International Journal of Electrical and Computer Engineering (IJECE)
Vol. 12, No. 3, June 2022, pp. 2636~2644
ISSN: 2088-8708, DOI: 10.11591/ijece.v12i3.pp2636-2644

Optimum consultation for serial distributed detection systems

Nedal Al-Ababneh[1], Hasan Aldiabat[2]
[1]Department of Electrical Engineering, Jordan University of Science and Technology, Irbid, Jordan
[2]Department of Telecommunications Engineering, Yarmouk University, Irbid, Jordan

Article Info

Article history:

Received Aug 8, 2019
Revised Nov 7, 2021
Accepted Nov 30, 2021

Keywords:

Decision fusion
Serial distributed detection systems
Statistical signal processing
Wireless sensor networks

ABSTRACT

This paper considers a distributed detection system which consists of N sensors that are connected in series. The observations of each sensor in this system design are considered to be statistically independent of all other sensors. In contrast to the popular serial decision fusion systems, we assume that consultations are allowed in a serial manner between successive sensors that make up the system. In addition, the paper demonstrates the similarity between the proposed consulting serial system and the optimal serial one in terms of detection probabilities for a give probability of false alarm. However, it should be emphasized that the proposed system has the benefit of conditional nonrandom consultation among the sensors. Consequently, its survivability is higher than that of serial systems. Numerical evaluations for the cases of two and three sensors are provided and compared with those of the serial as well as the centralized schemes.

Corresponding Author:

Nedal Al-Ababneh
Department of Electrical and Computer Engineering, Jordan University of Science and Technology
Irbid, P.O. Box 21110, Jordan
Email: nedalk@just.edu.jo

1. INTRODUCTION

Conventional detection systems used in communication and surveillance models utilize one sensor (detector) for signals detection. This sensor in such detection systems is supposed to have all of the observations of the participating sensors, which might be connected in serial, parallel, consulting, or hybrid topology, in a single place. Therefore, a large bandwidth and considerable processing time are required. To overcome these limitations, several researches have investigated this topic [1]–[25]. To just name a few, Tenney and Sandell [1] considered the design of the theoretical framework of the hypothesis testing problem at the sensors for the two-sensor distributed detection system. In addition, the authors provided a tradeoff between the performance of centralized and decentralized detection systems for a range of signal to noise ratio levels. In [2], an optimal data fusion rule based on decisions of individual participating sensors and threshold comparisons was presented. Hoballah and Varshney [3] considered using the criterion of Neyman Pearson for several sensors including the design of local sensor decision rules given the fusion rule and the design of the fusion rule given the local decision rules. In [4], serial and parallel distributed detection schemes using N sensors were considered. In addition, local sensor decision rules were provided in [4] using Neyman Pearson and Bayesian tests.

In this article, we present a distributed detection scheme of multiple sensors that are connected in series. The system in this scheme contains several local detectors/sensors as demonstrated in Figure 1(a). In this scheme, the decision of the first detector, u_1, is determined based on its observation, Z_1, and this decision (u_1) is passed to the second sensor. Therefore, the second sensor decides its decision based on its observation, Z_2, and the received decision u_1. This process is continued until the last, N^{th}, sensor decides using the $(N-1)^{th}$ decision, u_{N-1}, and its own observation, Z_{th}. The decision of the last, N^{th}, sensor is named as the global or the final decision $\left(i.e.u_N = u_f\right)$. For the instance of two sensors connected in series, Thomopoulos *et al.* [5]–[7] explored the communications costs associated with any consultation between the sensors of the distributed detection system in details. The sensors were classified as a primary sensor and a consulting sensor. The primary sensor is the one that is responsible for generating the global decision, and the consulting sensor is the one that relays its decision to the primary upon request. For every decision making scenario, an optimization problem with a set of constraints that the designers consider crucial is constructed. The work presented in this paper is an extension of the work in [5] where more than two sensors are considered herein, as shown in Figure 1(b). Moreover, the paper shows analytically that by properly choosing local detectors thresholds, the performance of the introduced detection scheme is equivalent to that of the optimum serial system. Besides, the paper demonstrates the consultation rate for the case of two sensors and shows that its value is relatively low, which helps in making the system less detectable and hence suitable for operation in a hostile environment.

The rest of the paper is organized as follows: in Section 2, we introduce the proposed consulting detection scheme for the case of two local sensors. We consider the proposed detection scheme for the case of three local sensors in Section 3. In Section 4, we provide numerical results that demonstrate the performance in terms of detection probability of the proposed scheme for both cases of two and three sensors. In addition, consultation rates for the case of two sensors is provided. Finally, the conclusions of the paper are drawn in Section 5.

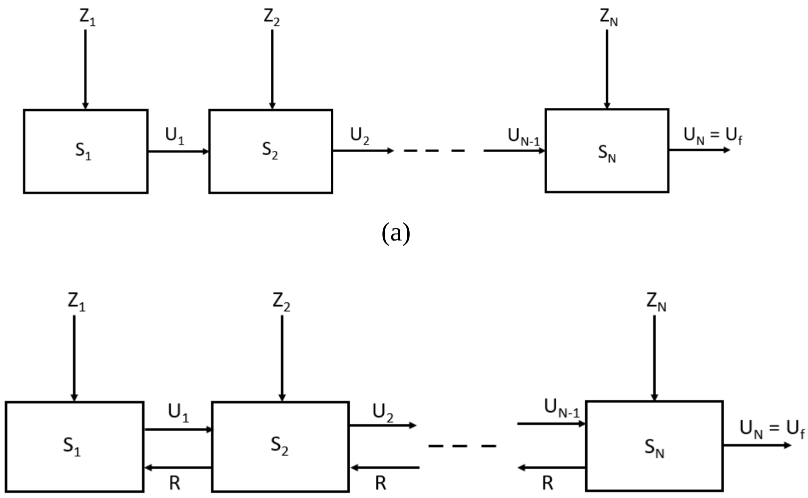

Figure 1. Schematic diagrams of distributed (a) serial detection and (b) serial consulting detection systems

2. THE PROPOSED CONSULTING DETECTION SCHEME USING TWO SENSORS

For the sake of the study presented in this paper, we consider the schematic of the serial detection system shown in Figure 1(a) and let $N = 2$. In addition, the observations Z_1 and Z_2 are assumed to be statistically independent under the binary hypotheses H_1 (signal is present) and H_0 (signal is absent). Define $P_{Z_i/H_j}(Z_i/H_j)$ to be the the probability density function (PDF) of Z_i given H_j is true for $i = 1, 2$ and $j = 0, 1$. The optimal serial test for maximizing the detection probability, P_D, for a given probability of false alarm, P_F can be stated as the following [17]: the test is initiated at sensor S_1 by comparing the likelihood ratio (LHR) $\Lambda(Z_1)$ with a single threshold $t_{11}^s = t_{10}^s$ as in (1).

$$\Lambda(Z_1) = \frac{p_{Z_1/H_1}(Z_1/H_1)}{p_{Z_1/H_0}(Z_1/H_0)} \begin{cases} > t_{11}^s \Rightarrow u_1 = 1 \\ \leq t_{11}^s \Rightarrow u_1 = 0 \end{cases} \tag{1}$$

The decision of sensor one (S_1), u_1 is transferred to sensor two (S_2). As a result, S_2 computes the likelihood ratio $\Lambda(Z_2, u_1)$ and performs the test

$$\Lambda(Z_2, u_1) = \frac{p_{Z_2,u_1/H_1}(Z_2,u_1/H_1)}{p_{Z_2,u_1/H_0}(Z_2,u_1/H_0)} \begin{cases} > t_2^s \Rightarrow H_1 \\ \leq t_2^s \Rightarrow H_0 \end{cases} \tag{2}$$

The observation Z_2 and the decision u_1 are statistically independent, therefore, we can write

$$p_{Z_2,\frac{u_1}{H_j}}\left(Z_2,\frac{u_1}{H_j}\right)=p_{\frac{Z_2}{u_1},H_j}\left(\frac{Z_2}{u_1},H_j\right)\cdot p_{\frac{u_1}{H_j}}\left(\frac{u_1}{H_j}\right)= p_{Z_2/H_j}(Z_2/H_j)\ \cdot p_{u_1/H_j}(u_1/H_j) \tag{3}$$

Substituting (3) in (2), we obtain

$$\Lambda(Z_2,u_1)=\Lambda(Z_2)\cdot\Lambda(u_1)\begin{cases}> t_2^s \Rightarrow H_1\\ \leq t_2^s \Rightarrow H_0\end{cases}, \tag{4}$$

where, the values that the random variable (RV) $\Lambda(u_1)$ can assume are demonstrated in [8] and given by (5):

$$\Lambda(u_1)=\begin{cases}\frac{P_{D_1}}{P_{F_1}} & \text{when } u_1=1\\ \frac{1-P_{D_1}}{1-P_{F_1}} & \text{when } u_1=0\end{cases}, \tag{5}$$

where, P_{D1} and P_{F1} are defined by (6) and (7):

$$P_{D_1}=p_r\{\Lambda(Z_1/H_1)>t_{11}^s\}=\int_{>t_{11}^s} dp_{\Lambda(Z_1/H_1)}\big(\Lambda(Z_1/H_1)\big) \tag{6}$$

and

$$P_{F_1}=p_r\{\Lambda(Z_1/H_0)>t_{11}^s\}=\int_{>t_{11}^s} dp_{\Lambda(Z_1/H_0)}\big(\Lambda(Z_1/H_0)\big), \tag{7}$$

where, $P_{\Lambda(Z_1/H_j)}\Lambda(Z_1/H_j)$ is the distribution of the LHR $\Lambda(Z_1)$ under the hypothesis H_j. For $u_1=1$, substituting (5) in (4), we arrive at the test

$$\Lambda(Z_2)\begin{cases}> \frac{P_{F_1}}{P_{D_1}}\cdot t_2^s=t_{21}^s \Rightarrow \text{decide } H_1(u_2=1)\\ \leq \frac{P_{F_1}}{P_{D_1}}\cdot t_2^s=t_{21}^s \Rightarrow \text{decide } H_0(u_2=0)\end{cases}. \tag{8}$$

Similarly, when $u_1=0$, substituting (5) in (4), we obtain the test

$$\Lambda(Z_2)\begin{cases}> \frac{1-P_{F_1}}{1-P_{D_1}}\cdot t_2^s=t_{20}^s \Rightarrow \text{decide } H_1(u_2=1)\\ \leq \frac{1-P_{F_1}}{1-P_{D_1}}\cdot t_2^s=t_{20}^s \Rightarrow \text{decide } H_0(u_2=1)\end{cases}. \tag{9}$$

It should be emphasized that the tests $\Lambda(Z_2)$ in (8) and (9) are final. Clearly, the observation space is partitioned by the tests in (1), (8), and (9) as illustrated in Figure 2(a). The hypothesis H_1 is declared in the region $\{\Lambda(Z_1)>t_{11}^s,\Lambda(Z_2)>t_{21}^s\}$ or the region $\{\Lambda(Z_1)<t_{11}^s,\Lambda(Z_2)>t_{20}^s\}$. Similarly, the hypothesis H_0 is declared in the region $\{\Lambda(Z_1)>t_{11}^s,\Lambda(Z_2)<t_{21}^s\}$ or the region $\{\Lambda(Z_1)<t_{11}^s,\Lambda(Z_2)<t_{20}^s\}$. In serial detection systems, the optimization amounts to specify t_{11}^s, t_{21}^s, and t_{20}^s such that P_{Ds2} is maximum for a given P_{Fs2}. The probability of detection, P_{Ds2}, is given by 10.

$$P_{D_s2}=p_r\big(u_f=1/H_1\big)=\sum_{u_1}p_r(u_2=1/u_1,H_1)\ =p_r(u_2=1/u_1=1,H_1)p_r(u_1=1/H_1)+p_r(u_2=1/u_1=0,H_1)p_r(u_1=0/H_1) \tag{10}$$

Expressing P_{Ds2} in terms of the likelihood ratios $\Lambda(Z_1)$ and $\Lambda(Z_2)$, we get

$$P_{D_s2}=\int_{>t_{21}^s} dp_{\Lambda(Z_2/H_1)}\big(\Lambda(Z_2/H_1)\big)\int_{>t_{11}^s} dp_{\Lambda(Z_1/H_1)}\big(\Lambda(Z_1/H_1)\big)+\int_{>t_{20}^s} dp_{\Lambda(Z_2/H_1)}\big(\Lambda(Z_2/H_1)\big)\int_{<t_{11}^s} dp_{\Lambda(Z_1/H_1)}\big(\Lambda(Z_1/H_1)\big) \tag{11}$$

Similarly, the false alarm probability, P_{Fs2}, is given by (12)

$$P_{F_s2} = \int_{>t_{21}^s} dp_{\Lambda(Z_2/H_0)}(\Lambda(Z_2/H_0)) \int_{>t_{11}^s} dp_{\Lambda(Z_1/H_0)}(\Lambda(Z_1/H_0)) + \int_{>t_{20}^s} dp_{\Lambda(Z_1/H_0)}(\Lambda(Z_1/H_0)) \int_{<t_{11}^s} dp_{\Lambda(Z_1/H_1)}(\Lambda(Z_1/H_1)) \quad (12)$$

If we consider the consulting detection system illustrated in Figure 1(b) and assume $N = 2$, the procedures of the test can be described as follows: The test is initiated at sensor two (S_2) as given by (13).

$$\Lambda(Z_2)\begin{cases} > t_{20}^c & \Rightarrow u_2 = 1 \\ \le t_{21}^c & \Rightarrow u_2 = 0. \\ \text{otherwise} & \Rightarrow u_2 = I \end{cases} \quad (13)$$

When $u_2 = 1, 0$, the test is terminated and the hypotheses H_1, H_0 are declared, respectively. Nevertheless, when $u_2 = I$, the primary sensor (S_2) declares ignorance and consults the consulting sensor S_1 [8]. No information is communicated from S_2 to S_1 expect the fact that $u_2 = I$ (consultation). Upon consulted, the sensor S_1 performs a likelihood ratio test based on its own observations and the fact that it was consulted as given by (14).

$$\Lambda(Z_1)\begin{cases} > t_{11}^c & \Rightarrow u_1 = 1 \\ \le t_{11}^c & \Rightarrow u_1 = 0 \end{cases} \quad (14)$$

The decision u_1 as specified in (14) is transferred to sensor two (S_2) which in turn announces it as a global or a final decision. The tests demonstrated in (13) and (14) partition the observation space of the detecting system as shown in Figure 2(b). The hypothesis H_1 is declared in the region $\{\Lambda(Z_2) > t_{20}^c\}$, or the region $\{\Lambda(Z_1) > t_{11}^c, t_{21}^c < \Lambda(Z_2) < t_{20}^c\}$. Similarly, the hypothesis H_0 is declared in the region $\{\Lambda(Z_2) < t_{21}^c\}$, or the region $\{\Lambda(Z_1) < t_{11}^c, t_{21}^c < \Lambda(Z_2) < t_{20}^c\}$. Once again, the optimization of the distributed detection system using consultation amounts to specifying t_{21}^c, t_{20}^c, and t_{11}^c such that P_{Dc2} is maximum for a given P_{Fc2}. The probability of detection, P_{Dc2}, is given by (15).

$$P_{D_c2} = p_r\left(u_f = \frac{1}{H_1}\right) = \sum_{u_2} p_r\left(u_f = \frac{1}{u_2}, H_1\right) = p_r\left(u_f = \frac{1}{u_2=1,H_1}\right) p_r\left(u_2 = \frac{1}{H_1}\right) + p_r(u_f = 1/u_2 = 0, H_1)p_r(u_2 = 0/H_1) + p_r(u_f = 1/u_2 = I_2, H_1)p_r(u_2 = I_2/H_1) \quad (15)$$

The second part of the right-hand side of (15) equal zero, whereas the first and third parts can be manipulated to given by (16).

$$P_{D_c2} = p_r(u_2 = 1/H_1) + p_r(u_1 = 1/u_2 = I_2, H_1) \cdot p_r(u_2 = I_2/H_1) \quad (16)$$

Likewise, the probability of false alarm, P_{Fc2}, is formulated as in (17).

$$P_{F_c2} = p_r(u_2 = 1/H_0) + p_r(u_1 = 1/u_2 = I_2, H_0) \cdot p_r(u_2 = I_2/H_0) \quad (17)$$

Expressing P_{Dc2} and P_{Fc2} in terms of the likelihood ratios $\Lambda(Z_1)$ and $\Lambda(Z_2)$ gives the same results as in (11) and (12) by replacing t_{ij}^s with t_{ij}^c.

By a close comparison between the observation spaces of the optimal consulting and optimal serial schemes shown in Figure 2(a) and Figure 2(b), respectively, one can clearly notice that the two schemes have the same observation spaces. This implies that the two schemes have the same performance when composed of two sensors each. In particular, if we assume that $t_{11}^s = t_{11}^c, t_{21}^s = t_{21}^c$, and $t_{20}^s = t_{20}^c$, the decision regions of the two schemes are identical.

The probabilities that sensor two (S_2) will consult sensor one (S_1) under the hypotheses H_1 and H_0 are given by (18) and (19),

$$p_r\left(\frac{c_{21}}{H_1}\right) = p_r\left\{t_{21}^c < \Lambda(Z_2) < \frac{t_{20}^c}{H_1}\right\} = \int_{t_{21}^c}^{t_{20}^c} dp_{\Lambda\left(\frac{Z_2}{H_1}\right)}\left(\Lambda\left(\frac{Z_2}{H_1}\right)\right) \quad (18)$$

and

$$p_r(c_{21}/H_0) = p_r\{t_{21}^c < \Lambda(Z_2) < t_{20}^c/H_0\} = \int_{t_{21}^c}^{t_{20}^c} dp_{\Lambda(Z_2/H_0)}(\Lambda(Z_2/H_0)) \quad (19)$$

It is evident from (18) and (19) that $p_r(c_{21}/H_j)$ can be determined once the distribution of the LHR $\Lambda(Z_2/H_j)$, $p\left(\Lambda(Z_2/H_j)\right)$, conditioned on the hypothesis $H_j, j = 0, 1$ is specified.

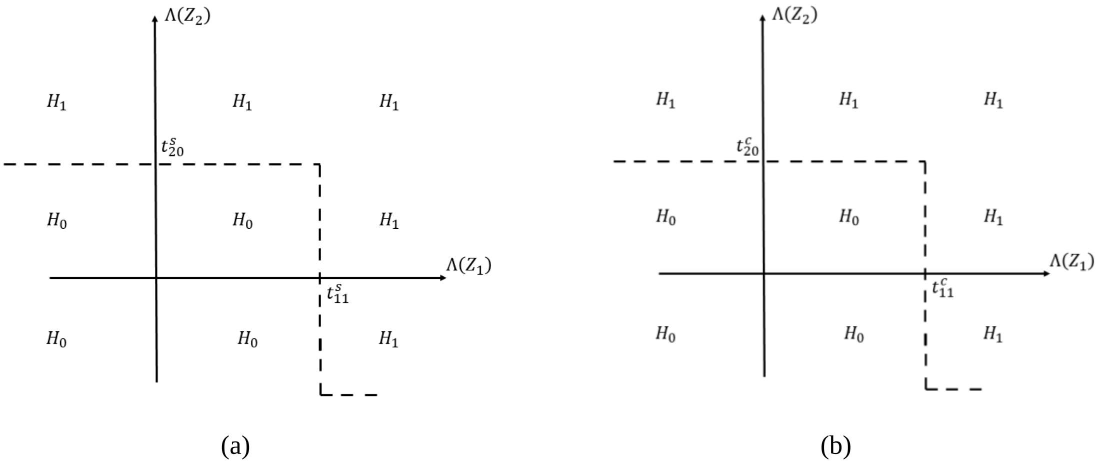

Figure 2. Observation space for (a) serial and (b) serial consulting distributed detection systems with $N = 2$

3. THE PROPOSED CONSULTING DETECTION SCHEME USING THREE SENSORS

Once more, we consider the schematic of the serial detection system shown in Figure 1(a) but with $N = 3$. The optimal serial detection test for maximizing the detection probability, P_D, for a given probability of false alarm, P_F is similar to the one mentioned in Section 2 and can be described as the following [8]: the test is initiated at sensor S_1 by comparing the likelihood ratio (LHR) $\Lambda(Z_1)$ with a single threshold t_{11}^s as in (20).

$$\Lambda(Z_1)\begin{cases} > t_{11}^s & \Rightarrow u_1 = 1 \\ \leq t_{11}^s & \Rightarrow u_1 = 0 \end{cases} \tag{20}$$

The decision of sensor one (S_1), u_1, is transferred to sensor two (S_2). As a result, S_2 implements the test given by (21),

$$\Lambda(Z_2)\begin{cases} > t_{21}^s & \Rightarrow u_2 = 1 \\ \leq t_{21}^s & \Rightarrow u_2 = 0 \end{cases} \tag{21}$$

or

$$\Lambda(Z_2)\begin{cases} > t_{20}^s & \Rightarrow u_2 = 1 \\ \leq t_{20}^s & \Rightarrow u_2 = 0 \end{cases} \tag{22}$$

Depending on whether $u_1 = 1$ or $u_1 = 0$, respectively. The test at sensor three (S_3) is performed once the decision u_2 is available and is given by (23),

$$\Lambda(Z_3)\begin{cases} > t_{31}^s & \Rightarrow \text{decide } H_1 \\ \leq t_{31}^s & \Rightarrow \text{decide } H_0 \end{cases} \tag{23}$$

or

$$\Lambda(Z_3)\begin{cases} > t_{30}^s & \Rightarrow \text{decide } H_1 \\ \leq t_{30}^s & \Rightarrow \text{decide } H_0 \end{cases} \tag{24}$$

When $u_2 = 1$ or $u_2 = 0$, respectively. The functional relationship between t_{i0} and t_{i1} is known, $i = 2, 3$ [8]. It should be noticed that the optimization process of the serial detection system necessitates the determination of all thresholds included in the system t_{ij}, ($i = 1, 2, 3$), ($j = 0, 1$), such that P_{Ds3} is maximum for a given P_{Fs3}, [8]. The probability of detection is given by (25).

$$P_{D_s3} = p_r(u_3 = 1/H_1) = \sum_{u_2} p_r(u_3 = 1/u_2, H_1) = p_r(u_3 = 1/u_2 = 1, H_1) p_r(u_2 = 1/H_1) + p_r(u_3 = 1/u_2 = 0, H_1) p_r(u_2 = 0/H_1) \quad (25)$$

Expressing P_{Ds3} in terms of the likelihood ratios $\Lambda(Z_1)$, $\Lambda(Z_2)$ and $\Lambda(Z_3)$, we get (26).

$$P_{D_s3} = P_{D_s2} \int_{>t_{31}^s} dp_{\Lambda(Z_3/H_1)}\big(\Lambda(Z_3/H_1)\big) + \big(1 - P_{D_s2}\big) \int_{>t_{30}^s} dp_{\Lambda(Z_3/H_1)}\big(\Lambda(Z_3/H_1)\big) \quad (26)$$

Where P_{Ds2} is given in (11). Equivalently, the false alarm probability, P_{Fs3}, cab be written as (27).

$$P_{F_s3} = P_{F_s2} \int_{>t_{31}^s} dp_{\Lambda(Z_3/H_0)}\big(\Lambda(Z_3/H_0)\big) + \big(1 - P_{F_s2}\big) \int_{>t_{30}^s} dp_{\Lambda(Z_3/H_0)}\big(\Lambda(Z_3/H_0)\big), \quad (27)$$

Where P_{Fs2} is given in (12). Since the observation space is a three-dimensional space, it is difficult to illustrate all probable hypotheses schematically. Therefore, we summarize them in Table 1.

Table 1. Summary of all possible events in the case of three serial sensors

$\Lambda(Z_1)$	$\Lambda(Z_2)$	$\Lambda(Z_3)$	Final decision
$> t_{11}^s$	$> t_{21}^s$	$> t_{31}^s$	H_1
$> t_{11}^s$	$> t_{21}^s$	$< t_{31}^s$	H_0
$> t_{11}^s$	$< t_{21}^s$	$> t_{30}^s$	H_1
$> t_{11}^s$	$< t_{21}^s$	$< t_{30}^s$	H_0
$< t_{11}^s$	$> t_{20}^s$	$> t_{31}^s$	H_1
$< t_{11}^s$	$> t_{20}^s$	$< t_{31}^s$	H_0
$< t_{11}^s$	$< t_{20}^s$	$> t_{30}^s$	H_1
$< t_{11}^s$	$< t_{20}^s$	$< t_{30}^s$	H_0

If we consider the consulting detection system illustrated in Figure 1(b) and assume $N = 3$, the procedures of the test can be described as follows: First the LHR $\Lambda(Z_3)$ is computed and tested as given by (28).

$$\Lambda(Z_3)\begin{cases} > t_{30}^s & \Rightarrow u_3 = 1(H_1) \\ \le t_{31}^s & \Rightarrow u_3 = 0(H_0) \\ \text{otherwise} & \Rightarrow u_3 = I_3(\text{consult } S_2) \end{cases} \quad (28)$$

Observe that when $u_3 = 1, 0$, the test is terminated by deciding H_1, H_0, respectively. Therefore, no consultation is necessary. However, when $u_3 = I_3$, S_2 is consulted and the test given by (29)

$$\Lambda(Z_2)\begin{cases} > t_{20}^c & \Rightarrow u_2 = 1(H_1) \\ \le t_{21}^c & \Rightarrow u_2 = 0(H_0) \\ \text{otherwise} & \Rightarrow u_2 = I_2(\text{consult } S_1) \end{cases} \quad (29)$$

is performed. Note that the events $u_2 = 1$ and $u_2 = 0$ corresponds to test termination with decisions H_1 and H_0, respectively. However, when $u_2 = I_2$, the test is continued by means of allowing sensor two (S_2) to have a consultation with sensor one (S_1) for assistance. Upon consultation, S_1 implements the test

$$\Lambda(Z_1)\begin{cases} > t_{11}^c & \Rightarrow u_1 = 1(H_1) \\ \le t_{11}^c & \Rightarrow u_1 = 0(H_0) \end{cases} \quad (30)$$

and the whole process is over.

Table 2 shows a summary of all probable events that might appear as outcomes of a consultation scheme composed of three sensors. One clear remark from form Table 2 is that when $\Lambda(Z_3) > t_{30}^c$, the decision is H_1 regardless of $\Lambda(Z_1)$ and $\Lambda(Z_2)$ values. Also, when $\Lambda(Z_3) \le t_{31}^c$, the final decision is H_0. Thus, consultations are not necessary. In addition, when S_2 is consulted $\Lambda(Z_2) > t_{20}^c$ ($\Lambda(Z_2) \le t_{21}^c$), the decision is $H_1(H_0)$ irrespective of $\Lambda(Z_1)$. Therefore, consultation between S_3 and S_2 is sufficient. Finally, when $t_{21}^c \le \Lambda(Z_2) \le t_{20}^c$ tc, consultation must be conducted between S_2 and S_1 also.

Table 2. Summary of all possible events in the case of three serial consulting sensors

$\Lambda(Z_1)$	$\Lambda(Z_2)$	$\Lambda(Z_3)$	Final decision remarks
$> t_{11}^s$	$> t_{21}^s$	$> t_{31}^s$	H_1 Consult S_2 and S_1
$> t_{11}^s$	$> t_{21}^s$	$< t_{31}^s$	H_0 No Consultation
$> t_{11}^s$	$< t_{21}^s$	$> t_{30}^s$	H_1 No Consultation
$> t_{11}^s$	$< t_{21}^s$	$< t_{30}^s$	H_0 No Consultation
$< t_{11}^s$	$> t_{20}^s$	$> t_{31}^s$	H_1 No Consultation
$< t_{11}^s$	$> t_{20}^s$	$< t_{31}^s$	H_0 No Consultation
$< t_{11}^s$	$< t_{20}^s$	$> t_{30}^s$	H_1 No Consultation
$< t_{11}^s$	$< t_{20}^s$	$< t_{30}^s$	H_0 Consult S_2 and S_1

The probability of detection, P_{Dc3}, is given by (31).

$$P_{D_c3} = p_r(u_f = 1/H_1) = \sum_{u_3} p_r(u_f = 1/u_3, H_1) = p_r(u_f = 1/u_3 = 1, H_1)p_r(u_3 = 1/H_1) + p_r(u_f = 1/u_3 = 0, H_1)p_r(u_3 = 0/H_1) + p_r(u_f = 1/u_3 = I_3, H_1)p_r(u_3 = I_3/H_1). \quad (31)$$

The second part of the right-hand side of (15) equal zero, whereas the first and third parts can be manipulated to yield

$$P_{D_c3} = p_r(u_3 = 1/H_1) + P_{D_c2} \cdot p_r(u_3 = I/H_1) \quad (32)$$

Likewise, the false alarm probability, P_{Fc2}, is given by (33).

$$P_{F_c3} = p_r(u_3 = 1/H_0) + P_{F_c2} \cdot p_r(u_3 = I/H_0) \quad (33)$$

Expressing P_{Dc2} and P_{Fc2} in terms of the likelihood ratios $\Lambda(Z_1)$, $\Lambda(Z_2)$ and $\Lambda(Z_3)$ gives the same results as in (26) and (27) by replacing t_{ij}^s with t_{ij}^c.

Now, if we assume that the thresholds of the serial and serial consulting schemes are the same $(i.e.\, t_{ij}^s = t_{ij}^c,\ j = 0, 1; i = 1, 2, 3)$, the results given in Table 1 and Table 2 would be the same. Consequently, the performance of a consulting scheme composed of three sensors is the same as one of an optimal serial scheme composed of three sensors as well. It is worth pointing out that an improvement is attained due to less utilization of sensors S_2 and S_1. The probabilities that sensors S_2 and S_1 will receive a consultation conditioned on H_j are respectively, given by (34) and (35).

$$p_r(c_2/H_j) = p_r\{t_{31}^c < \Lambda(Z_3) < t_{30}^c/H_j\} = \int_{t_{31}^c}^{t_{30}^c} dp_{\Lambda(Z_3/H_j)}\left(\Lambda(Z_3/H_j)\right) \quad (34)$$

$$p_r(c_1/H_i) = p_r\{t_{31}^c < \Lambda(Z_3) < t_{30}^c, t_{21}^c < \Lambda(Z_2) < t_{20}^c/H_j\} = \int_{t_{31}^c}^{t_{30}^c} dp_{\Lambda(Z_3/H_j)}\left(\Lambda(Z_3/H_j)\right) \int_{t_{21}^c}^{t_{20}^c} dp_{\Lambda(Z_2/H_j)}\left(\Lambda(Z_2/H_j)\right) \quad (35)$$

It is evident from (34) and (35) that $p_r(c_1/H_j)$ is less than $p_r(c_2/H_j)$. This gives S_1 a higher chance of survival than S_2. Furthermore, the two sensors are less visible in a consulting scheme than in a serial one.

4. NUMERICAL RESULTS

In this section, we present an example as a performance and a comparison study. The example considers detecting a given signal corrupted by white Gaussian noise (WGN) [9]. In addition, throughout this example the false alarm probability is assumed to be 0.001 $(i.e., P_F = 0.001)$ and all local sensor thresholds are equal. Utilizing matrix laboratory (MATLAB) algorithms for optimization, we obtain the optimum thresholds of the detection system when using two and three sensors. Then, we compute the detection probabilities for the two identical detection systems and compare the values as a function of the signal to noise ratio (SNR) in dB as illustrated in Figure 3. For the sake of comparison, we demonstrate the detection probability of the centralized/optimal detection system [10] in Figure 3 as well. It is clear from that the performance of the proposed scheme is identical to that of the conventional serial scheme. In Figure 4 the consultation rate for the case of two sensors is shown. It is clear from the figure that for some values of SNR the consultation rate is low.

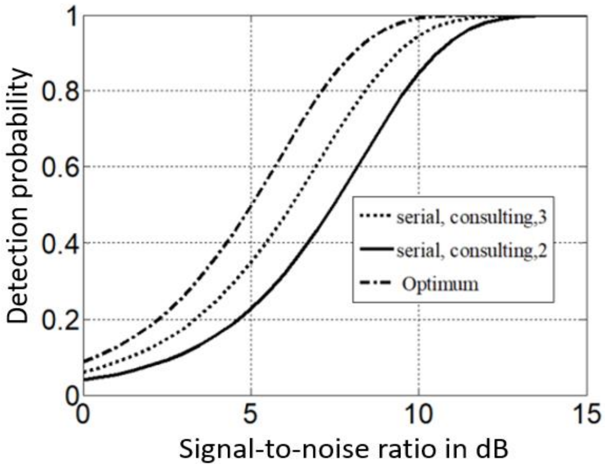

Figure 3. Probability of detection versus signal to noise ratio for the case of two and three sensors

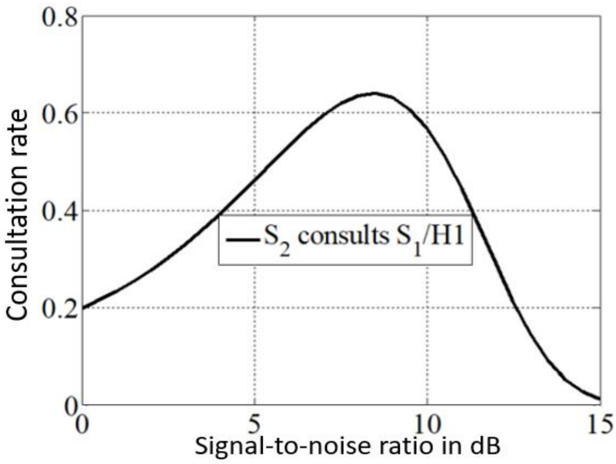

Figure 4. Probability of consultation versus signal to noise ratio for the case of two sensors

5. CONCLUSION

In this paper, we proposed an analytical model of a global optimal serial consulting detection system. In addition, we presented numerical results that demonstrate the equivalency between the proposed serial consulting detection system and the optimal serial detection system. The consultation rate among the detectors in the proposed system is relatively low when utilizing white Gaussian noise channels. Therefore, the system best suites the operations in a hostile environment. In particular, for high values of detection probabilities (P_D) and small values of false alarm probabilities (P_F), the proposed system has higher survivability than other detection systems available in the literature, such as the parallel detection system.

REFERENCES

[1] R. Tenney and N. Sandell, "Detection with distributed sensors," *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-17, no. 4, pp. 501–510, Jul. 1981, doi: 10.1109/TAES.1981.309178.

[2] Z. Chair and P. K. Varshney, "Optimal data fusion in multiple sensor detection systems," *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-22, no. 1, pp. 98–101, Jan. 1986, doi: 10.1109/TAES.1986.310699.

[3] I. Hoballah and P. Varshney, "Neyman-Pearson detection wirh distributed sensors," in *1986 25th IEEE Conference on Decision and Control*, Dec. 1986, pp. 237–241, doi: 10.1109/CDC.1986.267214.

[4] R. Viswanathan, S. C. A. Thomopoulos, and R. Tumuluri, "Optimal serial distributed decision fusion," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 24, no. 4, pp. 366–376, Jul. 1988, doi: 10.1109/7.7178.

[5] S. C. A. Thomopoulos and N. N. Okello, "Distributed detection with consulting sensors and communication cost," *IEEE Transactions on Automatic Control*, vol. 37, no. 9, pp. 1398–1405, 1992, doi: 10.1109/9.159581.

[6] N. Al-Ababneh, "Optimization of a distributed detection system with two consulting sensors using an improved detection scheme," *AEU-International Journal of Electronics and Communications*, vol. 108, pp. 158–162, Aug. 2019, doi: 10.1016/j.aeue.2019.06.024.

[7] M. M. Al-Ibrahim and N. K. Al-Ababneh, "Consultation scheme based on serial distributed detection system," in *Proceedings of the 1998 American Control Conference. ACC (IEEE Cat. No.98CH36207)*, 1998, pp. 3121–3123 vol.5, doi: 10.1109/ACC.1998.688435.
[8] W. L. Rubin and J. V. DiFranco, *Radar detection*. Institution of Engineering and Technology, 2004.
[9] P. F. Swaszek, "On the performance of serial networks in distributed detection," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 29, no. 1, pp. 254–260, 1993, doi: 10.1109/7.249133.
[10] I. Bahceci, G. Al-Regib, and Y. Altunbasak, "Serial distributed detection for wireless sensor networks," in *Proceedings. International Symposium on Information Theory, 2005. ISIT 2005.*, 2005, pp. 830–834, doi: 10.1109/ISIT.2005.1523453.
[11] N. Al-Ababneh, "Optimum consultation for parallel distributed detection systems," *Wireless Networks*, vol. 26, no. 7, pp. 5101–5111, Oct. 2020, doi: 10.1007/s11276-020-02365-5.
[12] Z. Geng, R. Xu, H. Deng, and B. Himed, "Fusion of radar sensing and wireless communications by embedding communication signals into the radar transmit waveform," *IET Radar, Sonar and Navigation*, vol. 12, no. 6, pp. 632–640, Jun. 2018, doi: 10.1049/iet-rsn.2017.0405.
[13] S. Zhang, L. N. Theagarajan, S. Choi, and P. K. Varshney, "Fusion of correlated decisions using regular vine copulas," *IEEE Transactions on Signal Processing*, vol. 67, no. 8, pp. 2066–2079, Apr. 2019, doi: 10.1109/TSP.2019.2901379.
[14] B. Chen and P. K. Willett, "On the optimality of the likelihood-ratio test for local sensor decision rules in the presence of nonideal channels," *IEEE Transactions on Information Theory*, vol. 51, no. 2, pp. 693–699, Feb. 2005, doi: 10.1109/TIT.2004.840879.
[15] M. A. Al-Jarrah, R. Al-Jarrah, and N. Al-Ababneh, "Decision fusion in mobile wireless sensor networks using cooperative multiple symbol differential space time coding," *AEU-International Journal of Electronics and Communications*, vol. 80, pp. 127–136, Oct. 2017, doi: 10.1016/j.aeue.2017.06.016.
[16] R. Srinivasan, "Distributed radar detection theory," *IEE Proceedings F Communications, Radar and Signal Processing*, vol. 133, no. 1, pp. 55–60, 1986, doi: 10.1049/ip-f-1.1986.0010.
[17] E. Conte, E. D'Addio, A. Farina, and M. Longo, "Multistatic radar detection: synthesis and comparison of optimum and suboptimum receivers," *IEE Proceedings F Communications, Radar and Signal Processing*, vol. 130, no. 6, pp. 484–494, 1983, doi: 10.1049/ip-f-1.1983.0078.
[18] A. Reibman and L. Nolte, "Optimal detection and performance of distributed sensor systems," *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-23, no. 1, pp. 24–30, Jan. 1987, doi: 10.1109/TAES.1987.313355.
[19] S. Thomopoulos, R. Viswanathan, and D. Bougoulias, "Optimal decision fusion in multiple sensor systems," *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-23, no. 5, pp. 644–653, Sep. 1987, doi: 10.1109/TAES.1987.310858.
[20] R. Viswanathan and P. K. Varshney, "Distributed detection with multiple sensors I. Fundamentals," *Proceedings of the IEEE*, vol. 85, no. 1, pp. 54–63, 1997, doi: 10.1109/5.554208.
[21] P. K. Varshney, "Distributed Bayesian detection: parallel fusion network," in *Distributed Detection and Data Fusion*, New York, NY: Springer New York, 1997, pp. 36–118.
[22] F. Sadjadi, "Hypotheses testing in a distributed environment," *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-22, no. 2, pp. 134–137, Mar. 1986, doi: 10.1109/TAES.1986.310747.
[23] H. L. Van Trees, *Detection, estimation, and modulation theory, part III*. New York, USA: John Wiley & Sons, Inc., 2001.
[24] C. Yang, L. Feng, H. Zhang, S. He, and Z. Shi, "A novel data fusion algorithm to combat false data injection attacks in networked radar systems," *IEEE Transactions on Signal and Information Processing over Networks*, vol. 4, no. 1, pp. 125–136, Mar. 2018, doi: 10.1109/TSIPN.2018.2790361.
[25] Q. Cheng, B. Chen, and P. Varshney, "Detection performance limits for distributed sensor networks in the presence of nonideal channels," *IEEE Transactions on Wireless Communications*, vol. 5, no. 10, pp. 3034–3038, Nov. 2006, doi: 10.1109/TWC.2006.05147.

BIOGRAPHIES OF AUTHORS

Nedal Al-Ababneh received his BSc and MSc degrees in electrical engineering from Jordan University of Science and Technology in 1993 and 1996, respectively. He received his doctor of engineering degree in electrical engineering from the University of Massachusetts-Lowell, USA in 2004. He is currently a full professor in the Department of Electrical Engineering at Jordan University of Science and Technology. His research interests are in free-space optical interconnects. He can be contacted at email: nedalk@just.edu.jo. Phone: +962 2 7201000 EXT. 22552.

Hasan Aldiabat is an assistant professor with the Department of Telecommunications Engineering at Yarmouk University in Irbid 21163, Jordan. He received his B. Sc and M. Sc degrees in electrical engineering from Jordan University of Science and Technology in 2009 and 2011, respectively. He received his doctoral of engineering degree in electrical engineering from the University of Minnesota, Minneapolis, USA in 2019. His research interests are in free space optical interconnects, signal processing, and wireless communications. He can be contacted at email: hasan.aldiabat@yu.edu.jo.