

## Optimization of automobile active suspension system using minimal order

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### Article Info

#### Article history:

Received Sep 23, 2019

Revised Nov 8, 2021

Accepted Nov 30, 2021

#### Keywords:

Linear actuator

Linear quadratic regulator control

Minimal realization technique active suspension system linear DC motor

### ABSTRACT

This paper presents an analysis and design of linear quadratic regulator for reduced order full car suspension model incorporating the dynamics of the actuator to improve system performance, aims at benefiting: Ride comfort, long life of vehicle, and stability of vehicle. Vehicle's road holding or handling and braking for good active safety and driving pleasure and keeping vehicle occupants comfortable and reasonably well isolated from road noise, bumps, and vibrations are become a key research area conducted by many researchers around the globe. Different researchers were tested effectiveness of different controllers for different vehicle model without considering the actuator dynamics. In this paper full vehicle model was reduced to a minimal order using minimal realization technique. The entire system responses were simulated in MATLAB/Simulink environment. The effectiveness of linear quadratic regulator controller was compared for the system model with and without actuator dynamics for different road profiles. The simulation results were indicated that percentage reduction in the peak value of vertical and horizontal velocity for the linear quadratic regulator with actuator dynamics relative to linear quadratic regulator without actuator dynamics was 28.57%. Overall simulation results were demonstrated that proposed control scheme has able to improve the effectiveness of the car model for both ride comfort and stability.

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## 1. INTRODUCTION

The suspension system is a mechanism that physically separates the car body from the car wheels, and a complex vibration system having multiple degrees of freedom. It is the most important part of the vehicle which heavily affects the ride quality and used to isolate the vehicle structure from shocks and vibration due to irregularities of the road surface [1]. The main purpose of vehicle suspension system is to minimize the vertical displacement, velocity or acceleration transmitted to the passenger which directly provides ride comfort [2], [3]. Usually vehicle model could be quarter car, half car and full car models based on types of suspension system and could be passive, semi active, and active suspension system based on energy consumptions. In passive suspension system, the good ride quality is mainly achieved by an appropriate choice of the springs and dampers. The parameters are generally fixed, with values compromised to achieve a certain level of performance of the suspension system. Since the appropriate choice is depend on

the road surface, it has a limitation to satisfy to different types of road irregularities [4], [5]. Therefore, a means of controlling to different types of road surface automatically should be researched. In contrast to passive systems, active suspension systems can adjust their dynamic characteristics in response to varying road conditions in real time [6], [7]. In order to design linear quadratic regulator controller, all states should be measurable and are sensed by its own individual sensor. Fourteen sensors are used to get information from each state which is bulky and requires high cost [8], [9]. Thus, a means of technique to reduce the number of states and sensors to a minimal number without affecting the behavior of the original model system is investigated. To have a good control performance, a good modeling of the system is necessary. As improving the model from quarter car to half car model, from half car to full car model and including the sensors and actuator dynamics in the system model, it will be more accurate [10]–[12]. Most of the researchers were proven the effectiveness of different controllers for different car suspension system model without considering the actuator dynamics, without considering the state reachable, and without considering the state measurable in which the design of state feedback is impossible [13]–[18]. However, in this research paper, it is proposed to design linear quadratic regulator (LQR) for active suspension system of automobile vehicles based on active suspension system of automobile vehicles using reduced order car model by combing a linear direct control motor actuator to reduce the vehicle body vibration and to settle within short period of time so that the passengers feel comfort for more specific and concern [19]. At the end of this paper, the controller performance will be evaluated under full car suspension system reduced order for different road profiles conditions. The controller's dynamic study is performed by evaluating the transient response during the magnitude variations of road profile reference.

The remaining part of this paper is organized: section 2 describes mathematical model of full car model with governing equation. The design of linear quadratic regulator controller and system analysis carried out in section 3. Section 4 presents the results of simulation carried out for different road profile. Conclusion of the work will present in section 5.

## 2. MATHEMATICAL MODELING OF SUSPENSION SYSTEM

The vehicle active suspension system model utilized for numerical simulation was developed in the MATLAB/Simulink environment and the presenters of the vehicle active suspension system are extracted. The model, as shown in Figure 1 indicates that block diagrams of active suspension system, and its control mechanism, Figure 2 depicted the schematic diagram of the active suspension system of automobile vehicle, and Figure 3 utilizes three degree of freedoms for the motion of the car body in the space (pitch, roll, and bounce), and four degree of freedom for wheel displacement (relative to the vehicle chassis) and rotations. The mathematical models were developed based on some assumptions (the model considers only linearity's region, external factor (aerodynamics resistance) were not consider, and non-linearity properties of tires, actuators were not considered).

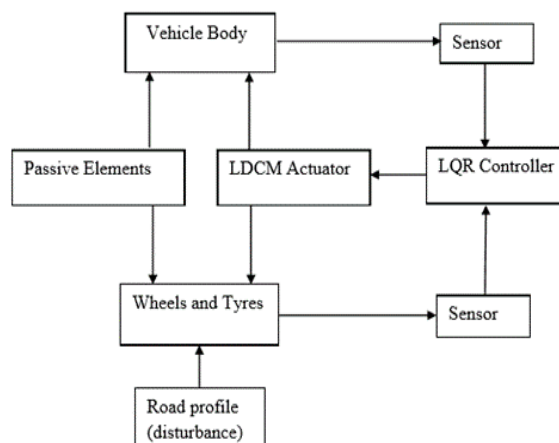


Figure 1. Block diagram representation of the active suspension system [10]

### 2.1. Basic structure of active suspension system

Active suspension systems are mechatronics systems that control the vertical movement of the vehicle body or chassis relative to the wheels. The active suspension systems considered in this paper is

applied for enhancing the ride comfort of automobile vehicles. Therefore, they are placed in between the chassis and the wheels through attachment of their two ends to the car body and the wheels. In active suspension system the vehicle's up and down motion is controlled by linear quadratic regulator feedback controller according to the road conditions; the controller output controls the actuator to compensate the road oscillations and increases the vehicle stability as well as ride comfort. As a result of the road disturbance, the vehicle body has been oscillated for some time. The sensors measure the amplitude of the vibration from the equilibrium position. The linear quadratic regulator controller processed the electrical signal information obtained from the sensors and it provides a control signal which controls the action of the actuator for fine response in real time. The passive elements and linear direct control motor actuator generate forces which counteract vertical, pitch and roll motions. Gradually, the purpose of the active suspension system is to replace the classical passive elements by a controlled system, which can supply a regulated force to the system. The active suspension system dynamically responds to the changing road surface due to its ability to supply energy, which is used to achieve the relative motion between the body and wheel [9]. As shown in Figure 2 the main purpose of suspension is supporting both roads holding and ride quality. Moreover, suspension system affects on the vehicle handling too. Furthermore, it is very important to keep the road wheel in contact with the road surface, for the suspension. There are different ways of attaching the wheels of the car so that they can move up and down on their springs and dampers. The design of front suspension and design rear suspension have some differences to the ability of opposite wheels to move independently of each other. For front-wheel drive cars, rear suspension has few constraints and a variety of beam axles and independent suspensions are used. For rear-wheel drive cars, rear suspension has many constraints and the development of the superior but more expensive independent suspension layout has been difficult. Four-wheel drive cars often have suspensions that are similar for both the front and rear wheels.

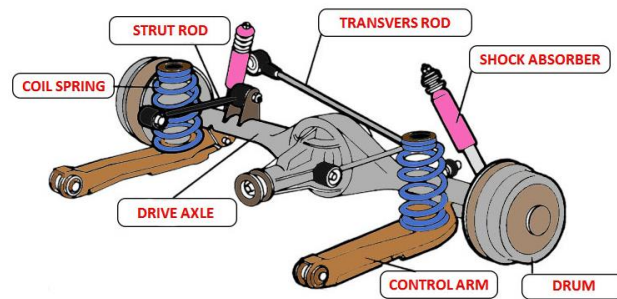


Figure 2. Schematic diagram of the active suspension system of automobile vehicle [10]

## 2.2. Mathematical models

Various types of car models such as quarter car model, half car model and full car model have been used to simulate the performance of suspension systems. In the research studies the quarter car model is frequently used because of its simplicity, however, the half car model shows more appropriate vertical motion, including either the pitch or the roll effects. The full car model is the best accurate one, but it requires more computation than the others as a result very few studies have been carried out based on it [20]. Lot of common vehicles today uses passive suspension system to control the dynamics of a vehicle's vertical motion as well as spinning (pitch) and tilting (roll) [21]. The design of a vehicle suspension is an issue that needs a series of mathematical calculations. To study the vibrational characteristics of the vehicle and to design the controller appropriately, a mathematical model of a dynamic system is defined as a set of various equations that represents dynamics of the system accurately or at least, well. By applying Newton's second law motion and using the static equilibrium position as the origin, for the linear vertical displacement, pitch angular displacement, and roll angular displacement of the vehicle body  $Y_0$ ,  $\theta$ , and  $\phi$  respectively from the center of gravity the equations of motion for the system can be formulated. Once a mathematical model of the system is obtained, various analytical and computer tool, MATLAB, can be used for design of linear quadratic regulator controller and analysis.

## 2.3. Equations of motion

The free body diagram of the passive and active suspension systems is shown in Figure 3. While modeling the system components the spring and the damper are assumed linear, i.e. the model is based on elements of linear dynamic systems theory. Therefore, the overall equations are linear.

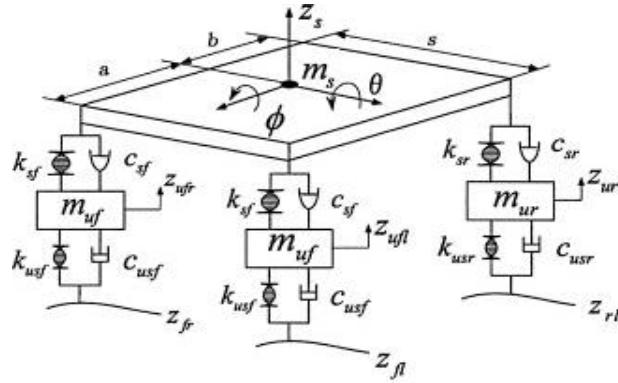


Figure 3. Full vehicle model [10]

For rolling motion of the sprung mass system

$$\begin{aligned}
 I_{sx}\varphi_{sx} &= -b_f T_f(z_{s1} - z_{u1}) + b_f T_f(z_{s2} - z_{u2}) \\
 &- b_r T_r(z_{s3} - z_{u3}) + b_r T_r(z_{s4} - z_{u4}) \\
 &- k_f T_f(z_{s1} - z_{u1}) + k_f T_f(z_{s2} - z_{u2}) \\
 &- k_r T_r(z_{s3} - z_{u3}) + k_r T_r(z_{s4} - z_{u4})
 \end{aligned} \tag{1}$$

For pitching motion of the sprung mass system

$$\begin{aligned}
 I_{sz}\varphi_{sz} &= -b_f a(z_{s1} - z_{u1}) + b_f a(z_{s2} - z_{u2}) \\
 &- b_r b(z_{s3} - z_{u3}) + b_r b(z_{s4} - z_{u4}) \\
 &- k_f a(z_{s1} - z_{u1}) + k_f a(z_{s2} - z_{u2}) \\
 &- k_r b(z_{s3} - z_{u3}) + k_r b(z_{s4} - z_{u4})
 \end{aligned} \tag{2}$$

For bouncing motion of the sprung mass system

$$\begin{aligned}
 M_s Z_s &= -b_f(z_{s1} - z_{u1}) + b_f(z_{s2} - z_{u2}) \\
 &- b_r(z_{s3} - z_{u3}) + b_r(z_{s4} - z_{u4}) \\
 &- k_f(z_{s1} - z_{u1}) + k_f(z_{s2} - z_{u2}) \\
 &- k_r(z_{s3} - z_{u3}) + k_r(z_{s4} - z_{u4})
 \end{aligned} \tag{3}$$

For each side of the wheel motion (vertical direction)

$$M_{uf}\ddot{z}_{u1} = b_r(\dot{z}_{s1} - \dot{z}_{u1}) + k_f(z_{s1} - z_{u1}) - k_{tf}z_{u1} + k_{tf}z_{r1} \tag{4}$$

$$M_{uf}\ddot{z}_{u2} = b_r(\dot{z}_{s2} - \dot{z}_{u2}) + k_f(z_{s2} - z_{u2}) - k_{tf}z_{u2} + k_{tf}z_{r2} \tag{5}$$

$$M_{uf}\ddot{z}_{u3} = b_r(\dot{z}_{s3} - \dot{z}_{u3}) + k_f(z_{s3} - z_{u3}) - k_{tf}z_{u3} + k_{tf}z_{r3} \tag{6}$$

$$M_{uf}\ddot{z}_{u4} = b_r(\dot{z}_{s4} - \dot{z}_{u4}) + k_f(z_{s4} - z_{u4}) - k_{tf}z_{u4} + k_{tf}z_{r4} \tag{7}$$

where

$$\begin{aligned}
 z_{s1} &= T_f\varphi_s + a\theta_s + z_s, & \dot{x}_1 &= \dot{\varphi}_s \approx x_8 \\
 \dot{z}_{s1} &= T_f\dot{\varphi}_s + a\dot{\theta}_s + \dot{z}_s, & \dot{x}_2 &= \dot{\theta}_s \approx x_9 \\
 z_{s2} &= T_f\varphi_s + a\theta_s + z_s, & \dot{x}_3 &= \dot{z}_s \approx x_{10} \\
 \dot{z}_{s2} &= T_f\dot{\varphi}_s + a\dot{\theta}_s + \dot{z}_s, & \dot{x}_4 &= \dot{z}_{u1} \approx x_{11} \\
 z_{s3} &= -T_f\varphi_s + a\theta_s + z_s, & \dot{x}_5 &= \dot{z}_{u2} \approx x_{12} \\
 \dot{z}_{s3} &= -T_f\dot{\varphi}_s + a\dot{\theta}_s + \dot{z}_s, & \dot{x}_6 &= \dot{z}_{u3} \approx x_{13} \\
 z_{s4} &= -T_f\varphi_s + a\theta_s + z_s, & \dot{x}_7 &= \dot{z}_{u4} \approx x_{14} \\
 \dot{z}_{s4} &= -T_f\dot{\varphi}_s + a\dot{\theta}_s + \dot{z}_s
 \end{aligned} \tag{8}$$

The values of are obtained by back substitution into the equation from the (1) to (8). After obtained the state equation, we put into state matrix:

$$\dot{x} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{21} \end{bmatrix} x(t) + Bu(t) + F(t)$$

where

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{21} \end{bmatrix} x(t) = [14 \times 1], u(t) = [4 \times 1],$$

$$B = [14 \times 1], F = [14 \times 4], C = [eye_2 \times 14], D = [zeros_2 \times 4]$$

### 3. SYSTEM ANALYSIS AND CONTROLLER DESIGN

The active suspension system model has been developed in section two. As derived and stated in section two, the model has fourteen (14) state variables. The state matrix (A) of the state space representation of the system is fourteen by fourteen (14×14). Therefore, it needs 14 sensors to measure each state which is not feasible practically. This is costly, so the system should be described with a minimal number of states and this is achieved using a minimal realization principle. In addition, it is difficult to manipulate the system. Therefore, to handle the system, the order of the system state space representation is reduced to eighth (8th) by using minimal realization technique without affecting the characteristics of the original model system. This is proved to be true by checking the response of the 14th order system and the 8th order system for the same input disturbance of single bump as shown in the Figures 4(a) and 4(b). Due to the more state variables are used to describe the system (redundancy of state variables), too much symmetry the system and the system has physically uncontrollable components. It is difficult (not feasible) to implement the system physically in real time application. Therefore, these problems are reduced using minimal realization technique. A minimal realization principle is a means of describing the system with a small number of states. Therefore, the system is implemented with a minimal number of components. Minimal realization technique eliminates uncontrollable or unobservable state in state-space models, or cancels pole-zero pairs in transfer functions. It describes the system with the minimum number of states. Thus, the obtained minimal realization model has minimal order and the same response characteristics as the original model system. A minimal realized system is both controllable and observable [22], [23]. When we split out the Figure 4(a) it looks as displayed in the Figure 4(b). So, our conclusion was that there is no any deviation between the original system and Minimal realization system. This implies that we can use the realization system instead of original system.

As it can be observed in Figures 4(a) and 4(b) the simulation results of both the original model system (14×14 matrixes) and minimal realized system (8×8 matrix) are the same. Therefore, this shows that minimal realization technique doesn't alter the behavior of the original model system. The six states (velocities of the four wheels, and positions of coupling wheels) were removed, since they were not affect the entire suspension system model due to velocities of the wheels are the same to the vehicle suspension system, and the position of the coupling wheels (both front wheels, and rear wheels) are the same. The difference of relative motion between them are zero. Therefore, the new state space model becomes as follow. A=[8×8], B=[8×4], C=[2×8], D=[zeros(2)×8]. In order to design a linear quadratic regulator controller, the system must be fully controllable. This is verified by determining the rank of the controllability matrix (A, B). Therefore, the controllability matrix (A, B) of the active suspension system as shown in the appendix B has full rank (8), which makes it fully controllable. It is also known that if the system is minimal realized, then it is fully accessible and observable. Stability is an important property that a system is required to have. It is usually not desirable that a small change in the input, initial condition, or parameters of the system produces a very large change in the response of the system. If the response increases indefinitely with time, the system is said to be unstable. The open-loop response of the system to a road profile 1 can be used to verify stability of the system.

From the Figure 5(a) it can be observed that the response of the suspension system without feedback controller is bounded. This shows that the system is stable because, for the bounded input the system is producing bounded output. Furthermore, Figure 5(a) displayed that open-loop body displacement of the suspension system model for road single bump road profile Figure 5(b), and open-loop body, pitch, roll, and yaw displacement of the suspension system model for road single bump road profile. As it is observed in Figures 5(a) and 5(b), the suspension system without active controller is stable, however, it needs an improvement to be more comfort for passengers. Therefore, in order to enhance the performance a suitable

controller must be designed. The controller generates an appropriate control signal to maintain the car body at the desired equilibrium position in response to road disturbances. The controller designed will help the system to be insensitive to the road disturbances. Therefore, the controller will try to make the system stable and perform well regardless of the disturbance. The linear quadratic regulator is the extension of pole placement technique that tends to find the control input so as to place the poles of the system at a desired optimal position. The main idea in linear quadratic regulator control design is to minimize the quadratic cost function of J given in (9) [24], [25].

$$J = \frac{1}{2} \int_{t_0}^{t_f} [x^T(t)Q(t)x(t) + u^T(t)R(t)u(t)] dt \tag{9}$$

The LQR should minimize this cost function (performance index) while obtaining the state feedback gains K that drives the system to the desired operating point. It turns out that regardless of the values of Q and R, the cost function has a unique minimum that can be obtained by solving the following Algebraic Riccati equation [25].

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{10}$$

By solving the above Riccati equation the positive-definite matrix P is obtained, thus the optimal gain (K) and controller (u) are determined as (11):

$$\begin{aligned} k &= R^{-1}B^T P \\ u(t) &= -kx(t) \end{aligned} \tag{11}$$

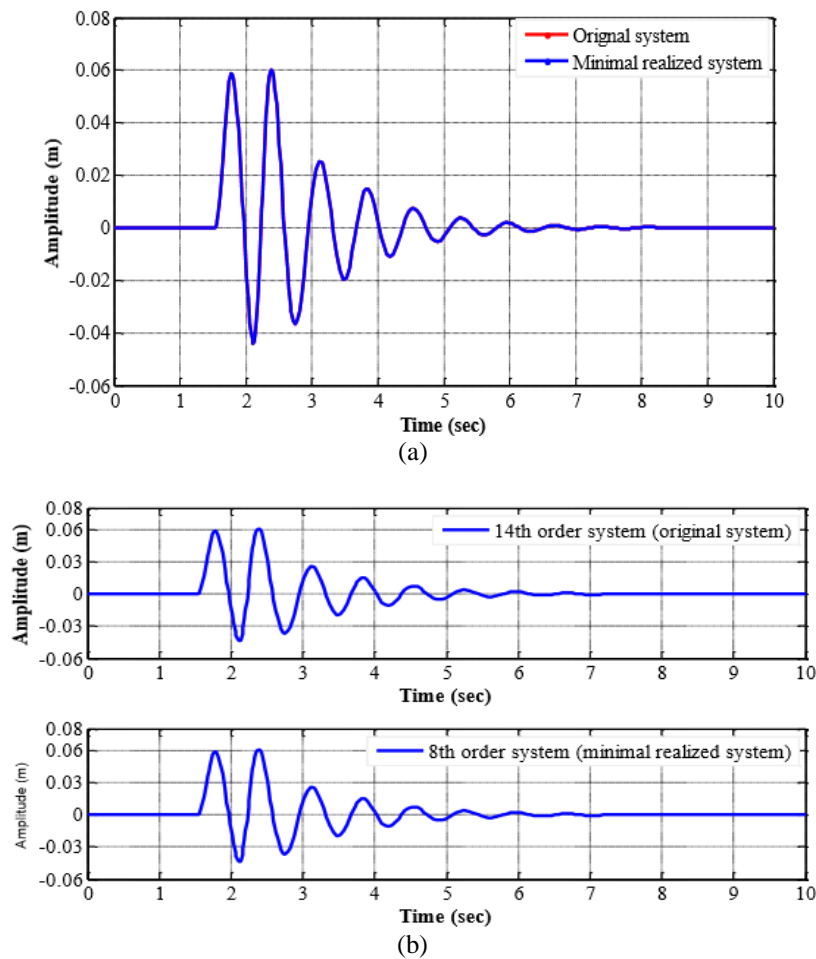


Figure 4. Comparing the original and reduced automobile suspension system (a) comparing the original and minimal realization system and (b) comparison the response of original and minimal realized systems

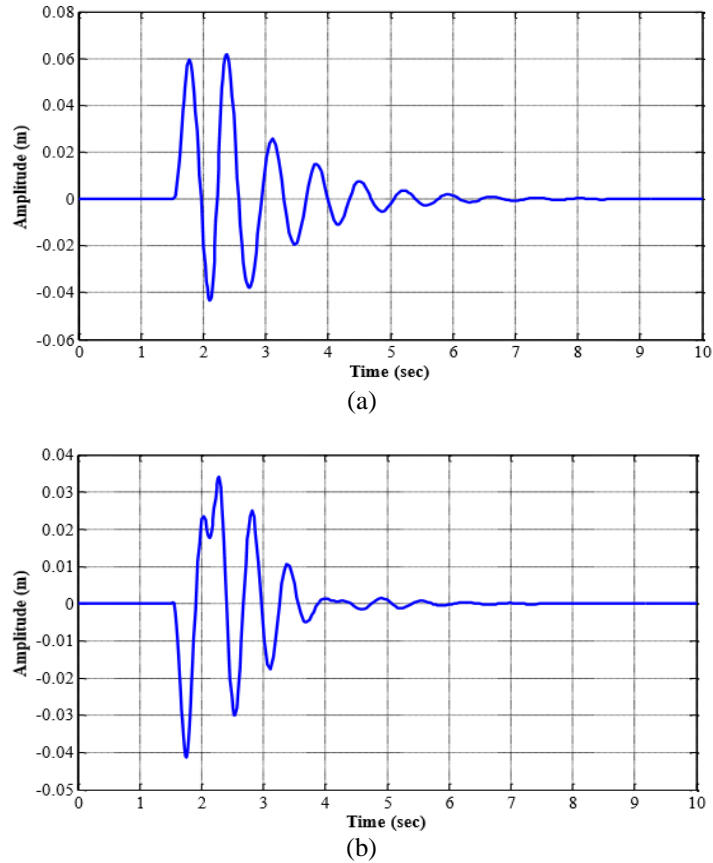


Figure 5. Observing the effects of (a) open-loop body displacement of the suspension system model for road profile 1 and (b) open-loop pitch angle response of the suspension system model for road profile 1

From Figure 5, we can observe that the effects of Figure 5(a) open-loop body displacement of the suspension system model for road profile, and Figure 5(b) open-loop pitch angle response of the suspension system model for road profile. The control signal given by (11) is the optimal control law. Therefore, if the matrix  $K$  are determined so as to minimize the performance index,  $J$ , then (11) is optimal for any initial state (0). The block diagram of the optimal LQR controller is displayed in the Figure 6.

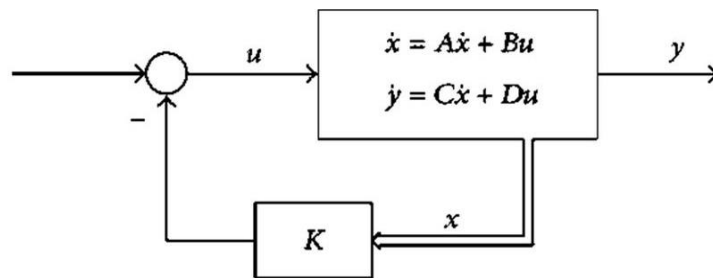


Figure 6. Block diagram of LQR control scheme

As seen in the cost function,  $J$ , given in (8) in addition to the states  $x(t)$  and control signals  $u(t)$  there are the weighting matrices  $Q$  and  $R$ . The parameters  $Q$  and  $R$  can be used as design parameters to penalize the state variables and the control signals. The larger these values are, the more you penalize these signals. The parameter  $Q$  penalizes the deviation of system states from the equilibrium. Basically, selecting a large value for  $Q$  means you try to stabilize the system with the least possible changes in the states and large  $Q$  implies less concern about the changes in the states. The parameter  $R$  penalizes the use of input control

signal. If you choose a large value for R means you try to stabilize the system with less (weighted) energy. This is usually called expensive control strategy. On the other hand, choosing a small value for R means you don't want to penalize the control signal (cheap control strategy). Q and R are weighting matrices and should be positive semi-definite and positive definite, respectively. They are also symmetric matrices  $Q=Q^T$ ;  $R=R^T$ . Normally, the Q and R matrices are chosen as diagonal matrices such that the quadratic performance index is a weighted integral of squared error. The sizes of Q and R matrices depend on the number of state variables and input variables respectively. If the system A, B is controllable, then we can place the Eigen values of the closed loop system anywhere we want. That is extremely powerful, but in practice it is sometimes not usefulness. Therefore, there is problem with placing system Eigen values. The main problem is we do not have a great sense of the input that is required to accomplish the Eigen values of the closed loop system to place anywhere. For example, if you try to make your system super-fast (i.e. placing of poles far way to the left of the s-plane) it can require huge input, which the real physical system could not achieve. So just placing system Eigen values from this fundamental reason to the appropriate position is required. Since the controller is a regulator, it tries to drive each state to a constant set point. In this thesis, the controller drives each state to zero. So, any value bigger than zero weather it is positive or negative is bad in this case. Therefore, the controller drives each state (t) to zero as time tends to infinity. By choosing the value of Q and R we can change the relative weightings of one state versus another. Since the number of state variables are eight, the value of Q can be represented by the following eight by eight ( $8 \times 8$ ) matrix  $Q=10000 \cdot [\text{eye}(8 \times 8)]$ ;  $R=0.001 \cdot [\text{eye}(8 \times 8)]$ . All the diagonal elements penalize their correspondence state individually. The off diagonal elements penalize combination of the states. Therefore, since the outputs of the system are combination of these individual states, by penalize each individual state independently using its respective Q value and observing the combination effect on the outputs the required performance can be achieved. In fact, penalize one state has an effect on another but it is small. So, the value of Q is selected carefully and systematically. The Figure 7(a) demonstrated the responses of four tires for each state variable for the Q and R values, whereas, the Figure 7(b) showed the responses of roll, pitch, and yaw for each state variable for the Q and R values respectively.

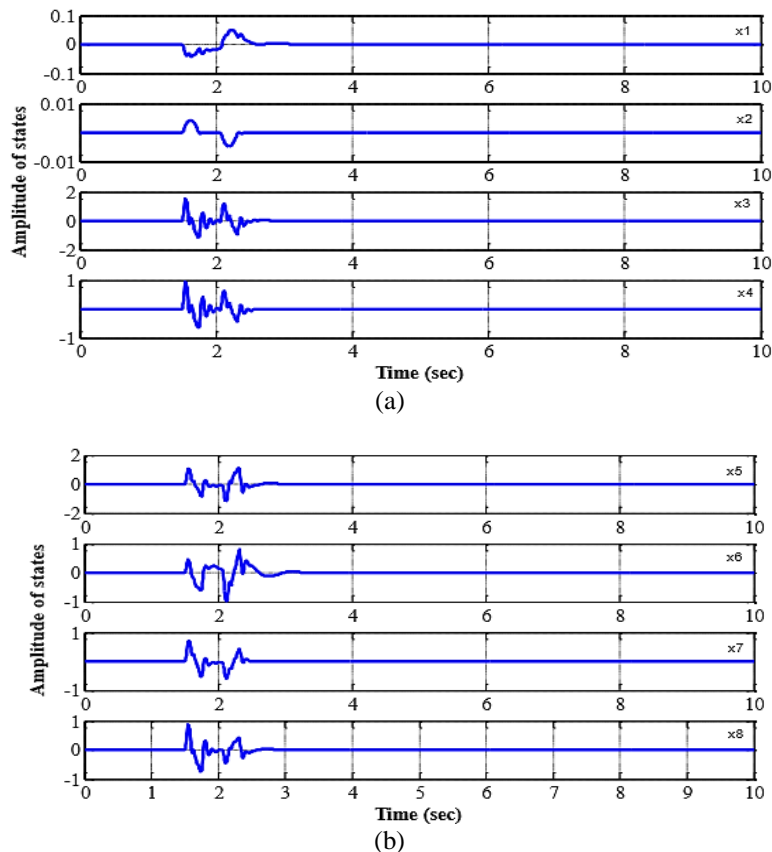


Figure 7. Response of each state variable for the Q and R values, (a) responses of four tires for each state variable for the Q and R values and (b) responses of roll, pitch, and yaw for each state variable for the Q and R values



#### 4. SIMULATION RESULTS AND DISCUSSION

This section discusses about simulation results of passive and active suspension systems for different scenarios. Performance of the suspension system in terms of ride quality will be observed, where car body displacement, velocity, pitch angle and angular velocity are considered as output parameters. Using the vehicle data's, the exact physical characteristics of that car can be determined by simulating the mathematical model with the help of MATLAB software. In order to show the performance of the LQR with actuator dynamics with tuned values Q and R matrices, different bump disturbances are applied to the system to observe for different scenarios. In order to study the dynamic behavior of the vehicle and to analysis the performance of the suspension system an external excitation input for the model is required. In this study, a different type of sinusoidal function road profiles is used as excitation for simulation purpose. The road bump profile in Figure 8 is appearing for  $1.5 \leq t \leq 1.75$  sec for front right and left wheels and  $2.06 \leq t \leq 2.31$  sec for rear right and left wheels. The width of each bump  $\Delta t$  in this case 0.25 sec indicates the duration of the road bump at each wheel. From Figure 9, it can be seen that the peak value response of chassis displacement for passive suspension system is 0.06 m: It can be also observed that the peak value response of the chassis displacement for active suspension excluding actuator dynamics in the system model is 0.055 m while that for the active suspension with actuator dynamics included in the system model is 0.045 m for the same road input and the same controller gains. Furthermore, the blue curve indicates the linear quadratic regulator with actuator dynamics, the rose color showed the passive suspension system, and the black dotted curve tells the linear quadratic regulator without actuator dynamics at single bump road profile. The reduction (improvement) in percentage for the displacement of the chassis can be calculated as (12), (13):

$$rp = \left( \frac{pv - av}{pv} \right) * 100\% \quad (12)$$

$$re = \left( \frac{ev - av}{ev} \right) * 100\% \quad (13)$$

where rp=reduction in peak value from passive pv=passive value av=active (LQR) with actuator dynamics value re=reduction in peak value from active excluding actuator dynamics ev=active (LQR) excluding actuator dynamics value.

$$rp = \left( \frac{0.06 - 0.045}{0.06} \right) * 100\% = 25\%; re = \left( \frac{0.055 - 0.045}{0.055} \right) * 100\% = 18.18\%$$

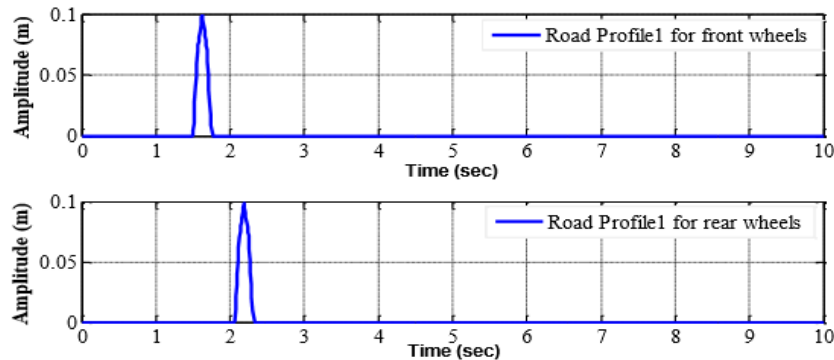


Figure 8. Road input disturbance of a single bump

Thus, the chassis displacement (peak value) is reduced by 25% and 18.18% in case of an active suspension system with actuator dynamics which is included in the system model. This is a direct indication of the superiority of active suspension system using LQR with actuator dynamics over passive suspension system and active suspension system without actuator dynamics. The settling time, as we can observe from Figure 9 is 4.95 sec, 3.45 sec and 1.9 sec for passive suspension, active suspension excluding actuator dynamics and active suspension including actuator dynamics respectively. Thus, reductions (improvements) in settling time in active suspension including actuator dynamics in the system model are 61.61% and 44.93% as compared to passive suspension and LQR excluding actuator dynamics respectively. Moreover, the Table 1 comparison of passive suspension system, linear quadratic regulator without actuator dynamics, and linear quadratic regulator with actuator dynamics for displacement.

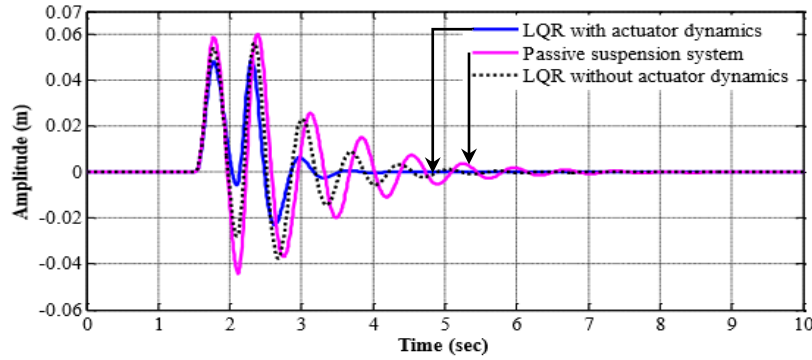


Figure 9. Simulation result of comparison for body displacement using a single bump

Table 1. Comparison of PSS, LQR without actuator dynamics, and LQR with actuator dynamics for displacement

PS	PSS (rpm)	LQRA	LQRAO	% LQRA over PSS	% LQRA over LQRAO
PPR(m)	0.06	0.045	0.055	25%	18.18%
Ts(sec)	4.95	1.9	3.45	61.61%	44.93%
VPR(m/s)	0.65	0.4	0.56	38.46%	28.57%
Ts(sec)	4.95	1.7	3.45	65.66%	50.72%
OPR(rad)	0.0388	0.0263	0.0338	32.22%	22.19%
Ts(sec)	3.95	1.57	2.45	62.25%	35.92%
AVPR(rad/s)	0.3375	0.2625	0.2875	22.22%	8.7%
Ts(sec)	3.95	1.57	2.45	60.25%	35.95%

Where PS=performance specifications, PSS=passive suspension system, LQRA=LQR with actuator dynamics, LQRAO=LQR without actuator dynamics, PPR=position peak response, (amplitude)(m)=Ts(sec) settling time(sec), VPR(m=sec)=velocity peak response (amplitude)(m=s); OPR(rad) orientation peak response (amplitude)(rad), AVPR(rad/sec)=angular velocity peak response (amplitude)(rad/s). As observing from the simulation result of Figure 10, the peak overshoot of sprung mass velocity for passive suspension system is 0.65 m/s. For the LQR without actuator dynamics and LQR with actuator dynamics are 0.56 m/s and 0.4 m/s respectively. From these values, it is found that for active suspension system (LQR) with actuator dynamics the peak value of the velocity of the sprung mass is reduced by 38:46% as compared to passive suspension system. As compared to active suspension system (LQR) without actuator dynamics the reduction is 28:57%. The passive suspension system and LQR without actuator dynamics have the same settling time as of the displacement whereas the settling time for the LQR with actuator dynamics is 1:7 sec. Therefore, the reduction settling time in active controller (LQR) with actuator dynamics which is included in the system model is 65:66% as compared with passive system while compared with LQR without actuator dynamics is 50:72%. Furthermore, the blue curve indicates the linear quadratic regulator with actuator dynamics, the rose color showed the passive suspension system, and the black dotted curve tells the linear quadratic regulator without actuator dynamics at single bump road profile.

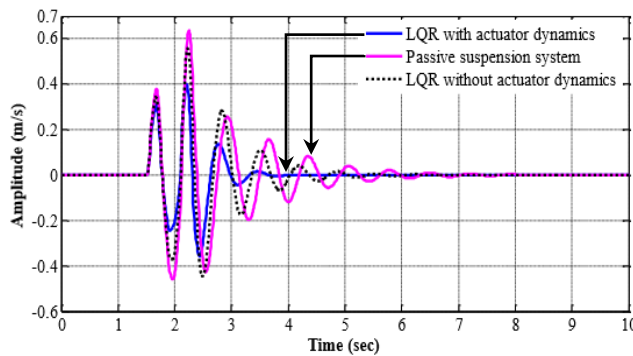


Figure 10. Simulation result of comparison for body velocity using a single bump

Similarly, in Figure 11 we can observe that, the peak values of the pitch angle for sprung mass of passive suspension, LQR without actuator dynamics and LQR with actuator dynamics are 0.0388 rad, 0.0338 rad and 0.0263 rad respectively. 32.22% and 22.19% are the peak value reductions in LQR with actuator dynamics as compared to passive suspension system and LQR without actuator dynamics respectively. From the Figure 11, the settling time for the passive suspension system is 3.95 sec. For the LQR without actuator dynamics the settling time is 2.45 sec. and for the LQR with actuator dynamics, it is 1.57 sec. Thus, the settling time reductions (improvements) are 60:25% and 35:92% for LQR with actuator dynamics as compared to the passive and LQR without actuator dynamics respectively. From the Figure 11, the settling time for the passive suspension system is 3:95 sec. For the LQR without actuator dynamics the settling time is 2:45 sec and for the LQR with actuator dynamics, it is 1:57 sec: Thus, the settling time reductions (improvements) are 60:25% and 35:92% for LQR with actuator dynamics as compared to the passive and LQR without actuator dynamics respectively. The Figure 12(a) showed that the force generated two wheels from front tires (actuators) in active suspension system at single road profile (bump). Further, Figure 12(b) demonstrated that the force generated two wheels from front tires (actuators) in active suspension system at single road profile (bump). Moreover, all the response of the tires is settles about 3.5 second; this showed that the proposed control algorithm is best fitting on the active suspension system. The time interval 1.7 second to 3.5 second tell us the four tires, and roll, pitch, yaw is in the condition of unstable. With sometimes later at 3.5 second all the active suspension systems are well settle.

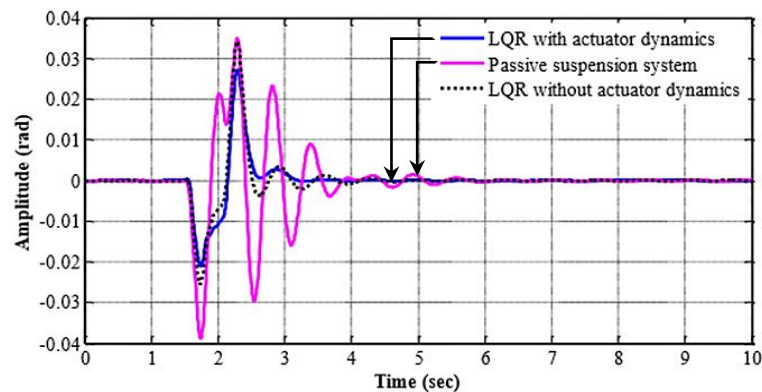


Figure 11. Simulation result of comparison for pitch angular velocity using upward single bump

Figure 13 displayed that the different road profile applied on the front and rear wheels respectively. The top most indicates that the road profile for the front wheels whereas the bottom most demonstrated that the rear wheels. Furthermore, it means that Speed humps are in widespread use around the world. Despite their effective performance in increasing safety, they cause considerable damage to vehicles and discomfort to drivers and passengers. So, this road profiles are used as input for the system.

As it can be seen in Figures 14 for a two bumps input road profile with different amplitudes, the amplitude and settling time of the results are different. For the high amplitude input road profile both the amplitude and settling time of the simulation results are higher than as the input is the low amplitude road profile which is expected. In addition to, the simulation results indicate that for existence of road disturbances the vehicle body vibrates up and down from its equilibrium position which is zero whereas for the smooth (absence) of road input disturbance the vehicle body will be remained in its equilibrium position.

Figure 15(a) displayed that the comparison of suspension system with actuator dynamics, without actuator dynamics, and passive suspension system for angular position at double road bump profile, whereas Figure 15(b) indicates that comparison of suspension system with actuator dynamics, without actuator dynamics, and passive suspension system for angular velocity at double road bump profile. From the Figures 15 we generalized as the proposed control is best fit for the regulation and optimization of the vehicle suspension with actuator dynamics, and without actuator dynamics, compared to passive suspension system. Furthermore, the proposed controller that is linear quadratic regulator is more effective on the suspension system with actuator dynamics, as we compared to without actuator dynamics. From Figure 16 we infer those effects of forces on actuators are demonstrated as Force generated from front actuators in active suspension system using double bumps displayed in Figure 16(a) whereas Figure 16(b) indicated that Force generated from rear actuators in ASS using double bumps.

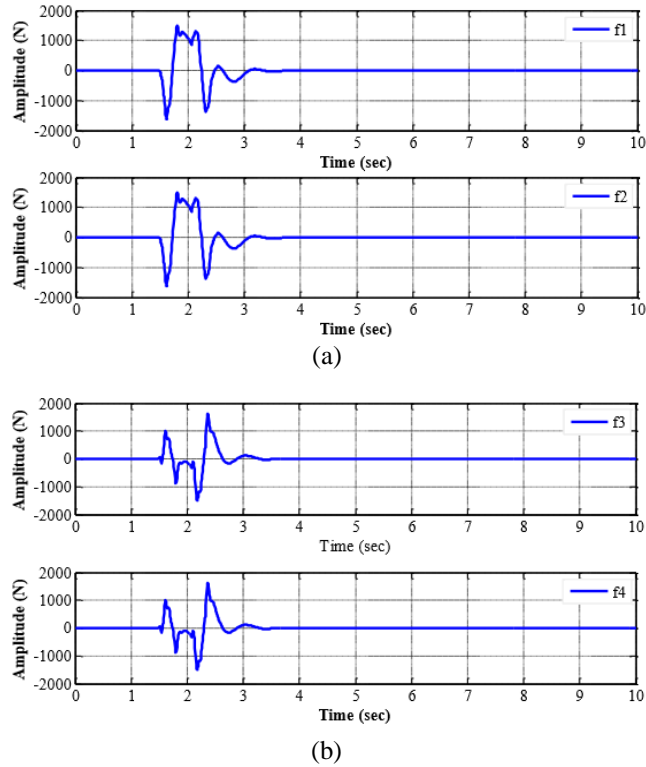


Figure 12. Effects of the forces on actuator (a) force generated from front actuators in active suspension system using a single bump and (b) force generated from rear actuators in active suspension system using a single bump

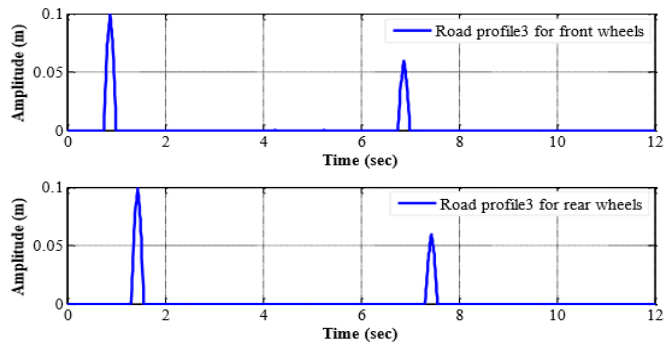


Figure 13. Road input disturbance of two road bumps

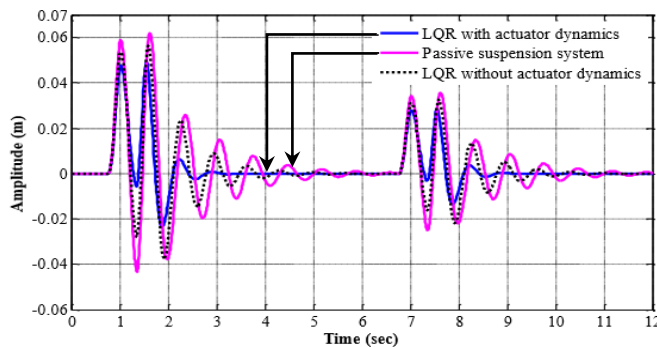


Figure 14. Comparison of the vehicle suspension system by considering actuator and without considering actuator dynamics at double bump road profile

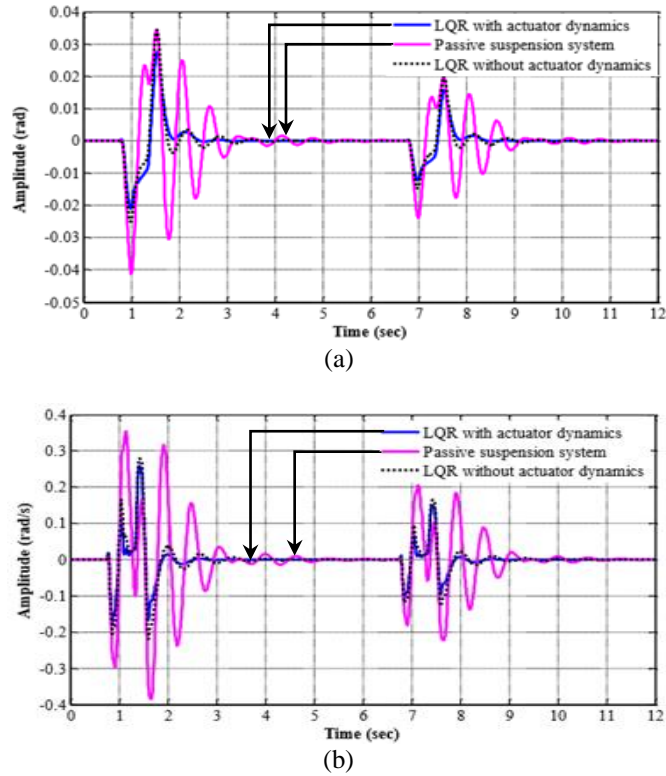


Figure 15. Effects of automobile vehicle suspension system on double bump road profile, (a) simulation result of comparison for pitch angle using two bumps and (b) simulation result of comparison for pitch angular velocity using two bumps

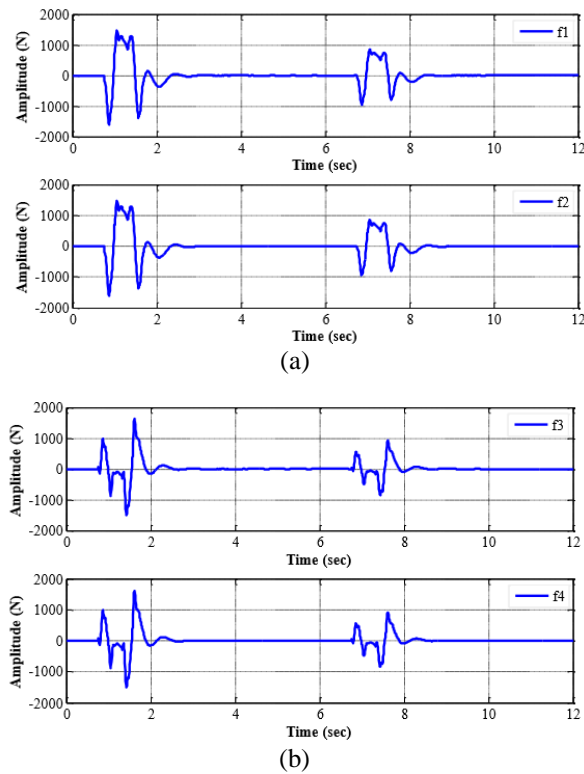


Figure 16. Effects of forces on actuator, (a) force generated from front actuators in ASS using double bumps and (b) force generated from rear actuators in active suspension system using double bumps

## 5. CONCLUSION

Active suspension system is one part of the essential mechatronic system of a vehicle system. In this research paper, with some assumptions, the model of the active suspension system is developed and the state feedback controller LQR is designed. By comparing the performance of the passive, LQR without actuator dynamics and LQR with the actuator dynamics, the simulation results clearly indicate that LQR with actuator dynamics can give lower amplitude and faster settling time. The reduced value of peak response will result in less sprung-mass travel and hence, the reduced vibrations felt by the passenger. The less settling time will quickly suspend the oscillations induced in the car body which will ensure better comfort to the passenger. Therefore, the proposed LQR controller with actuator dynamics which is included in the system model is more effective in the vibration isolation of the car body than the passive suspension system and LQR without actuator dynamics. So, the proposed LQR controller with the selected weighting matrices Q and R is acceptable. The proposed LQR control with actuator dynamics that is included in the system model gives 25% and 18.18% reduction in the peak value of vertical displacement as compared to passive and LQR controller without actuator dynamics respectively, for the same road input and the same controller gains, thus improving passenger comfort. It is found that for LQR controller with actuator dynamics, the peak value of the velocity of the sprung mass is reduced by 38.46% compared to passive suspension system while compared to LQR without actuator dynamics the reduction is 28.57% which guarantee better ride comfort.




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