

Three-dimensional structure from motion recovery of a moving object with noisy measurement

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ABSTRACT

In this paper, a Nonlinear Unknown Input Observer (NLUIO) based approach is proposed for three-dimensional (3-D) structure from motion identification. Unlike the previous studies that require prior knowledge of either the motion parameters or scene geometry, the proposed approach assumes that the object motion is imperfectly known and considered as an unknown input to the perspective dynamical system. The reconstruction of the 3-D structure of the moving objects can be achieved using just two-dimensional (2-D) images of a monocular vision system. The proposed scheme is illustrated with a numerical example in the presence of measurement noise for both static and dynamic scenes. Those results are used to clearly demonstrate the advantages of the proposed NLUIO.

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1. INTRODUCTION

The progress in structure and motion estimation (a.k.a. structure-from-motion) research has been hectic, stimulated by recent breakthroughs in computer vision, the advent of digital photography and the augmented reality [1-6]. This progress has the potential to substantially increase the use of the structure from motion technique for a variety of applications, for example the growing application of unmanned aerial vehicles for remote surveying for a numerous of ecological domain [7]. Wide-reaching marine assessments using this technique have recently become possible in some cases like in [8, 9] with drone-based application. The structure from motion technique can be used for topographic data collection in field and laboratory studies [10] and as a means of digital preservation and for documenting archaeological excavations, cultural material and architecture [11]. On the other side, structure from motion can be a good low-cost alternative to generate high resolution topography [12], where light detection and ranging data is unaffordable or scarce. Recently in the area of agriculture [13], the use of unmanned aerial systems (UAS) based on the structure from motion technique as remote-sensing platforms have massive potential for obtaining detailed of crop features. The structure and motion field of research is worried with the recovery of 3-D geometry of the dynamic scene (the structure) when observed through a moving camera (the motion). Basically, structure from motion involves three main steps: First extracting features in images and matching these features between images, then modeling the camera-object relative motion and finally recovery of the 3-D structure using the estimated motion and features.

Keeping in view the above literature, several works have addressed structure estimation observer based approach where full velocity parameters feedback of the calibrated camera was provided. Such as in [14] where authors designed a nonlinear observer to estimate an unmeasurable state called depth with known dynamics. That last one has been experimented on a mobile robot with an on-board camera. Authors in [15] have introduced a nonlinear observer for a particular case of feature points on the object moving with constant velocities and have approved in many practical scenarios. Although, in [16] a nonlinear observer is defined to recover structure and motion with less restrictive assumptions on the moving object motion. A reduced-order nonlinear observer is presented in [17] to estimate the range from a moving camera to a feature point on a static scene. Furthermore, a design of complete order observers based on nonlinear contraction theory and synchronization is given in [18] where angular and linear velocity measurements are also noisy.

The information of the camera motion parameters has been unavoidable in the preceding cited references. Various studies on structure from motion estimation are also available where the camera motion is not known. Starting with [19], sliding mode observers were presented to estimate the motion parameters and the structure of a moving object with the aid of a charge-coupled device (CCD) camera. The advantage presented by the proposed observers is that both rigid and affine motion parameters, constant or time-varying, can be estimated correctly. In the uniqueness context, [20] introduced a developed nonlinear reduced order observer which only requires one camera linear velocity to estimate a stationary object seen by a calibrated camera. The methods described in [21, 22] present nonlinear observers based on a Robust Integral Signed Error method (RISE) to estimate the unknown distance between the camera and the object and the moving camera velocities. This problem was also investigated in [23] where a nonlinear reduced-order observer is proposed to recover the feature point depth and camera linear velocity. Only the camera's angular velocity is assumed to be known. Authors described in [24] a new approach based on Extended Kalman Filter to simultaneously recover camera pose and the structure of non-rigid extensible surfaces. In order to extend the problem to a deformable domain, authors defined the object's surface mechanics by means of Navier's equations. A recent paper [25] addresses the case where a novel complete-order observer is designed to estimate the unknown motion parameters and feature depth in the presence of measurement noise. The observer is derived from a differentiator based on the sliding-mode technique.

This paper, tackles the problem of motion and structure recovery for a class of system consisted on a moving camera moving object. Naturally, motions are constructed in continuous time settings and the motion parameters are assumed to be all time varying. The 3D position is estimated by using a set of image data observed through a dynamic camera with varying focal length. The contributions of this paper are first the analysis of the extent to which a scheme can be developed that is guaranteed to converge by observing a single point and having an unknown object motion. In addition, for a more accurate treatment, this paper extensively validates this approach for both static and dynamic object in the presence of measurement noise.

The remainder of this paper is organized as follows: Necessary preliminaries and state dynamics formulation are sought in Section 2. Section 3 presents the design of the Nonlinear Unknown Input Observer NLUIO to estimate structure of a feature point where LMI-based formulation is developed to prove asymptotic convergence. In Section 4 the simulation results are demonstrating the robustness of the approach in the presence of measurement noise. Finally, concluding remarks are drawn in Section 5.

2. STATE DYNAMIC FORMULATION

In this section an overview of the perspective relationships and basic kinematic is given modeling a camera which moves and observes a moving object. Most of the concepts can be found, for example, in [21] and [26]. Consider a scenario in Figure 1 where the motion of a single moving object is viewed by a moving camera undergoing rotation and translation. The equation of a feature point in the object can be presented in the reference frame as

$$\dot{x} = [\omega] \cdot x + v \quad (1)$$

Where, the state vector $x(t) = [x_1(t) \ x_2(t) \ x_3(t)]^T \in \mathbb{R}^3$ is defined rather as $x = [X/Z, Y/Z, 1/Z]^T$, With : X, Y and Z are the unknown Euclidean coordinates of feature point in the camera's inertial frame. $x_3(t)$ being perpendicular to the camera's image plane is the inverse of an unmeasurable focal distance.

$[\omega] \in \mathbb{R}^{3 \times 3}$ represent a skew symmetric matrix made from the angular motion of the moving camera $\omega(t) = [\omega_1(t) \ \omega_2(t) \ \omega_3(t)]^T$ and given by

$$[\omega] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{2}$$

$v(t)$ Is a relative camera linear velocity, such that $v = v_c - v_p$, where $v_c = [v_{c1} \ v_{c2} \ v_{c3}]^T$ is the linear camera velocity and $v_p = [v_{p1} \ v_{p2} \ v_{p3}]^T$ is the feature point velocity. Note that velocities v_c and v_p are bounded and continuously differentiable.

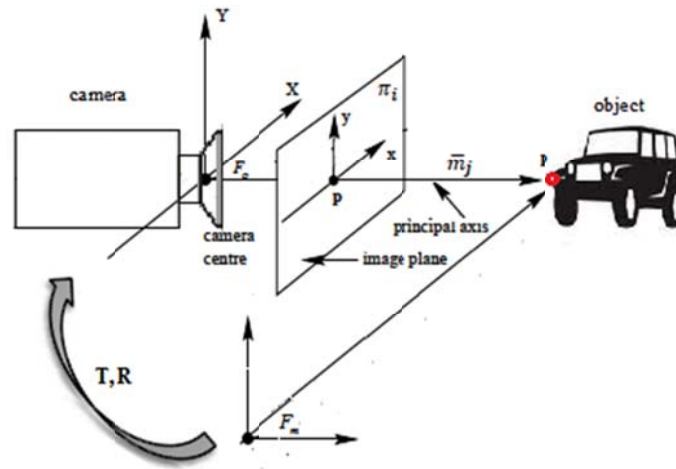


Figure 1. Camera object motion model

The next development aimed to design of NLUIO observers is to estimate the structure of moving object with respect to a moving camera. The relative motion of the feature point can be given by substituting $\omega(t)$ and $v(t)$ into (1) as follows

$$\begin{aligned} \dot{x}_1 &= -x_1 x_2 \omega_1 + \omega_2 + x_1^2 \omega_2 - x_2 \omega_3 + (v_{c1} - x_1 v_{c3}) x_3 - v_{p1} x_3 + x_1 v_{p3} x_3 \\ \dot{x}_2 &= -\omega_1 - x_2^2 \omega_1 + x_1 x_2 \omega_2 + x_1 \omega_3 + (v_{c2} - x_2 v_{c3}) x_3 - v_{p2} x_3 + x_2 v_{p3} x_3 \\ \dot{x}_3 &= -v_{c3} x_3^2 - (x_2 \omega_1 - x_1 \omega_2) x_3 + v_{p3} x_3^2 \\ y &= [x_1 \ x_2]^T \end{aligned} \tag{3}$$

The equations above are composed of unmeasurable coordinate x_3 and unknown motion information of the moving scene. To recuperate the 3D structure, the state vector should be estimated. That's why the unknown motion parameters of both camera and object are separated in the following. We define

$$\begin{aligned} G_1(u, y) &\square -x_1 x_2 \omega_1 + \omega_2 + x_1^2 \omega_2 - x_2 \omega_3 \\ G_2(u, y) &\square -\omega_1 - x_2^2 \omega_1 + x_1 x_2 \omega_2 + x_1 \omega_3 \end{aligned} \tag{3.1}$$

$$\begin{aligned} F_1(x, u) &\square (v_{c1} - x_1 v_{c3}) x_3 \\ F_2(x, u) &\square (v_{c2} - x_2 v_{c3}) x_3 \\ F_3(x, u) &\square -v_{c3} x_3^2 - (x_2 \omega_1 - x_1 \omega_2) x_3 \end{aligned} \tag{3.2}$$

$$\begin{aligned}
 D_1 &= -v_{p1}x_3 + x_1v_{p3}x_3 \\
 D_2 &= -v_{p2}x_3 + x_2v_{p3}x_3 \\
 D_3 &= v_{p3}x_3^2
 \end{aligned}
 \tag{3.3}$$

With $G_2(u, y)$, $F_1(x, u)$, $F_2(x, u)$, $F_3(x, u)$, $d_1(x)$, $d_2(x)$ and $d_3(x) \in \mathbb{R}$ and $u = [v_c \ \omega]^T$ is a measurable input constituted by the angular and the linear camera velocities. Consequently, the dynamics in (3) can be rewritten in nonlinear system form as follows

$$\begin{cases} \dot{x} = f(x, u) + g(y, u) + Hd(t) \\ y = Cx \end{cases}
 \tag{4}$$

Where $x(t) \in \mathbb{R}^n$ is the state of the normalized Euclidean coordinates, $d(t) \in \mathbb{R}^q$ is an unknown input which contains the point velocities, $y(t) \in \mathbb{R}^p$ the output, $g(y, u) \in \mathbb{R}^3$ and $f(x, u): \mathbb{R}^3 \rightarrow \mathbb{R}^3$ are nonlinear functions. $H \in \mathbb{R}^{n \times q}$ is the unknown input matrix and $C \in \mathbb{R}^{p \times n}$ is the output matrix.

Remark 1 (Observability) [27]: The nonlinear system in (4) is not observable if all the linear and angular velocities of the camera are null. That means $v_c(t) = 0$ and $\omega(t) = 0$. In addition, where the moving camera and the feature point of the moving object follow the same ray, it means that $v_{c1} - v_{p1} - x_1(v_{c3} - v_{p3}) = v_{c2} - v_{p2} - x_2(v_{c3} - v_{p3}) = 0$

In our case, the dynamic system in (4) can be rewritten as follows

$$\begin{cases} \dot{x} = Ax + \tilde{f}(x, u) + g(y, u) + Hd \\ y = Cx \end{cases}
 \tag{5}$$

Where $\tilde{f}(x, u) = f(x, u) - Ax$ and $A \in \mathbb{R}^{3 \times 3}$.

Note that the nonlinear function $f(x, u)$ is a Lipschitz function [28] and \hat{x} is the estimated state vector such that for the same positive constant α independent of x

$$\|f(x, u) - f(\hat{x}, u)\| \leq \alpha \|x - \hat{x}\|
 \tag{6}$$

When the camera is moving with faster velocities the Lipschitz constant α is large. As regards to the nonlinear function $\tilde{f}(x, u)$, it is also a Lipschitz function so there exists a positive Lipschitz constant β such that

$$\|f(x, u) - f(\hat{x}, u) - A(x - \hat{x})\| \leq (\alpha + \beta) \|x - \hat{x}\|
 \tag{7}$$

Given an observable perspective system, the design purpose is to estimate the coordinates $x(t)$ from the linear and the angular camera velocities.

The overall structure of the proposed method is shown in Figure 2. After the state dynamic formulation, the NLUIO for the structure estimation will be achieved in the following section

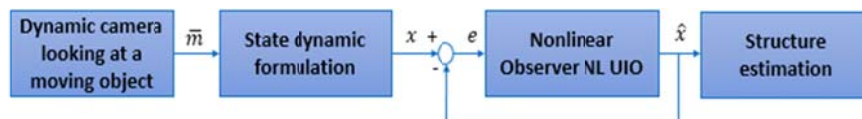


Figure 2. Structure of the proposed structure estimation method

3. OBSERVER FORMULATION

In this section, an asymptotically converging NLUIO is constructed, the state of which follows the state of the dynamics system given in (5) as closely as possible further in the presence of an unknown input. For the rest of the study it is going to be assumed that the following conditions [29] are satisfied:

- H is assumed to be column rank matrix
- $\text{rank}(CH) = \text{rank}(H) = q$.

Where q the number of the unknown input.

With above conditions, the NLUIO for system represented by (5) can be shown as follows

$$\begin{cases} \dot{z} = Nz + Ly + M\tilde{f}(\hat{x}, u) + Mg(y, u) \\ \hat{x} = z - Ey \end{cases} \quad (8)$$

Where $\hat{x}(t) \in \mathbb{R}^n$ is an estimate of $x(t)$ and $z(t) \in \mathbb{R}^n$ is the state observer. Matrices $N \in \mathbb{R}^{n \times n}$, $L \in \mathbb{R}^{n \times p}$, $E \in \mathbb{R}^{n \times p}$ and $M \in \mathbb{R}^{n \times n}$ are determined to design the observer such that $x(t)$ eventually tends to $\hat{x}(t)$ in the face of unknown input. Observer gain matrices equations can be expressed in alternate form

$$\begin{cases} N < 0 \\ M = I + EC \\ N = MA - KC \\ L = K(I + CE) - MAE \end{cases} \quad (9)$$

Where K and E are gain matrices of suitable dimensions subsequently designed.

The error equation for system (5) and NLUIO (8) is defined as follows

$$e(t) = \hat{x}(t) - x(t) = z - Ey - x = z - Mx \quad (10)$$

By substituting the system output presented in (5) into the error equation in (10), the dynamic error $\dot{e}(t)$ will have the following form

$$\dot{e}(t) = \dot{z} - (I + EC)\dot{x} \quad (11)$$

Then substituting (5) and (8) into (11), the dynamic error can be expressed as follows

$$\begin{aligned} \dot{e} &= Ne + N(I + EC)x + LCx + M(\tilde{f}(\hat{x}, u) - \tilde{f}(x, u)) - MAx - MHd \\ \dot{e} &= Ne + (NM + LC - MA)x + M(\tilde{f}(\hat{x}, u) - \tilde{f}(x, u)) - MHd \end{aligned} \quad (12)$$

To obtain matrices, the following steps should be followed: First using (9) the equation $NM + LC - MA = 0$ is satisfied, and if the matrix E satisfies (13)

$$MD = (I + EC)D = 0 \quad (13)$$

Then the equation of the error dynamics in (12) yields to

$$\dot{e} = Ne + M(\tilde{f}(\hat{x}, u) - \tilde{f}(x, u)) \quad (14)$$

The condition in (13) can be written as

$$ECH = -H \quad (15)$$

After that, a solution exists for matrix E using generalized inverse as follows

$$\begin{aligned} E &= -H(CH)^+ + Y(I_q - (CH)(CH)^+) \\ E &= F + YG \end{aligned} \quad (16)$$

Where $Y \in \mathbb{R}^{n \times q}$ an arbitrary matrix, $F = -H(CH)^+$ and $G = (I_2 - (CH)(CH)^+)$.

Finally, by substituting E into (9) the only unknowns are matrices K and Y . The following section presents a theorem that gives a sufficient condition for choosing them.

3.1. LMI sufficient condition

Theorem: The error $e(t)$ will converge asymptotically to 0 for any initial value $e(0)$ and the NLUIO in (8) is exponentially stable such that $\|e(t)\| \leq \|e(t_0)\| \exp(-\lambda t)$, where $\lambda \in \mathbb{R}^+$, if there exists P a positive symmetric matrix $P > 0$ satisfying the following condition [8]

$$N^T P + PN + (\alpha^2 + \beta^2)PMM^T P + 2I < 0 \quad (17)$$

Proof: Let's define the Lyapunov function candidate $V : \mathbb{R}^3 \rightarrow \mathbb{R}$ as follows

$$V = e^T P e \quad (18)$$

This Lyapunov function verify the inequality below

$$\lambda_{\min}(P)\|e\|^2 \leq V \leq \lambda_{\max}(P)\|e\|^2 \quad (19)$$

Where λ_{\min} and λ_{\max} are the minimum and the maximum Eigen values of P . By expanding the Lyapunov candidate function of (18) along the error equation in (14) the following expression is obtained

$$\begin{aligned} \dot{V} &= e^T (N^T P + PN)e + 2e^T PM (\tilde{f}(\hat{x}, u) - \tilde{f}(x, u)) \\ \dot{V} &= e^T (N^T P + PN)e + 2e^T PM (f(\hat{x}, u) - f(x, u)) - 2e^T PMA(\hat{x} - x) \\ \dot{V} &\leq e^T (N^T P + PN)e + 2\|e^T PM\| \|A\| \|e\| + 2\|e^T PM\| \|f(\hat{x}, u) - f(x, u)\| \\ \dot{V} &\leq e^T (N^T P + PN)e + 2\|e^T PM\| \alpha \|e\| + 2\|e^T PM\| \beta \|e\| \end{aligned}$$

then using the bellow inequality, where $\alpha \in \mathbb{R}^+$ and $\beta \in \mathbb{R}^+$.

$$2\alpha \|e^T PM\| \|e\| \leq \alpha^2 \|e^T PM\|^2 + \|e\|^2 \text{ And } 2\beta \|e^T PM\| \|e\| \leq \beta^2 \|e^T PM\|^2 + \|e\|^2$$

after simplification, \dot{V} may be reconstructed as

$$\begin{aligned} \dot{V} &\leq e^T (N^T P + PN)e + (\alpha^2 + \beta^2)e^T PMM^T P e + 2e^T e \\ \dot{V} &\leq e^T (N^T P + PN + (\alpha^2 + \beta^2)PMM^T P + 2I)e \end{aligned}$$

Define the matrix $Q < 0$ by $Q = N^T P + PN + (\alpha^2 + \beta^2)PMM^T P + 2I$, hence the time derivative \dot{V} is presented as

$$\dot{V} \leq e^T Q e \quad (20)$$

Using (19) and (20), the upper bound for $V(t)$ can be written as $V(t) \leq V(t_0) \exp(-\lambda t)$ where $\lambda = \frac{\lambda_{\max}(Q)}{\lambda_{\min}(P)}$ and the upper bound for $e(t)$ is expressed by $\|e(t)\| \leq \varphi \|e(t_0)\| \exp(-\lambda t)$ where $\varphi = \lambda_{\max}(P) / \lambda_{\min}(P)$.

The condition $\text{rank} \begin{bmatrix} sI_n - A & D \\ C & 0 \end{bmatrix} = n + q, \forall s \in \mathbb{C}$ leads to the fact that (MA, C) is observable.

In consequence, the matrix K can be obtained such that $N = MA - KC$ is Hurwitz equality.

Note that there is no systematic way to obtain the adaptable NLUIO parameters directly from condition (10) and the expression in the theorem given by (17). This allows to reformulate them as LMIs. Substituting N and M from (9) into (17) the following relationship can then be established

$$(MA - KC)P + P(MA - KC) + 2I + (\alpha^2 + \beta^2)P(I + EC)(I + EC)^T P < 0 \quad (21)$$

Using the solution in (16), the inequality (21) becomes

$$\begin{aligned} A^T(I + FC)^T P + P(I + FC)A + A^T C^T G^T P_1^T + P_1 G C A - C^T P_2^T - P_2 C \\ + 2I + (\alpha^2 + \beta^2)(P + PFC + P_1 GC)(P + PFC + P_1 GC)^T < 0 \end{aligned} \quad (22)$$

Variables $P_1 = PY$ and $P_2 = PK$ are given to make the resolution of the nonlinear matrix inequalities easier. Exponential convergence to the object coordinates is achieved.

3.2. LMI formulation

For the NLUIO synthesis the following LMIs (23) have feasible solutions for P , K and Y invoking the inequality in (22) transformed with schur's complement.

$$\begin{bmatrix} X & \gamma W \\ \gamma W^T & -I \end{bmatrix} < 0 \quad (23)$$

Where $W = (P + PFC + P_1 GC)$,

$$X = A^T(I + FC)^T P + P(I + FC)A + A^T C^T G^T P_1^T + P_1 G C A - C^T P_2^T - P_2 C + 2I$$

$$\gamma = (\alpha^2 + \beta^2).$$

Using solution of LMI i.e. feasible values of $Y = P^{-1}P_1$ and $K = P^{-1}P_2$, observer matrices satisfying the requisite conditions are found. The LMI feasibility can be solved using standard LMI approach [30].

4. RESULTS AND DISCUSSIONS

In contrast with previous research that assume noise-free measurements and demand prior knowledge of the object and camera motion, the proposed method assume that the object velocity is unknown. In the following, the performance of the NLUIO is validated through different numerical simulations in the presence of measurement noise for both static and dynamic scenes. As the current simulation results are restricted to tracking a single point feature. Two different object motion models are considered, and the proposed NLUIO performance is evaluated for both cases. Whereas the usual speed of the monocular camera is 30 frames/s, the NLUIO is valid for a continuous-time system. For the simulation results, SIMULINK is used with sampling period $t_s = 10^{-3} s$. The initial Euclidean coordinates of the object feature are $x(t_0) = [5 \ 2 \ 1]^T$ (m). Since initial target feature point is not known at the NLUIO, thus the system and observer start from different initial conditions. Initial condition for the observer is taken as $\hat{x}(t_0) = [-1 \ -0.5 \ -0.2]^T$ (m).

Matrices A , C and H are given by

$$A = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{And } D = [1 \ 0 \ 0]^T$$

Note, the third component x_3 of the state, which is the unmeasurable distance between the camera and the moving object. Clearly the estimation of the three-dimensional Euclidean coordinates can yield the distance estimation. The comparison of RMS error values obtained with the proposed NLUIO with different values of measurement noise was used to demonstrate the proposed method.

3.1. Static scene

In this case, the object and the camera velocities parameters are chosen respectively by

$$v_c = \begin{bmatrix} 1 \\ 2 \\ 0.2\sin(\pi t / 4) \end{bmatrix} (m/s), \omega = [0 \ 0 \ -\pi/30]^T (\text{rad/s}) \text{ and } v_p = \begin{bmatrix} 0.5 \\ 0 \\ 0 \end{bmatrix} (m/s)$$

Figure 3 shows the structure estimation of the object position in the single camera images. Figure 4 presents the error in the position estimation of the static object. Notice that the transient performance of the proposed scheme is significantly less than 4 second. The RMS error values obtained by the proposed NLUIO are given as follows

$$e_1 = 0.1578, e_2 = 0.0789 \text{ and } e_3 = 0.0776.$$

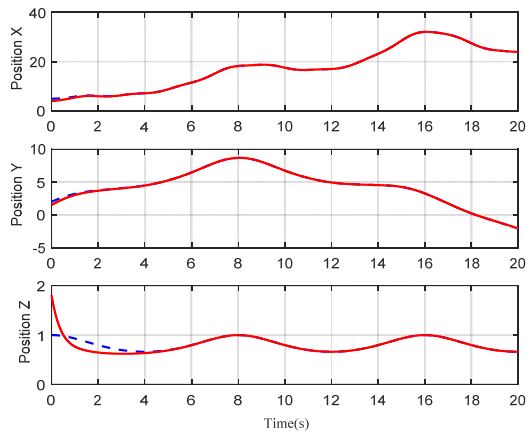


Figure 3. Time histories of the static object position in the single-camera images; solid line: estimated and Dashed-dotted line: real state

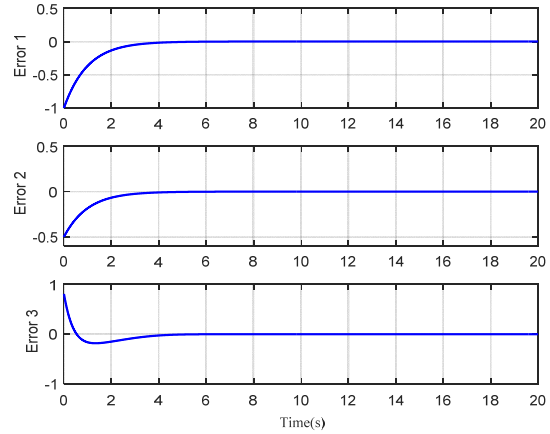


Figure 4. Static object estimation error of the proposed method

Next, the measurements of V_c as shown in Figure 5 is assumed to be corrupted by adding a Band Limited White Noise (BLWGN) with 5% of power, a correlation time of 0 and a covariance of infinity. Figure 6 shows the structure estimation of the static object position in the single camera images in the presence of measurement noise. The error in the position estimation of the static object is described in Figure 7. Only the third component of RMS error is changed $e_3 = 0.0816$.

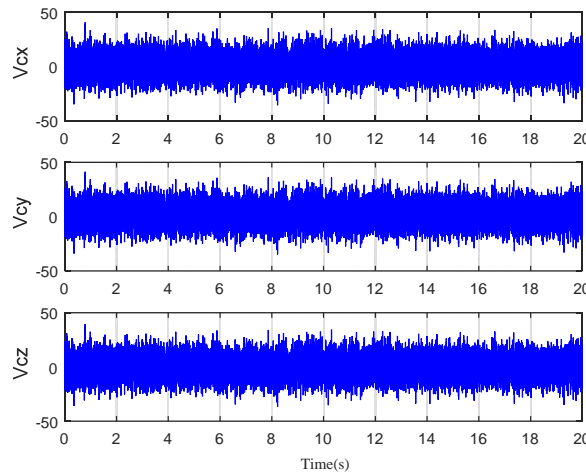


Figure 5. The measurements of the camera velocities in the presence of noise

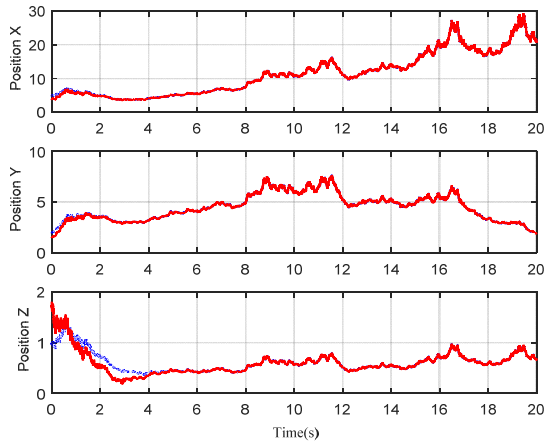


Figure 6. Time histories of the static object position in the single-camera images in the presence of noisy camera velocity; solid line: estimated and Dashed-dotted line: real state

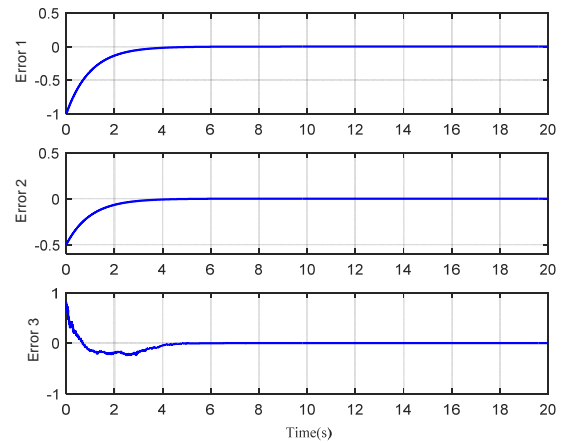


Figure 7. Static object estimation error of the proposed method in the presence of noisy camera velocity

In addition to the previous measurement noise of V_c , the proposed observer is validated for robustness by the addition of a Band Limited White Noise (BLWGN) with 5% of power to the object velocity. Figure 8 shows the structure estimation of the dynamic object coordinates with noisy object and camera velocities and Figure 9 describes the error in the position estimation of the dynamic. The NLUIO then yields uniformly asymptotically convergent estimates of the three-dimensional Euclidean coordinates of the feature point. In the presence of noise in the motion parameters, the estimated state \hat{x}_3 is corrupted directly by the source of noise, therefore the third component of RMS error increases and becomes $e_3 = 0.2048$.

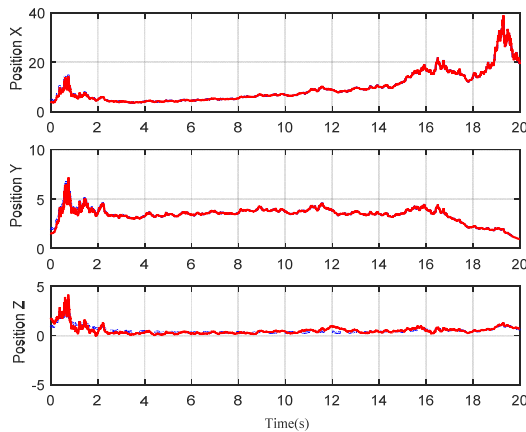


Figure 8. Time histories of the static object position in the single-camera images in the presence of noisy camera and object velocities; solid line: estimated and Dashed-dotted line: real state

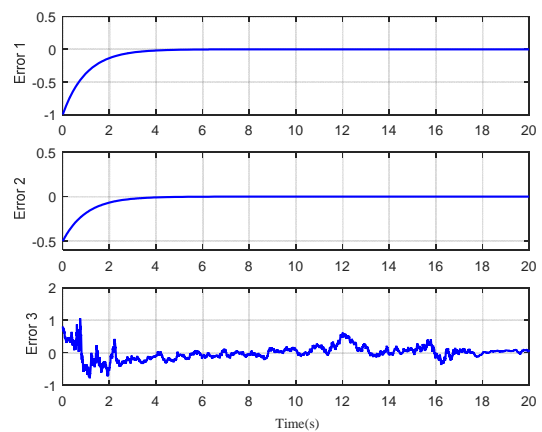


Figure 9. Static object estimation error of the proposed method in the presence of noisy camera and object velocities

3.2. Dynamic scene

In this case, only the performance in the presence of measurement noise for both camera and object velocity is studied. The same values of measurement noise are used. The object and the camera velocities are chosen respectively as:

$$v_c = \begin{bmatrix} 1 \\ 2 \\ 0.2\sin(\pi t / 4) \end{bmatrix} (m/s), \omega = [0 \ 0 \ -\pi/30]^T (\text{rad/s}) \text{ and } v_p = \begin{bmatrix} 0.2+\cos(2\pi t/4) \\ 0 \\ 0 \end{bmatrix} (m/s).$$

Figure 10 presents the structure estimation of the dynamic object position and Figure 11 shows the error in the position estimation of the object with noisy camera velocity. These results demonstrate that the proposed NLUIO based object structure estimation method can achieve satisfactory performance even with camera velocities. This observer gives better estimates for a significant level of noise even changing scene. RMS error values are given as follows:

$$e_1 = 0.1578, e_2 = 0.0789 \text{ and } e_3 = 0.0816.$$

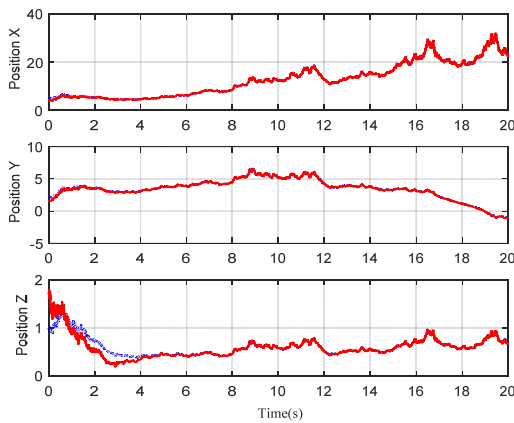


Figure 10. Time histories of the dynamic object position in the presence of noisy camera velocity; solid line: estimated and Dashed-dotted line: real state

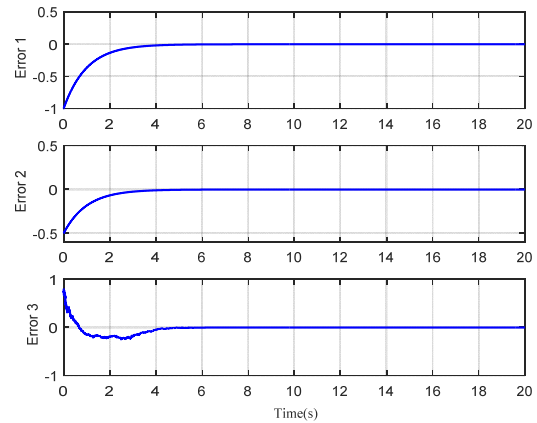


Figure 11. Dynamic object estimation error of the proposed method in the presence of noisy camera velocity

Figure 12 shows the structure estimation of the dynamic object position with noisy object and camera velocities and Figure 13 shows the structure estimation error of the dynamic object. Here again, only the third component of RMS error are changed $e_3 = 0.0992$. However, the presence of noise on both camera and object velocities can significantly degrade the performance of NLUIO. Therefore, it can be seen that the practical situation does require a more robust nonlinear observer for the considered problem.

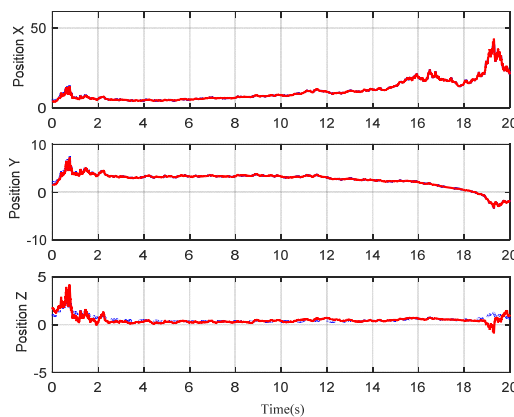


Figure 12. Time histories of the dynamic object position in the presence of noisy camera and object velocities; solid line: estimated and Dashed-dotted line: real state

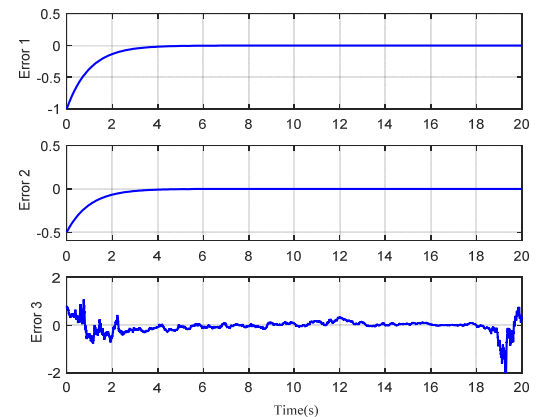


Figure 13. Dynamic object estimation error of the proposed method in the presence of noisy camera and object velocities

5. CONCLUSION

The robust NLUIO has been designed for a nonlinear camera-object system. Furthermore, the stability of the error systems has been demonstrated to estimate structure from motion of a feature point. The sufficient condition for existence of the designed nonlinear observer is derived. In order to facilitate the NLUIO design, the obtained condition is formulated in terms of LMIs. This paper extensively validates the proposed method for both static and dynamic scenes. Simulation results are promising and much has to be done to assess the performance of the proposed method against measurement noise. An interesting direction for future research is improving the proposed method to test it with experimental data and considering a trajectory of a moving object along a plane.

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