ball	0.1kg
k	Viscous friction coefficient	0.001N/m/s
g	Gravity acceleration	9.81m/s ²
a	Air gap when the levitated ball is in equilibrium	0.05m
L_0	Inductance due to levitated ball	0.01H
L_1	Electromagnetic coil inductance without the suspended ball	0.02H
R	Series resistance of the circuit	1Ω

3. CONTROLLER DESIGN

To demonstrate the enhanced performance of the proposed ASFC, a nonadaptive SFC

$$u_f = -Kx = [k_1 \quad k_2 \quad k_3] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (13)$$

Is designed to stabilize the closed loop system at $r = 0.04m$, which corresponds to the equilibrium point $(x_{ss}, u_{ss}) = \left(\begin{bmatrix} 0.04 \\ 0 \\ 5.6377 \end{bmatrix}, 5.6377 \right)$. For this equilibrium point, the linear system (12) becomes

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 218 & -0.1 & -3.4801 \\ 0 & 13.6178 & -39.13304 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 39.13304 \end{bmatrix} u \quad (14)$$

The gain matrix K is designed to locate the closed loop poles at positions so that the percentage overshoot is 1.5% and the settling time is 0.5s (5% criterion).

$$M_p = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} \Rightarrow \frac{1.5}{100} = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} \Rightarrow \xi = 0.8 \text{ and } T_s = \frac{3}{\xi\omega_n} \Rightarrow 0.5 = \frac{3}{0.8\omega_n} \Rightarrow \omega_n = 7.5 \frac{\text{rad}}{\text{s}}$$

Thus, the two complex conjugate poles are $-6 \pm j4.5$. To make the poles $s_{1,2}$ dominant, the third pole is selected such that $|\text{Re}(s_3)| \geq 5|\text{Re}(s_{1,2})|$; let $s_3 = -30$. Using Ackerman's formula, the gain matrix is

$$K = [0 \ 0 \ 1][B \ AB \ A^2B]^{-1}((A - (-6 + j4.5)I)(A - (-6 - j4.5)I)(A - (-30)I)) \\ = [-79.6114 \ -4.3064 \ 0.0731] \quad (15)$$

and the control law becomes

$$u = u_f + u_{ss} = -79.6114x_1 - 4.3064x_2 - 0.0731x_3 + 5.6377 \quad (16)$$

A block diagram of the MLS with SFC is shown in Figure 1. A drawback of this controller is that it assures the stabilization of the system and it achieves the required design specifications only in a certain neighborhood of the linearization-based point, i.e., the equilibrium point that corresponds to $r = 0.04m$. To stabilize the system at another position, the controller may fail to stabilize the system, or at least it will not achieve the required design specifications.

To overcome this problem, an adaptive state feedback controller is designed. This can be achieved by parameterizing the linear system (14) in terms of its equilibrium point, i.e., the quantities $\mathbf{x}_{ss} = [x_{1ss} \ x_{2ss} \ x_{3ss}]^T$ and $u_{ss} = V_{ss}$ are not given constant values; rather, they are considered as parameters, and system (14) can be rewritten as

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{x}_{ss}, u_{ss}) + \mathbf{B}(\mathbf{x}_{ss}, u_{ss})u \tag{17}$$

$$\mathbf{x}_{ss} = \begin{bmatrix} 1 \\ 0 \\ \sqrt{\frac{L_0 a}{2mg}} \end{bmatrix} r + \begin{bmatrix} 0 \\ 0 \\ \sqrt{\frac{L_0 a}{2mg}} a \end{bmatrix} = \mathbf{g}_x(r) \tag{18}$$

$$u_{ss} = R \sqrt{\frac{L_0 a}{2mg}} (a + r) = g_u(r) \tag{19}$$

and system (17) can be rewritten as

$$\dot{\mathbf{x}} = \mathbf{A}(r) + \mathbf{B}(r)u \tag{20}$$

and the ASFC is

$$u = -\mathbf{K}(r)\mathbf{x} = [k_1(r) \ k_2(r) \ k_3(r)] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{21}$$

where $\mathbf{K}(r)$ is given by

$$\mathbf{K}(r) = [0 \ 0 \ 1] \left[\mathbf{B}(r) \ \mathbf{A}(r) \mathbf{B}(r) \ (\mathbf{A}(r))^2 \ \mathbf{B}(r) \right]^{-1} \\ \left((\mathbf{A} - (-6 + j4.5)\mathbf{I})(\mathbf{A} - (-6 - j4.5)\mathbf{I})(\mathbf{A} - (-30)\mathbf{I}) \right) \tag{22}$$

The control law (21) is a family of controllers, i.e., an adaptive state feedback controller, whose parameters k_1 , k_2 , and k_3 are changed (designed) according to the value of the reference input r . A block diagram of the MLS with ASFC is shown in Figure 2.

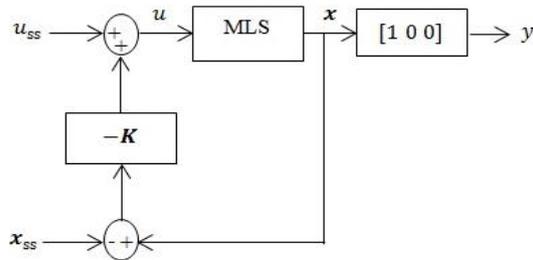


Figure 1. Block diagram of the MLS with SFC

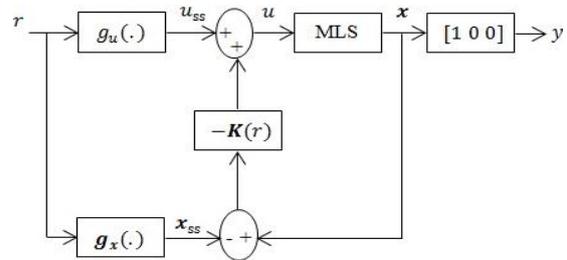


Figure 2. Block diagram of the MLS with ASFC

4. SIMULATION RESULTS AND DISCUSSION

A simulation of the closed loop MLS was carried out using script MATLAB program. Four cases were considered, regarding the operating range of the system. The first case is when the system operates in a range that lies relatively close to the equilibrium point that corresponds to $r = 0.04$ m; this range was achieved by taking an initial position $y_0 = 0.02$ m and a desired position $r = 0.06$ m. The second case is when the system operates in a range that deviates from the equilibrium point by a relatively moderate distance; this range was achieved by taking an initial position $y_0 = 0.06$ m and a desired position $r = 0.10$ m. The third case is when the system operates in a range that deviates from the equilibrium point by a relatively large distance; this range was achieved by taking an initial position $y_0 = 0.10$ m and a desired position $r = 0.14$ m. The fourth case is when the system operates in a wide range; this range was achieved by taking an initial position $y_0 = 0.01$ m and a desired position $r = 0.10$ m. Figure 3 shows the ranges of the operating points of the four cases relative to the linearization-based point, and Table 2 shows the performance of the system with the SFC and with the ASFC, for all cases.

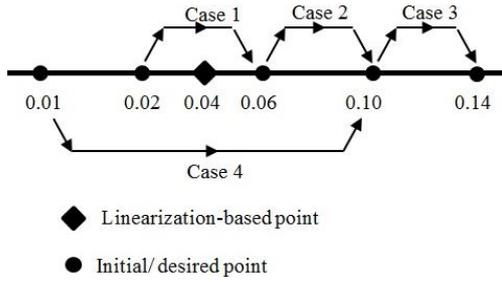


Figure 3. Ranges of operating points of the four cases

		Rise time (s)	Settling time (s)	Percentage overshoot
Case 1	SFC	0.26	0.94	3.30%
	ASFC	0.29	0.58	0.73%
Case 2	SFC	0.28	1.10	4.92%
	ASFC	0.31	0.54	0.47%
Case 3	SFC	0.30	1.21	5.08%
	ASFC	0.32	0.51	0.35%
Case 4	SFC	Unstable		
	ASFC	0.34	0.69	0.53%

The results given in Table 2 shows that as the operating point deviates from the linearization-based point, the performance of the SFC degraded (the rise time, the settling time, and the percentage overshoot are increased), and it became unstable in the fourth case. However, the ASFC showed better performance and robustness, since the controller gain matrix was adapted with every new reference input to maintain the same required design specifications of the system. The responses of both controllers for the four cases are shown in Figures 4-11.

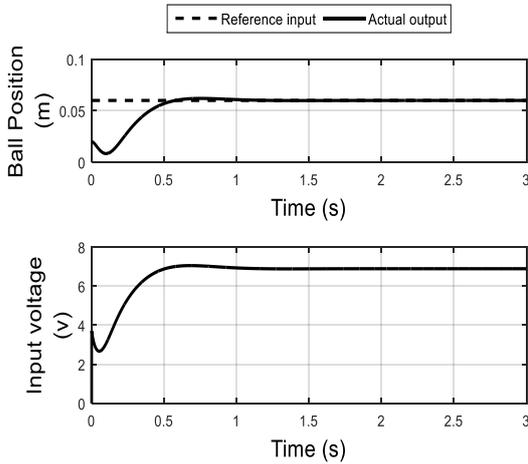


Figure 4. Step response of MLS with SFC: case 1

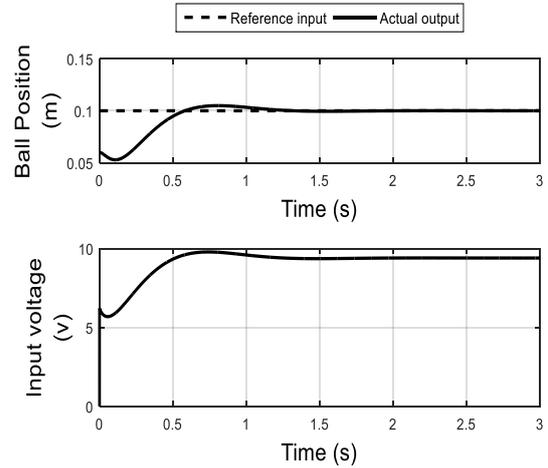


Figure 5. Step response of MLS with SFC: case 2

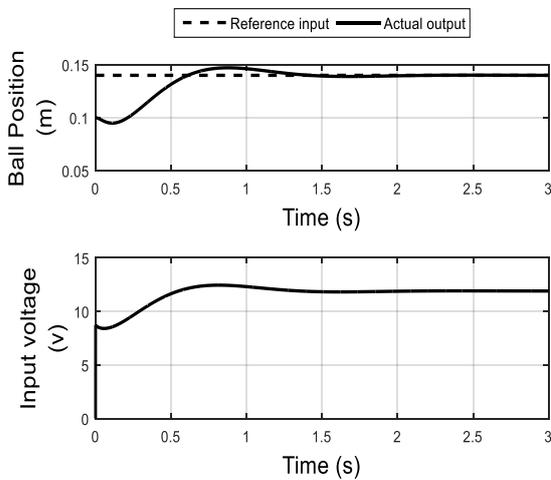


Figure 6. Step response of MLS with SFC: case 3

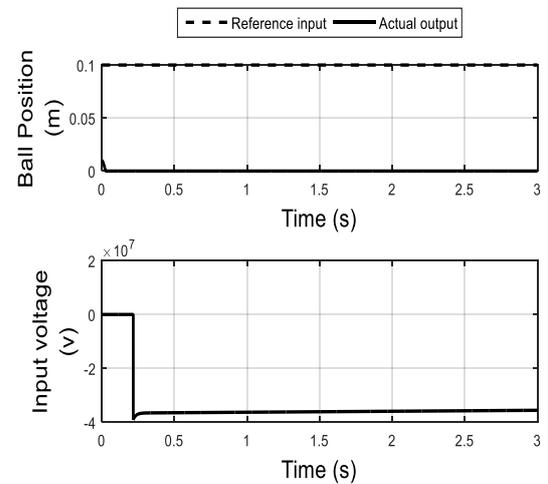


Figure 7. Step response of MLS with SFC: case 4,

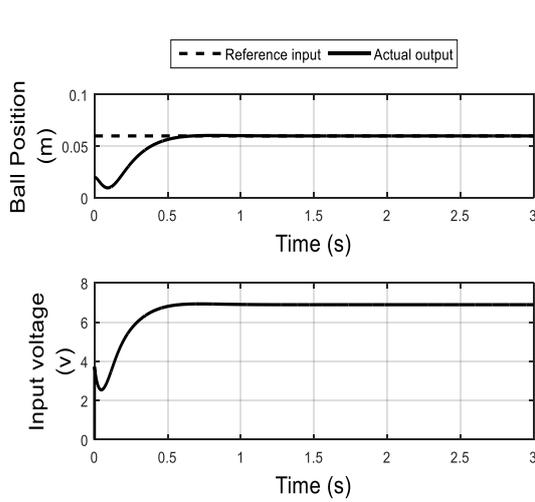


Figure 8. Step response of MLS with ASFC: case 1

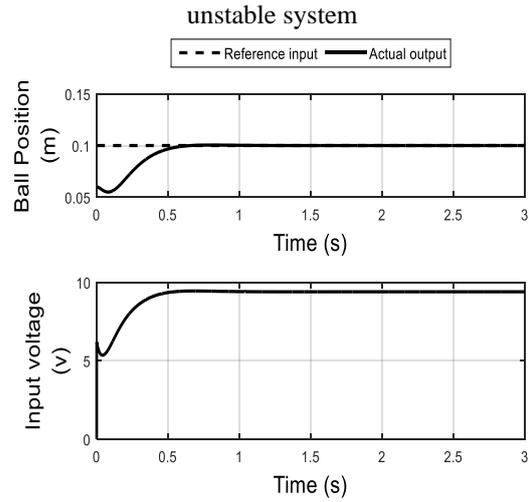


Figure 9. Step response of MLS with ASFC: case 2

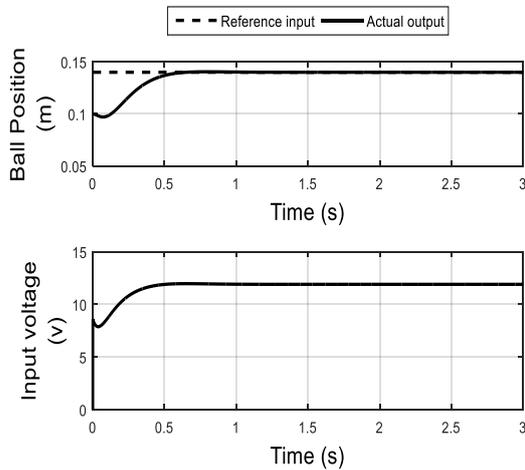


Figure 10. Step response of MLS with ASFC: case 3

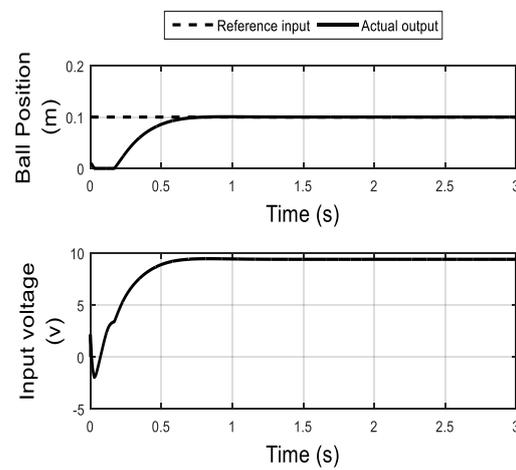


Figure 11. Step response of MLS with ASFC: case 4

5. CONCLUSION

In this paper, the design of an ASFC for a MLS has been proposed. The SFC was design by first linearizing the MLS about a selected equilibrium point, then the closed loop poles are positioned at locations so as to achieve certain design specification. However, when the reference input changed, the nonadaptive state feedback controller could no longer satisfy the closed loop design specifications and its performance degraded, or even it fails to stabilize the MLS, while the ASFC satisfied the closed loop design specifications for all reference inputs. Several conclusions can be drawn from the obtained results. First, the linearization design method has a limitation when applied to highly nonlinear system, such as the MLS. Second, the ASFC is a suitable solution to stabilize highly nonlinear system, and it outperforms the nonadaptive state feedback controller.

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