

The cubic root unscented kalman filter to estimate the position and orientation of mobile robot trajectory

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ABSTRACT

In this paper we introduce a cubic root unscented kalman filter (CRUKF) compared to the unscented kalman filter (UKF) for calculating the covariance cubic matrix and covariance matrix within a sensor fusion algorithm to estimate the measurements of an omnidirectional mobile robot trajectory. We study the fusion of the data obtained by the position and orientation with a good precision to localize the robot in an external medium; we apply the techniques of kalman filter (KF) to the estimation of the trajectory. We suppose a movement of mobile robot on a plan in two dimensions. The sensor approach is based on the CRUKF and too on the standard UKF which are modified to handle measurements from the position and orientation. A real-time implementation is done on a three-wheeled omnidirectional mobile robot, using a dynamic model with trajectories. The algorithm is analyzed and validated with simulations.

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1. INTRODUCTION

The mobile robots are equipped with sensors to measure the distance of the robot from the space of the objects where they move. Measurements which are always linked by noises are then fused together in a filter to obtain an estimate of the position and orientation of the mobile robot trajectory in the space and plan [1, 2]. The most important problems related to robots are situated in sensors powered by their battery which reduces the autonomy. Also there are situations where sensors cannot function simultaneously, when they use the same frequency band [3, 4]. The Kalman filter is an optimal linear estimator when the process noise and the measurement noise can be modeled by white Gaussian noise. Non-linear problems can be solved with the unscented Kalman filter (UKF) and we propose a new algorithm that is called the cubic root unscented Kalman filter (CRUKF) to solve also nonlinear problems. In our work, we use this technique to estimate the position and the angle of the robot in its displacement then a comparison between the performances of the two filters is presented to give an evaluation of more precise measurement and will appreciate better statistical properties [2, 5].

This paper is organized as follows; we present the modeling of the mobile robot with the equations of trajectory and angles of the robot. Next, we pass to the presentation of the UKF algorithm and we propose a CRUKF algorithm to improve the accuracy of the state estimate. Finally we present and discuss the simulation results and outline some possible extensions for future investigations.

2. MODELING OF MOBILE ROBOT

The robot takes the position trajectory T between two moments from sampling with the variation in the position and the direction then we suppose in our paper the omnidirectional mobile robot like shown in

Figure 1. Our robot have three wheels whoos effect on the robot speed because of their weight and friction on the ground, we suppose m_1, m_2 and m_3 the robot mass applied on the wheels. The omnidirectional mobile Robot has two independently speed-controlled wheels equipped with sensors, and a castor wheel [2, 6-12]. For such a robot, an approximated, discrete-time model is:

$$\begin{aligned} A_{k+1} &= A_k + v_k \cdot m_k \cdot t \cdot u_1 \cdot \cos(\theta_{k+1}) + w_1 \\ B_{k+1} &= B_k + v_k \cdot m_k \cdot t \cdot u_2 \cdot \sin(\theta_{k+1}) + w_2 \\ \theta_{k+1} &= \theta_k + \Delta\theta + w_3 \end{aligned} \tag{1}$$

where (A_k, B_k) is the position of the robot at time t_k , with $A_k = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix}, A_k = A_1, B_k = A_2, \theta_k = A_3,$

θ_k represente the angle between the robot axle and the x-axis, $w_k = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ are zero-mean uncorrelated

Gaussian noises and $u_k = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ are the angular velocities of wheels, $m_k = R \cdot (m_1 + m_2 + m_3)/3L$ is the mass of the robot in wheels. $v_k = R \cdot (u_1 + u_2)/2$ is the linear velocity of the robot, $t = t_{k+1} - t_k$ is the sampling period and $\Delta\theta = R \cdot (u_2 - u_1) \cdot t/2L$ is the rotation within $[t_k, t_{k+1}]$. R is the radius of the wheels and L is the length of the axes.

The omnidirectional mobile robot is equipped with sensors to measure its position and its orientation like shown in Figure 2. The figure also defines the environment of the robot, assumed perfectly known [7, 10, 13]. In our work we propose the trajectory T on which we apply the technical of Kalman filters CRUKF and UKF to estimate this trajectory.

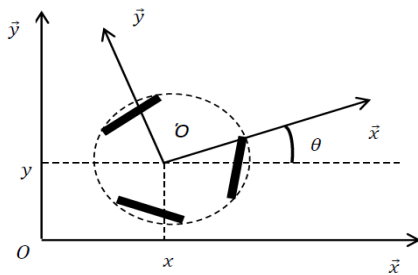


Figure 1. Omnidirectional mobile robot

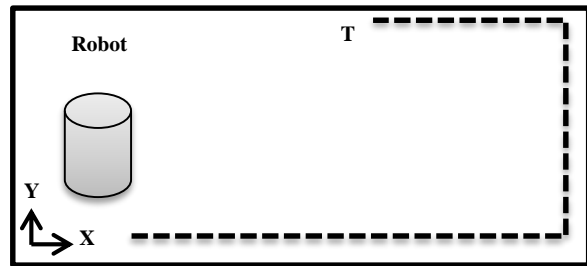


Figure 2. Omnidirectional mobile robot trajectory T

2.1. The equations of trajectory T

The trajectory of the robot is defined by the equation $B_{k+1} = f(A_{k+1})$ where $A_k = A_1 = 1; B_k = A_2 = 1; \theta_k = A_3 = 1; t = 1s; v_1 = v_2 = 0,02$ m/s; $u_1 = 0,2; u_2 = 0,1; \Delta\theta = \pi/2; m_1 = 0,3$ g; $m_2 = 0,3$ g; $m_3 = 0,3$ g; $R = 0,5 \cdot 10^{-2}$ m $L = 0,18$ m and $m_k = R \cdot (m_1 + m_2 + m_3)/3L$; then $m_k = 0,0083$ g.

We have our model:

$$\begin{cases} A_{k+1} = A_k + v_1 m_k t u_1 \cos(\theta_{k+1}) + w_1 \\ B_{k+1} = B_k + v_2 m_k t u_2 \sin(\theta_{k+1}) + w_2 \\ \theta_{k+1} = \theta_k + \Delta\theta + w_3 \end{cases}$$

then

$$\begin{cases} A_{k+1} - A_k - w_1 = v_1 m_k t u_1 \cos(\theta_{k+1}) \\ B_{k+1} - B_k - w_2 = v_2 m_k t u_2 \sin(\theta_{k+1}) \end{cases}$$

then

$$\frac{(A_{k+1} - A_k - w_1)}{v_1 m_k t u_1} = \cos(\theta_{k+1}) \tag{2}$$

$$\frac{(B_{k+1} - B_k - w_2)}{v_2 m_k t u_2} = \sin(\theta_{k+1}) \quad (3)$$

and we make square:

$$\frac{(A_{k+1} - A_k - w_1)^2}{(v_1 m_k t u_1)^2} = \cos^2(\theta_{k+1}) \quad (4)$$

$$\frac{(B_{k+1} - B_k - w_2)^2}{(v_2 m_k t u_2)^2} = \sin^2(\theta_{k+1}) \quad (5)$$

$$(4) + (5) \Rightarrow \frac{(A_{k+1} - A_k - w_1)^2}{(v_1 m_k t u_1)^2} + \frac{(B_{k+1} - B_k - w_2)^2}{(v_2 m_k t u_2)^2} = \cos^2(\theta_{k+1}) + \sin^2(\theta_{k+1})$$

then

$$\begin{aligned} \frac{(A_{k+1} - A_k - w_1)^2}{(v_1 m_k t u_1)^2} + \frac{(B_{k+1} - B_k - w_2)^2}{(v_2 m_k t u_2)^2} &= 1 \\ \Rightarrow \frac{(B_{k+1} - B_k - w_2)^2}{(v_2 m_k t u_2)^2} &= 1 - \frac{(A_{k+1} - A_k - w_1)^2}{(v_1 m_k t u_1)^2} \\ \Rightarrow (B_{k+1} - B_k - w_2)^2 &= (v_2 m_k t u_2)^2 \left(1 - \frac{(A_{k+1} - A_k - w_1)^2}{(v_1 m_k t u_1)^2}\right) \\ \Rightarrow B_{k+1} - B_k - w_2 &= \sqrt{(v_2 m_k t u_2)^2 \left(1 - \frac{(A_{k+1} - A_k - w_1)^2}{(v_1 m_k t u_1)^2}\right)} \end{aligned}$$

then

$$B_{k+1} = B_k + w_2 + v_2 m_k t u_2 \sqrt{\left(1 - \frac{(A_{k+1} - A_k - w_1)^2}{(v_1 m_k t u_1)^2}\right)}$$

For the trajectory T we take: $w_1 = 0$, $w_2 = 0$ and $w_3 = 0$

$$B_{k+1} = 1 + 0 + 0,02.1.0,0083.0,1. \sqrt{\left(1 - \frac{(A_{k+1} - 1 - 0)^2}{(0,02.10,0083..0,2)^2}\right)}$$

Then the final equation of T is:

$$B_{k+1} = 1 + 0,016.10^{-3} \sqrt{\left(1 - \frac{(A_{k+1}-1)^2}{0,256.10^{-6}}\right)}$$

2.2. The angle of robot θ_{k+1}

We have

$$\begin{cases} \frac{(A_{k+1} - A_k - w_1)}{v_1 m_k t u_1} = \cos(\theta_{k+1}) \\ \frac{(B_{k+1} - B_k - w_2)}{v_2 m_k t u_2} = \sin(\theta_{k+1}) \end{cases}$$

(2) $\Rightarrow \theta_{k+1} = \arccos\left(\frac{A_{k+1}-A_k-w_1}{v_1 m_k t u_1}\right)$ is the angle of robot with axis, or can we find this angle by (3):

$$(3) \Rightarrow \frac{(B_{k+1} - B_k - w_2)}{v_2 m_k t u_2} = \sin(\theta_{k+1}) \Rightarrow \theta_{k+1} = \text{asin}\left(\frac{B_{k+1} - B_k - w_2}{v_2 m_k t u_2}\right)$$

$\theta_k = \text{atan}\left(\frac{\text{imag}(\theta_{k+1})}{\text{real}(\theta_{k+1})}\right)$ is the angle of robot with wheels axis

3. UNSCENTED KALMAN FILTER AND HER ALGORITHM

The UKF has been developed in recent years to overcome two main problems of the EKF (extended Kalman filter), the poor approximation properties of the first order approximation and the requirement for the noises to be Gaussian. The basic idea behind the UKF is that of finding a transformation that allows approximating the mean and the covariance of a random vector of length n when it is transformed by a nonlinear map. This is done by computing a set of $2n + 1$ points, called sigma points, on the basis of the mean and variance of the original vector, transforming these points by the nonlinear map and then approximating the mean and variance of the transformed vector from the transformed sigma points. We refer to [2-5, 14-17] for the theoretical aspects.

As for approximating properties of the filter, it has been shown that, while the EKF estimate of the state is accurate to the first order, the UKF estimate is accurate to the third order in the case of Gaussian noises. The covariance estimate also is accurate to the first order for the EKF, and to the second order for the UKF and her algorithm is:

UKF Algorithm

• **Initialization:**

$$\begin{aligned} \hat{A}_0 &= E[A_0], P_{A_0} = E[(A_0 - \hat{A}_0)(A_0 - \hat{A}_0)^T] \\ \hat{A}_0^a &= E[\hat{A}_0^a] = [\hat{A}_0^T \quad \bar{v}_0^T \quad \bar{n}_0^T]^T \\ P_0^a &= E[(A_0^a - \hat{A}_0^a)(A_0^a - \hat{A}_0^a)^T] = \begin{bmatrix} P_{A_0} & 0 & 0 \\ 0 & R_v & 0 \\ 0 & 0 & R_n \end{bmatrix} \end{aligned}$$

For $k_u = 1, \dots, \infty$:

1. **Set** $t_u = k_u - 1$ (for unscented Kalman filter)

2. **Calculate sigma-points:**

$$A_{t_u}^{av} = [\hat{A}_{t_u}^a \quad \hat{A}_{t_u}^a + M \sqrt{P_{t_u}^a} \quad \hat{A}_{t_u}^a - M \sqrt{P_{t_u}^a}]$$

3. **Time-update equations:**

$$\begin{aligned} A_{k_u|t_u}^x &= f(A_{t_u}^x, A_{t_u}^v, u_{t_u}) \\ \hat{x}_{k_u}^s &= \sum_{i=0}^{2L} w_i^m A_{i,k_u|t_u}^x \\ P_{x_{k_u}}^s &= \sum_{i=0}^{2L} w_i^c (A_{i,k_u|t_u}^x - \hat{x}_{k_u}^s) (A_{i,k_u|t_u}^x - \hat{x}_{k_u}^s)^T \end{aligned}$$

4. **Measurement-update equations:**

$$\begin{aligned} B_{k_u|t_u} &= h(A_{k_u|t_u}^x, A_{t_u}^n) \\ \hat{B}_{k_u}^s &= \sum_{i=0}^{2L} w_i^m B_{i,k_u|t_u} \\ P_{\bar{B}_{k_u}} &= \sum_{i=0}^{2L} w_i^c (B_{i,k_u|t_u} - \hat{B}_{k_u}^s) (B_{i,k_u|t_u} - \hat{B}_{k_u}^s)^T \\ P_{x_{k_u} B_{k_u}} &= \sum_{i=0}^{2L} w_i^c (A_{i,k_u|t_u}^x - \hat{x}_{k_u}^s) (A_{i,k_u|t_u}^x - \hat{x}_{k_u}^s)^T \\ G_{k_u} &= P_{x_{k_u} B_{k_u}} P_{\bar{B}_{k_u}}^{-1} \\ \hat{x}_{k_u} &= \hat{x}_{k_u}^s + G_{k_u} (B_{k_u} - \hat{B}_{k_u}^s) \\ P_{x_{k_u}} &= P_{x_{k_u}}^s - G_{k_u} P_{\bar{B}_{k_u}} G_{k_u}^T \end{aligned}$$

4. THE CUBIC ROOT UKF (CRUKF) PROPOSITION

In our paper we propose the cubic root CRUKF algorithm which is based on square root UKF algorithm [18, 19]. So we use in our algorithm a proposition of new function $qq(x)$ and UKF algorithm [4]. We study the coefficients of covariance matrix to introduce the cubic root. In this technique, we based on the same principals of the UKF algorithm.

4.1. The function $qq(x)$ proposition

The new function $qq(x)$ is the quadrature quadrature or orthogonal orthogonal function for find and calculates two matrixes quadrature or orthogonal and its equation is:

$$qq = I_1 - \frac{2V_1V_1^T}{\|V_1\|} \cdot I_2 - \frac{2V_2V_2^T}{\|V_2\|}$$

I_1 and I_2 are identity matrix

V_1 and V_2 are vectors ; V_1^T and V_2^T are vectors transposes
 $\|V_1\|$ and $\|V_2\|$ vectors modules

4.2. The CRUKF algorithm

CRUKF Algorithm

• **Initialization:**

$$\hat{A}_0 = E[A_0], \quad S_{A_0} = \sqrt[3]{E[(A_0 - \hat{A}_0)(A_0 - \hat{A}_0)^T]}, \quad S_v = \sqrt[3]{R_v} \text{ and } S_n = \sqrt[3]{R_n}$$

$$\hat{A}_0^a = E[A^a] = [\hat{A}_0 \quad \bar{v} \quad \bar{n}]^T$$

$$S_0^a = \sqrt[3]{E[(A_0^a - \hat{A}_0^a)(A_0^a - \hat{A}_0^a)^T]}, \quad \text{then } S_0^a = \begin{bmatrix} S_{A_0} & 0 & 0 \\ 0 & S_v & 0 \\ 0 & 0 & S_n \end{bmatrix}$$

For $k_c = 1, \dots, \infty$:

1. **Set** $t_c = k_c - 1$; (for the cubic root unscented Kalman filter)

2. **Calculate sigma-points for time-update:**

$$A_{t_c}^a = [\hat{A}_{t_c}^a \quad \hat{A}_{t_c}^a + NS_{A_{t_c}}^a \quad \hat{A}_{t_c}^a - NS_{A_{t_c}}^a]$$

3. **Time-update equations:**

$$A_{k_c|t_c}^x = f(A_{t_c}^a, A_{t_c}^v, u_{t_c})$$

$$\hat{x}_{k_c}^s = \sum_{i=0}^{2L} w_i^m A_{i,k_c|t_c}^x$$

$$S_{x_{k_c}}^s = qq\{[\sqrt[3]{w_1^c} (A_{1:2L,k_c|t_c}^x - \hat{x}_{k_c}^s)]\} \quad ; \quad (\text{quadrature quadrature function})$$

$$S_{x_{k_c}}^s = \text{update}\{S_{x_{k_c}}^s, A_{0,k_c|t_c}^x - \hat{x}_{k_c}^s, w_0^{(c)}\} \quad ; \quad \text{time update}$$

$$B_{k_c|t_c} = h(A_{i,k_c|t_c}^x, A_{t_c}^n)$$

$$\hat{B}_{k_c}^s = \sum_{i=0}^{2L} w_i^m B_{i,k_c|t_c}$$

4. **Measurement-update equations:**

$$S_{B_{k_c}}^s = qq\{[\sqrt[3]{w_1^c} (B_{1:2L,k_c|t_c} - \hat{B}_{k_c}^s)]\}$$

$$S_{B_{k_c}}^s = \text{update}\{S_{B_{k_c}}^s, B_{0,k_c|t_c} - \hat{B}_{k_c}^s, w_0^{(c)}\}$$

$$P_{x_{k_c} B_{k_c}} = \sum_{i=0}^{2L} w_i^c (A_{i,k_c|t_c}^x - \hat{x}_{k_c}^s)(B_{i,k_c|t_c} - \hat{B}_{k_c}^s)^T$$

$$G_{k_c} = (P_{x_{k_c} B_{k_c}} / S_{B_{k_c}}^T) / S_{B_{k_c}}^s$$

$$\hat{A}_{k_c} = \hat{A}_{k_c}^s + G_{k_c} (B_{k_c} - \hat{B}_{k_c}^s)$$

$$U = G_{k_c} S_{B_{k_c}}^s$$

$$S_{x_{k_c}} = \text{update}\{S_{x_{k_c}}^s, U, -1\}$$

We have two filters UKF and CRUKF where: $M = \sqrt{L + \lambda}$, $N = \sqrt[3]{L + \lambda}$, $w_0^m = \lambda / (L + \lambda)$, $w_0^c = w_0^m + (1 + \alpha^2 + \beta)$, $w_i^c = w_i^m = 1 / 2(L + \lambda)$, for $i = 1 \dots 2L$, and $\lambda = \alpha^2(L + k)$ is a compound scaling parameter. L is the dimension of the augmented state-vector $0 < \alpha < 1$ is the primary scaling factor determining the extent of the spread of the sigma-points around the prior mean. Typical range for α is $1e - 3 < \alpha < 1$. β is a secondary scaling factor used to emphasize the weighting on the zeros sigma-point for the posterior covariance calculation. β can be used to minimize certain higher-order error terms based on known moments of the prior R_v . For Gaussian priors, $\beta = 2$ is optimal. k is a tertiary scaling factor and is usually set equal to 0. In general, the optimal values of these scaling parameters will be a specific problem [2, 18, 19, 20-25].

- General notes:

The augmented state vector and sigma-point vector is given by: $x^a = [x^T \ v^T \ n^T]^T$, $A^a = [(A^x)^T \ (A^v)^T \ (A^n)^T]^T$ where R_v and R_n are the process-noise and observation-noise covariance matrices.

- Linear-algebra operators:

$\sqrt[3]{\cdot}$: matrix cubic-root using lower triangular.

$qq(X)$: lower-orthogonal part of X matrix resulting from function *ortho*(X) decomposition of data-matrix X . *update* $\{R, U, \pm v\}$: N consecutive rank-1 updates of the lower-orthogonal factor q by the N columns of

$\sqrt{v}U$: / : Efficient least-squares pseudo inverse implemented using orthogonal QQ decomposition with pivoting finally this parameters for two filters CRUKF and UKF.

5. SIMULATION RESULTS

We compare the performance of CRUKF and UKF in our paper, we start from the point of trajectory T : $(A_0, B_0) = (1, 1)$ with $\theta_0 = 0$. In the chosen sample time $t = 1s$ and each trajectory has been completed in $k_f = 400 s$, imposing a profile to the angular velocities of the wheels. The parameters and weights used in the CRUKF and UKF are: $\alpha = 10^{-4}$; $\beta = 5$; $\kappa = (8 - n)/2$; $\lambda = \alpha^2(n + \kappa) - n$. $\beta = 3$ minimizes the error of the a posteriori covariance when the random vector is Gaussian.

- $W = diag\{0.123 \ 0.124 \ 0.121\}$, which corresponds to a standard deviation less than one centimeter on A_k and B_k and less than one degree on θ_k ;
- $V = diag\{0.110 \ 0.100 \ 0.111\}$, a standard deviation less that 0.1 centimeter for each sensor;
- $\hat{A}_{0/0} = [1.5000 \ 1.9821 \ 2.4123]'$, the initial estimate state of robot
- $A_0 = [1 \ 1 \ 0]'$, the initial state of robot
- $P_{0/0} = [0.5010 \ 0.5021 \ 0.5023]'$, a standard deviation of width and length on the position and standard orientation.

We compare the performance of the two filters we have computed

- The average error position index: $\varepsilon_x = \frac{1}{k_f+1} \sum_{k=0}^{k_f} \|[A_k - \hat{A}_{k/k}, B_k - \hat{B}_{k/k}]\|$
- The average orientation position index: $\varepsilon_\theta = \frac{1}{k_f+1} \sum_{k=0}^{k_f} |\theta_{k+1} - \hat{\theta}_{k+1/k}|$
- The estimation error covariance averaged along the whole trajectory has been computed: $\pi = \frac{1}{k_f+1} \sum_{k=0}^{k_f} tr(\sum P_{k/k})$

The comparison between CRUKF and UKF at our case as follows: we start with time, we take three values for k to each algorithm. Table 1 show the comparison and the simulation results show in different figures: Figures 3-6.

5.1. Estimation of position by CRUKF and UKF

Figures 3 and 4 show the simulation results using the UKF and CRUKF applied to the trajectory estimation and errors. Figure 3 shows the robot trajectory with its estimate using UKF (red) compared to the CRUKF (bleu), the pursuit was well done. Figure 4 presents the estimation error between the original trajectory and both of the two filters also the MSE (mean square error) of the algorithms, it can be seen the CRUKF was better then the UKF.

Table 1. Comparison between CRUKF and UKF

Algorithm	ε_x (m)			ε_θ (rad)			π		
CRUKF	0.0156	0.0258	0.0077	0.3550	0.4405	0.1815	0.0263	0.0116	0.0104
UKF	0.0082	0.0050	0.0074	0.1563	0.0716	0.0493	0.0015	0.0010	0.0003

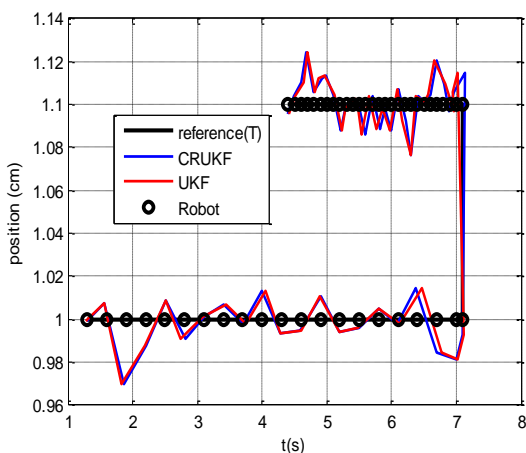


Figure 3. Estimate reference trajectory T by CRUKF and UKF

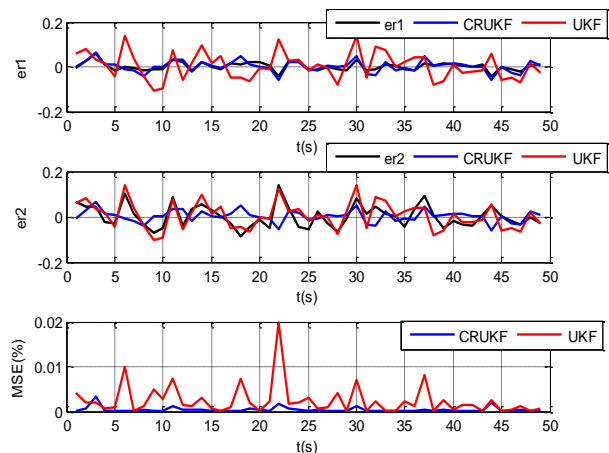


Figure 4. Estimate error er1, er2 and her estimation by CRUKF and UKF and MSE

5.2. Estimation of angle by CRUKF and UKF

Figure 5 shows the angle estimate of the robot using the two filters. Figure 6 exhibits estimation errors of these filters where the UKF was here better than the CRUKF.

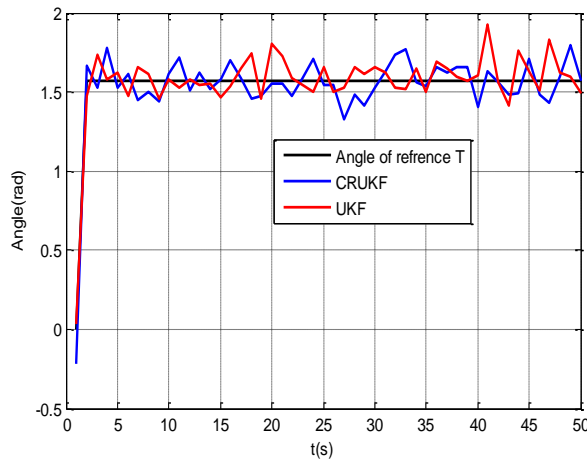


Figure 5. The estimate angle of trajectory T with axis by CRUKF and UKF

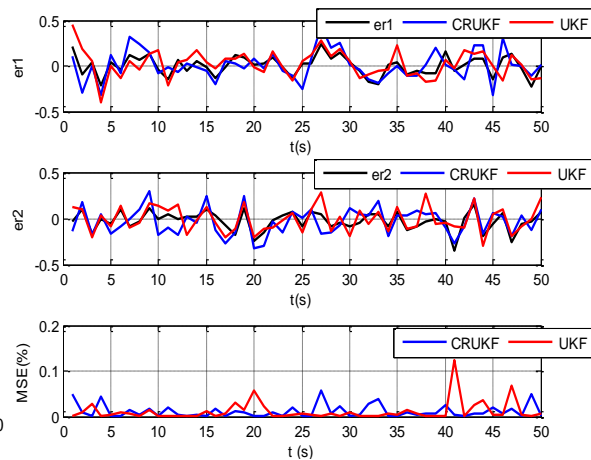


Figure 6. Estimate error er1,er2 of angle and her estimation by CRUKF and UKF and MSE

6. CONCLUSION

The analysis of the results in Table 1 is that the two filters perform comparably the position and orientation of the robot, we see that the estimation error of UKF is weak than those the estimation error of CRUKF in angle estimation contrary in the position where the CRUKF was better because the approximating properties of each filter. These two filters, always giving a value of the error covariance π of index which is better than those given by any sensor, and completely close to that obtained when all the sensors are used. Results shows the estimate reference trajectory T and estimate error by CRUKF and UKF too MSE of this case and the estimate of the robot angle and its estimate error by CRUKF and UKF too MSE of this case, we see that the estimations were optimal. In the near future this technique of work can be done and develop on other types of robots and other applications with less sensor but with a great measuring accuracy in real-time but its better to make one filter complete the other.

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