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Design and performance comparison of different adaptive control schemes for pitch angle control in a Twin-Rotor-MIMO-System

Winston Netto, Rohan Lakhani, S. Meenatchi Sundaram
Department of Instrumentation and Control Engineering, Manipal Institute of Technology,
Manipal Academy of Higher Education, India

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ABSTRACT

The Twin Rotor MIMO System is a higher order non-linear plant and is inherently unstable due to cross coupling between tail and main rotor. In this paper only the control of main rotor is considered which is non-linear and stable by using adaptive schemes. The control problem is to achieve perfect tracking for input reference signals while maintaining robustness and stability. Four adaptive schemes were implemented, two using Model Reference Adaptive Control under which MIT rule and Modified MIT rule are used. The other two using Adaptive Interaction, namely, Adaptive PID and Approximate Adaptive PID. It is observed that adaptive schemes fulfill all the three system performance requirements at the same time. Modified MIT rule was found to give superior performance in comparison to other controllers. Also Approximate Adaptive PID was able to stabilize the main rotor and cancel the effect of cross coupling between tail rotor and main rotor when operating simultaneously without the need for designing decouplers for the system. Thus the main rotor can be made independent from the state of the tail rotor by using Approximate Adaptive PID.

Corresponding Author:

Winston Netto,
Department of Instrumentation and Control Engineering,
Manipal Institute of Technology,
MAHE, Manipal, Udupi, Karnataka, 576104, India.
Email: winnetto@gmail.com

1. INTRODUCTION

The Twin Rotor MIMO System was developed by Feedback Instruments Ltd. and serves as a real-time model of nonlinear multidimensional system. To visualize the parts and motions of the TRMS together with the forces generated by the actuators, a model of the TRMS is seen in Figure 1. The TRMS consists of a tower with a beam attached by two bearings. These bearings allow the beam to move freely in the horizontal and vertical plane within some limits. At the two ends of the beam, rotors are attached which rotated 90 degrees from each other are allowing them to generate horizontal and vertical thrusts. The rotor generating vertical thrust is called the main rotor. This enables the model to pitch, which is rotation in the vertical plane around the horizontal axes. The rotor generating the horizontal thrust is called the tail rotor. This enables the model to yaw, which is rotation in the horizontal plane around the vertical axis [1].

The Twin Motor MIMO system is a highly non-linear plant in which there are certain states that cannot be measured, this makes designing of the controller a difficult task. The popular well known schemes will not give desired output characteristics if used for controlling the plant and will fail to stabilize it in most cases. Many control schemes have been developed for controlling the main rotor of the TRMS.

PID’s and linear controllers [2] were not able to guarantee global stability and fulfill desired response characteristics. Model predictive controller [3] does guarantee global stability but at a price of low degree of robustness and poor tracking performance. Robust PID obtained [4] using Kharitonov's theorem gave good robustness but poor tracking performance and remains stable only for small values of controller gains. Using Sliding Mode Controller [5] satisfactory tracking was obtained but due to chattering convergence rate of system states was very low also the non-robust reaching phase in SMC makes system unstable.

Figure 1. Twin Rotor MIMO System

Thus it can be concluded that the schemes implemented up until now had a tradeoff between robustness, tracking performance and global stability. In this paper the adaptive controller schemes implemented for TRMS main rotor satisfy all these three system characteristics together i.e. there is no compromise between robustness, stability and tracking performance. These adaptive schemes are also easy to implement and require minimum plant knowledge to work.

The main rotor's transfer function was obtained using black box identification, which is the only thing we need to know about the plant. This transfer function was used in the adaptive schemes as a reference model and as a part of the controller itself. The results obtain show high robustness, good tracking performance and guarantee absolute stability all at the same time which was previously not obtained.

2. METHODOLOGY

2.1. Identification

System was identified using black box identification. Cross coupling between the tail and main rotor was not considered. The tail rotor was kept at zero and only the main rotor was identified in form of a single transfer function. A PRBS (Pseudo Random Binary Signal) was given to the plant and the output was analyzed in MATLAB system identification toolbox as shown in Figure 2.

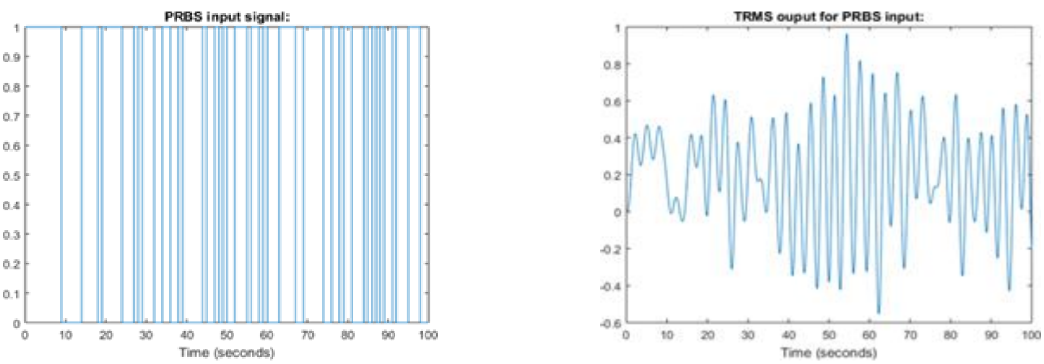


Figure 2. PRBS input applied to the TRMS and the output form the main rotor

The best estimate of the transfer function obtained is having an accuracy of 72.7%.

$$G(s) = \frac{0.01106s+0.3768}{0.258s^3+0.2528s^2+1.16s+1} \quad (1)$$

where G(s) represents the approximated transfer function of the non-linear main rotor. The degree to which G(s) is a faithful representation of the TRMS main rotor can be seen by Figure 3, where the step response of both the TRMS and the approximated model was compared. The transfer function obtained is a close approximation of the TRMS main rotor.

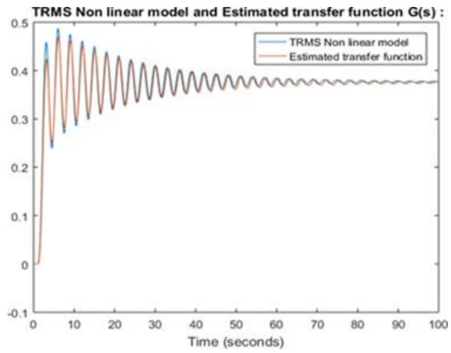


Figure 3. Model validation

2.2. Controller design

Adaptive schemes are employed here to control the main rotor of the TRMS. The advantage of adaptive schemes over conventional PID is that the values of the controllers are not fixed and update with time, it has knowledge of the states of the plant which it is controlling. Figure 4 shows the control structure where the parameters of the controllers are not fixed but evolve over time and how they evolve depends upon the adaptive law formulated.

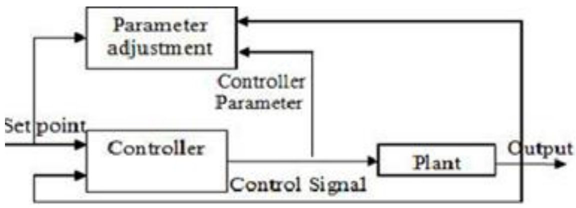


Figure 4. Adaptive control structure

2.2.1. The MIT rule

The MIT rule works on MRAC (Model Reference Adaptive Control) where the output of the plant is made to follow the output of a reference model as shown in Figure 5. Here the error $e(t, K_c)$ obtained between the output of the reference model and the plant is subjected to a cost function which is minimized using Gradient algorithm. Minimization algorithm used was proposed by Whitaker, the gradient method [6].

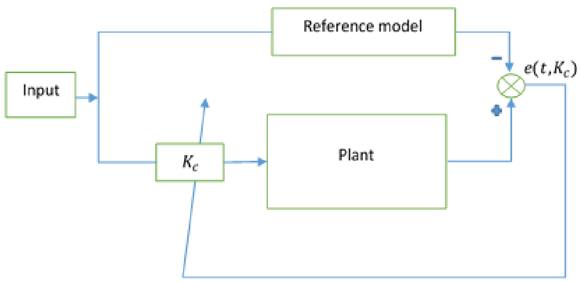


Figure 5. The MIT rule

$$\dot{K}c = -\gamma \frac{\partial\left[\frac{1}{2}e^2(t,Kc)\right]}{\partial Kc} \quad (2)$$

$$e = \left[y_p(t) - y_m(t)\right] \quad (3)$$

which is stated as [6].

$$\dot{K}_c = -\gamma\left[y_p(t) - y_m(t)\right]y_m(t) \quad (4)$$

where is γ the adaptive gain, $y_m(t)$ is the output from the reference model and $y_p(t)$ is the output from the plant. The adaptive law is given as:

$$\dot{K}_c = -\gamma e y_m(t) \quad (5)$$

- Perfect tracking and fast convergence of $\dot{K}_c$

$$\lim_{T\to\infty} inf\ \frac{1}{T}\int_0^T [Z_m(s)\{K_m r(t)\}]\left[Z_p(s)\left\{K_p r(t)\right\}\right]dt > 0 \quad (6)$$

where $Z_p(s)$ is the plant and $Z_m(s)$ is the reference model, K_m and K_p are the gains of the reference model and plant respectively. Theorem 1: Under the condition that $Z_m(s)$ and $Z_p(s)$ are strictly stable, that r(t) is bounded and (6) is satisfied, there exists a positive constant $\gamma *$ such that for all $\gamma \in (0, \gamma *)$ gain K_c adjusted by the MIT rule is bounded and converges exponentially fast to $K_c{}^*(t)$ as $t \to \infty$. Also Energy in r(t) should be localized where $Z_m(s)$ and $Z_p(s)$ have similar frequency responses. If these conditions are satisfied then perfect tracking and fast convergence of $\dot{K}_c$ is obtained [6].

- Stability with large adaptation gain-

$$\dot{K}_c = -g\left[Z_p(s)K_p K_c r(t) - Z_p(s)K_m r(t)\right]Z_p(s)K_m r(t) \quad (7)$$

where $Z_p(0) = 1$. (7) can be rewritten as

$$K_c(s) = \frac{gK_m{}^2R^2}{s+gK_mK_pR^2Z_p(s)} \quad (8)$$

Stability for large adaptation gain is proved with root locus technique where the boundedness of $K_c(t)$ is obtained. Theorem 2: The MIT rule with r(t)=R has infinite gain margin (i.e. for all positive values of g and R, the adaptive law is stable independent of K_p if and only if [6].

$$-\frac{\pi}{2} < \arg Z_p(jw) < \frac{3\pi}{2} \ \ \forall\, w \in \mathbb{R} \quad (9)$$

Stability will become independent of the adaptive gain, system remains stable for all adaptive gains if the above condition is satisfied. Block diagram for the MIT rule as shown in Figure 6.

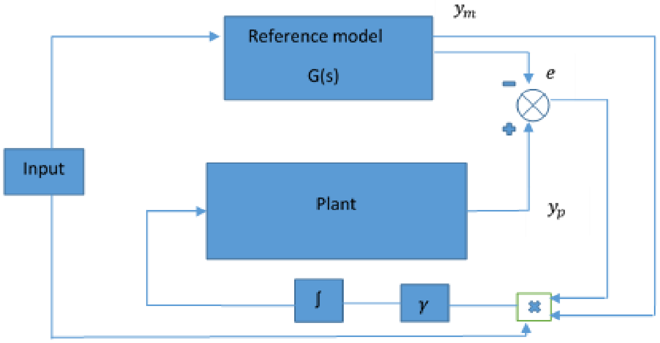


Figure 6. Block diagram for the MIT rule

2.2.2. Modified MIT rule

There are certain limitations when using MIT rule. The magnitude of the gradient changes as we descent to the minima. But if large number of saddle points are present then the derivative becomes zero and its magnitude also becomes zero. Thus it may take some time to escape these points making the convergence slower [7]. These limitations are overcome by Modified MIT rule in which normalized gradient method is used therefore the direction of the gradient is preserved but the magnitude is ignored. The Normalized MIT Rule using Normalized algorithm is given as

$$\frac{dK_c}{dt} = \frac{-\gamma e\varphi}{\alpha+\varphi'\varphi} \tag{10}$$

here $\varphi = \frac{\partial e}{\partial K_c}$ and $\alpha(\alpha > 0)$ which is a constant [8]

2.2.3. Adaptive PID

The Adaptive PID works by using adaptive interaction. Adaptive interaction works on the principal that a system can be broken down into number of subsystems (1, 2, 3,..,n) and the interaction of these system causes adaptation. According to adaptive interaction theory four subsystems are considered here.

- Proportional with output y_1
- Integral with output y_2
- Derivative with output y_3
- $G(s)$, estimated transfer function of the plant

With the Non-linear plant interaction the interaction of these subsystems will give rise to the adaptation. γ is the adaptive gain, e is the error signal, u is the reference input, g(t) is the impulse response of the system and α_1 is the weighing factor. Using the theory of adaptive interaction the PID controller algorithm becomes

$$\dot{K}_c = -\lambda_1 \frac{\partial E}{\partial y_o}\ o\ \dot{T}[u] o\ y_1 \tag{11}$$

$$\dot{K}_I = -\lambda_1 \frac{\partial E}{\partial y_o}\ o\ \dot{T}[u] o\ y_2 \tag{12}$$

$$\dot{K}_D = -\lambda_1 \frac{\partial E}{\partial y_o}\ o\ \dot{T}[u] o\ y_3 \tag{13}$$

where o denotes functional composition, λ_1 is the adaptation gain. y_1, y_2, y_3 represents the output of the proportional, the integral and derivative transfer function blocks, respectively. y_0 is the plant's output y_{in} is the command input. T is a causal functional relationship between plant's input and output. $\dot{T}[u] = \frac{dT}{du}$ is the Frechet derivative [9].

Theorem 3: Condition for adaptive interaction is that the input as well as the output should be an integrable signal. The application of adaptive interaction requires a critical condition that should be satisfied which is that the Frechet derivative of the impulse response of the system must exist.

$$\lim_{\|\Delta\|\to 0} \frac{\|T[u+\Delta]-T[u]-\dot{T}[u]\Delta\|}{\|\Delta\|} = 0 \tag{14}$$

For linear time invariant plant with transfer function G(s) the Frechet derivative is given as [9]

$$\dot{T}[u] o\ y_= \int_0^t g(t-\tau)h(\tau)d\tau = g(t) * h(t) \tag{15}$$

$$G(s)u(s) \tag{16}$$

where $G(s)$ is the estimated plant transfer function and satisfies these conditions hence it can be used for controlling the main rotor of TRMS. The adaptive laws are given as follows [9]

$$\begin{aligned} \dot{K}_p &= \gamma(e \times (\dot{g}(t) * y_1) - \alpha_1 u y_1) \\ \dot{K}_I &= \gamma(e \times (g(t) * y_2) - \alpha_1 u y_2) \\ \dot{K}_D &= \gamma(e \times (g(t) * y_3) - \alpha_1 u y_3) \end{aligned} \tag{17}$$

The scheme for adaptive PID is shown in Figure 7.

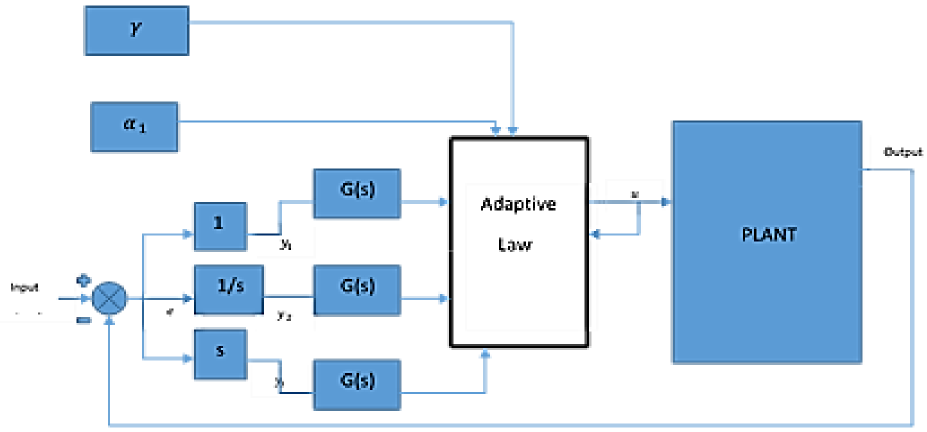


Figure 7. Block diagram for adaptive PID

2.2.4. Approximate adaptive PID

The controller can be designed without using G(s) i.e. knowledge of the plant. Theorem 4: The Frechet derivative can be approximate as [10]

$$\dot{T}[u]o\ y = h \tag{18}$$

where h is the impulse response substituting in (18) the adaptive laws become independent of $G(s)$. The approximate algorithm given as in [10] is

$$\begin{aligned} \dot{K}_P &= -\gamma e y_1 \\ \dot{K}_I &= -\gamma e y_2 \\ \dot{K}_D &= -\gamma e y_3 \end{aligned} \tag{19}$$

γ is the adaptive gain, e is the error signal and y_1, y_2, y_3 are outputs from first three subsystem.

2.3. Algorithm

Using the above adaptive laws simulations were performed in Matlab Simulink enviournment with the non-linear TRMS plant provided by Feedback Instruments Ltd. this model is a replica of the real plant and was designed using grey box modelling. Figure 8 shows the implementation of The MIT rule in Simulink which is done directly by using the adaptive law (5). Modified MIT rule is implemented in Figure 9. It is implemented using the adaptive law (10). Figure 10 shows the Adaptive PID, this can be implemented using (17) in Laplace domain. Figure 11 shows Approximate Adaptive PID implemented using (19) without the reference model.

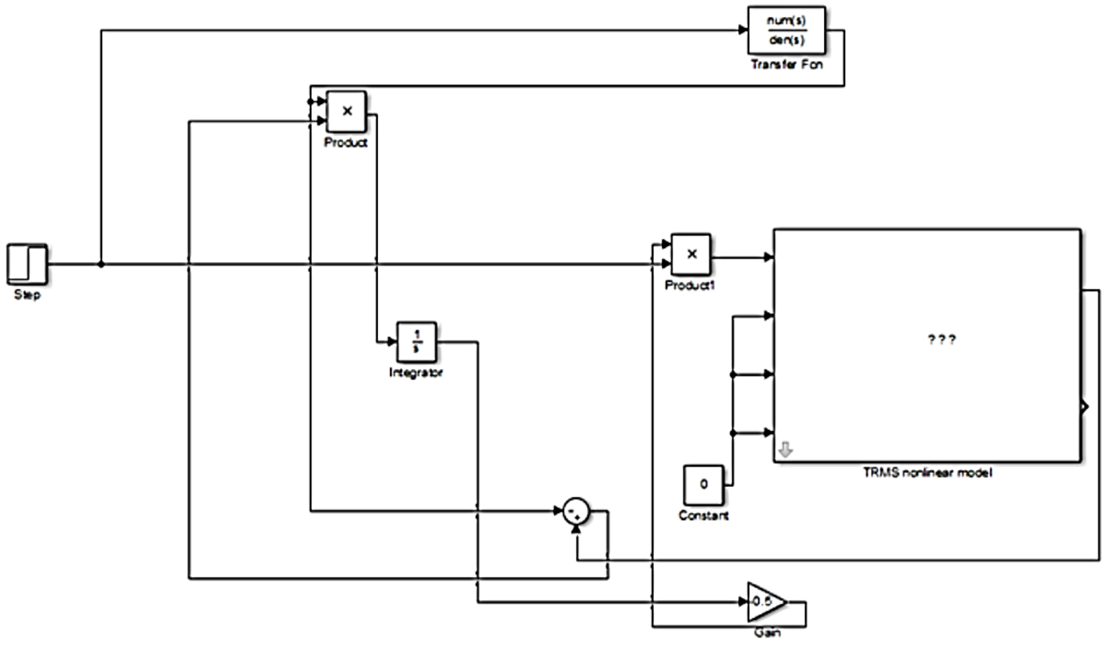


Figure 8. The MIT rule

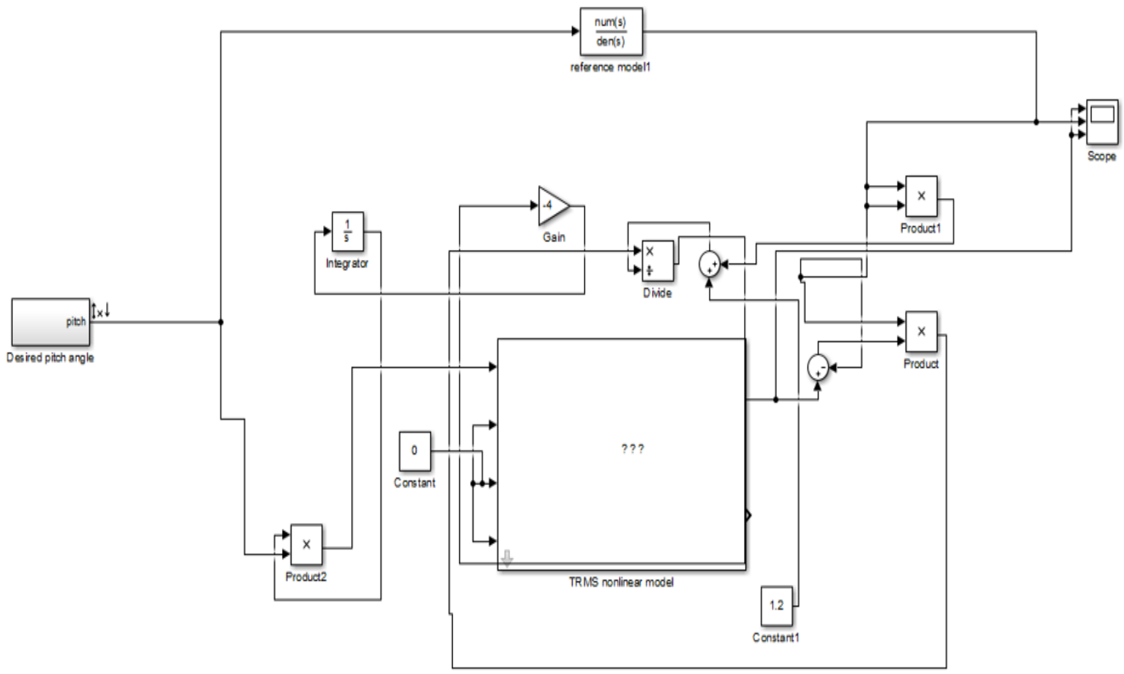


Figure 9. Modified MIT rule

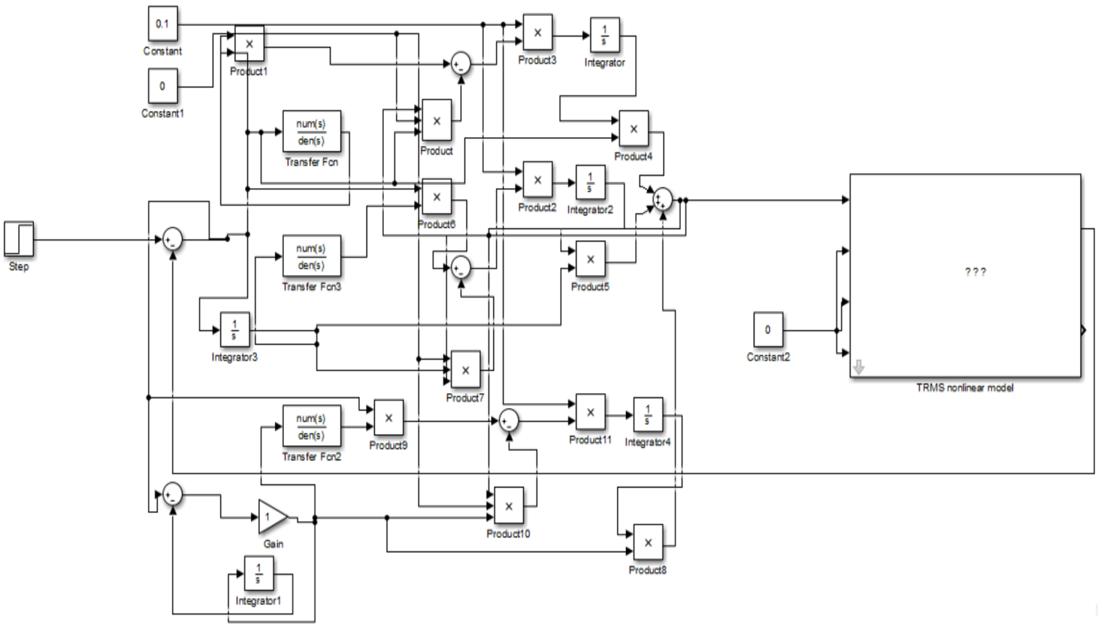


Figure 10. Adaptive PID

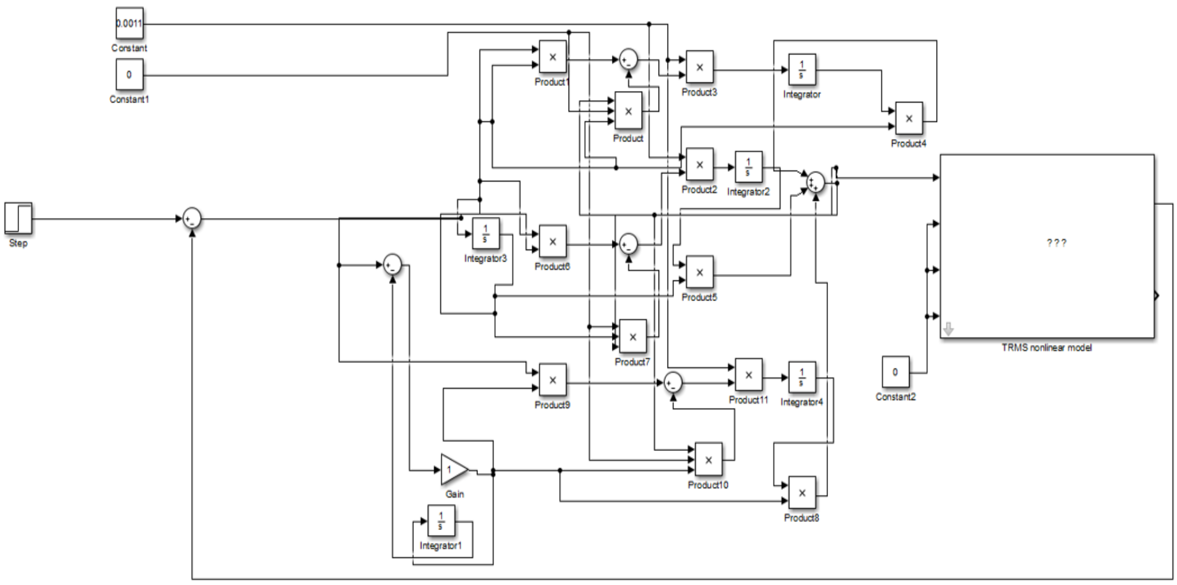


Figure 11. Approximate adaptive PID

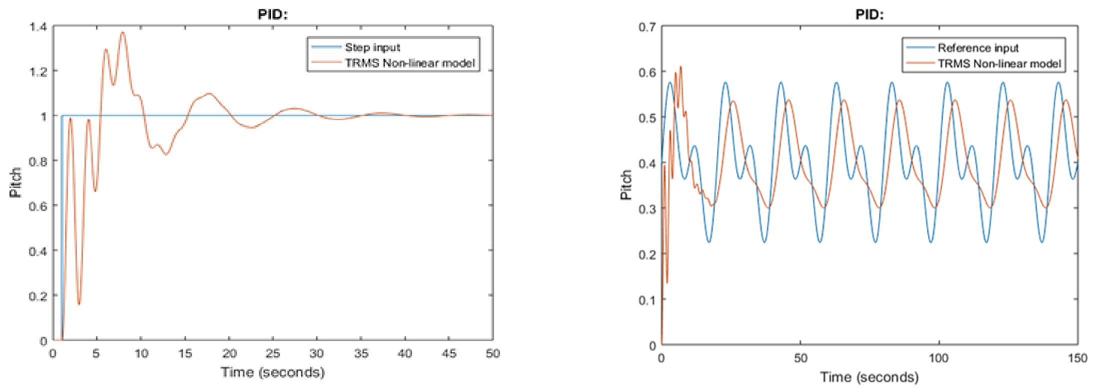
3. RESULTS AND ANALYSIS

All simulations were performed on the main rotor. Cross coupling has not been considered and tail rotor will be kept stationary until stated otherwise.

3.1. PID controller

The PID was tuned using Root locus technique for the main rotor. Figure 12 shows the PID step response and tracking response to reference input, a large overshoot with oscillatory behaviour to step input and an inablity to track reference inputs.

$$C(s) = \frac{3.9s^2+0.2s+2}{s} \tag{20}$$



(a) (b)

Figure 12. (a) Step response (b) Reference tracking with convertional PID

3.2. Adaptive controllers

3.2.1. Adaptive PID and approximate PID

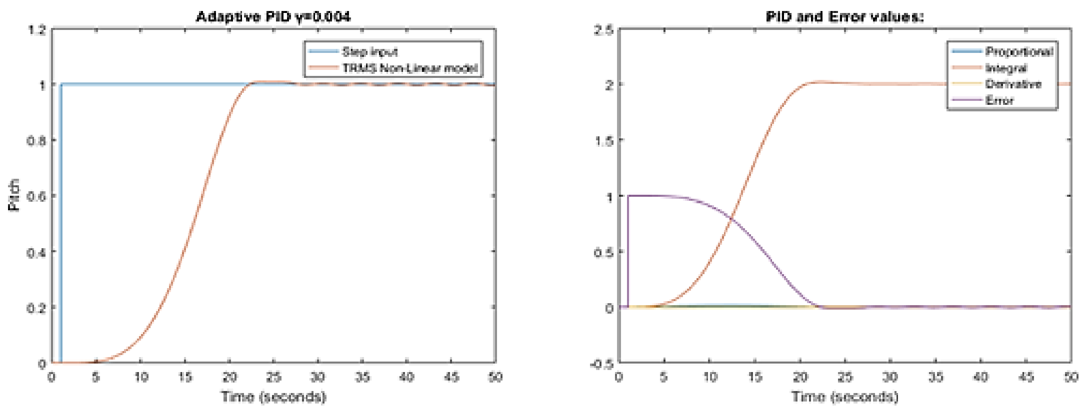
a) Step response

1) Adaptive PID

The variation of adaptive gain with α_1 and its effect on the system was studied as shown in Table 1, the range of values for which system remains stable or still has tracking capability. Step response for $\gamma = 0.004$ with its PID and error characterstics is shown in Figure 13, PID values converge to some final values with error reducing to zero.

Table 1. Variation in $\gamma\ with\ \alpha_1$ and its effect on the system

γ	α_1	System
0.004	0	Stable
0.009	0	Unstable
0.004	> 0	Tracking lost but Stable



(a) (b)

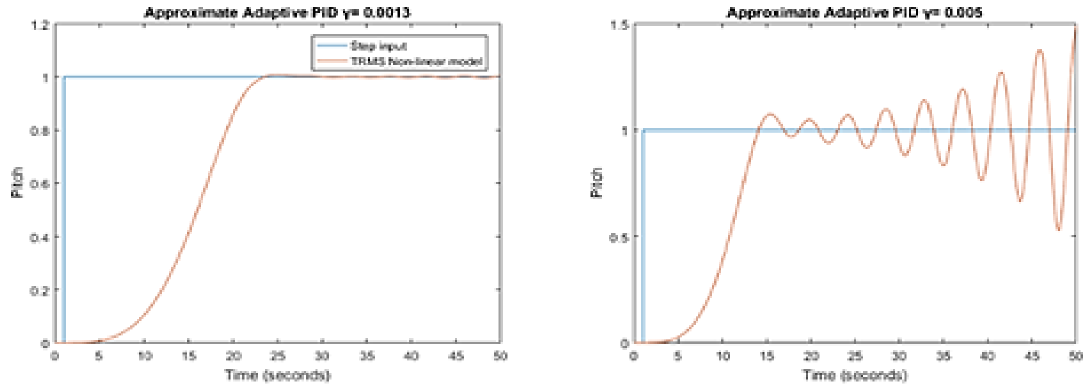
Figure 13. (a) Shows the stable step response, (b) The PID values with error for stable response

2) Approximate Adaptive PID

For a step input system becomes unstable for $\gamma > 0.0013$ as seen from Table 2. The stable and unstable step response for different values of γ is shown in Figure 14, for high value of adaptive gain the system breaks down into oscillations.

Table 2. Variation in γ and its effect on the system

γ	System
0.0013	Stable
0.005	Unstable



(a) (b)

Figure 14. (a) Shows stable and (b) unstable step responses

b) Reference tracking

Tracking for adaptive PID in Figure 15 and tracking for approx adaptive PID in Figure 16.

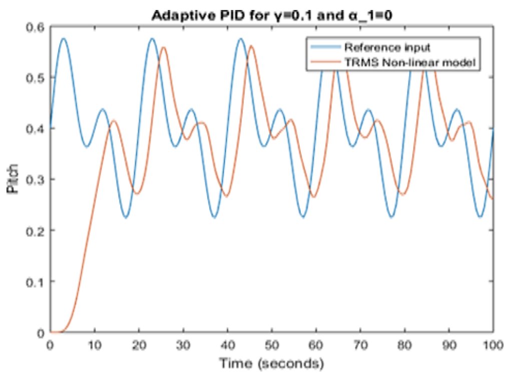


Figure 15. Reference tracking with Adaptive PID for γ=0.

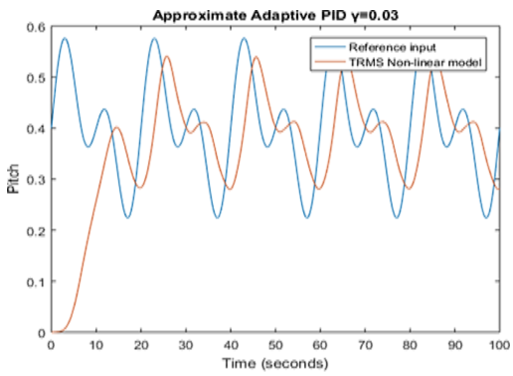


Figure 16. Reference tracking with Approximate Adaptive PID for γ=0.03

3.2.2. Adaptive controller using model reference adaptive control

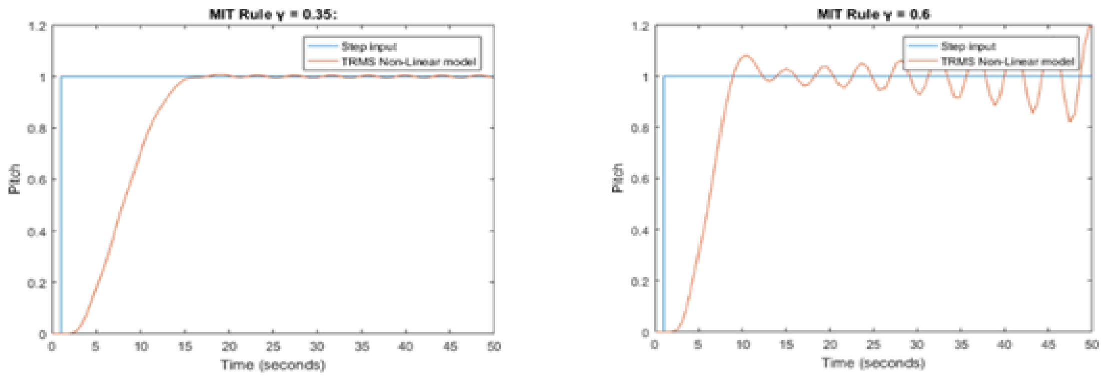
a) Step response

1) MIT Rule

The stability of the system for different values of γ is shown in Table 3, it can be said that the system remains stable for $-0.35 \leq \gamma \leq 0.35$. Stable and unstable step response for MIT Rule are shown in Figure 17 the values of γ are in accordance with Table 3.

Table 3. Variation in γ and its effect on the system

γ	System
0.35	Stable
0.6	Unstable
-0.35	Stable with inverse response



(a) (b)

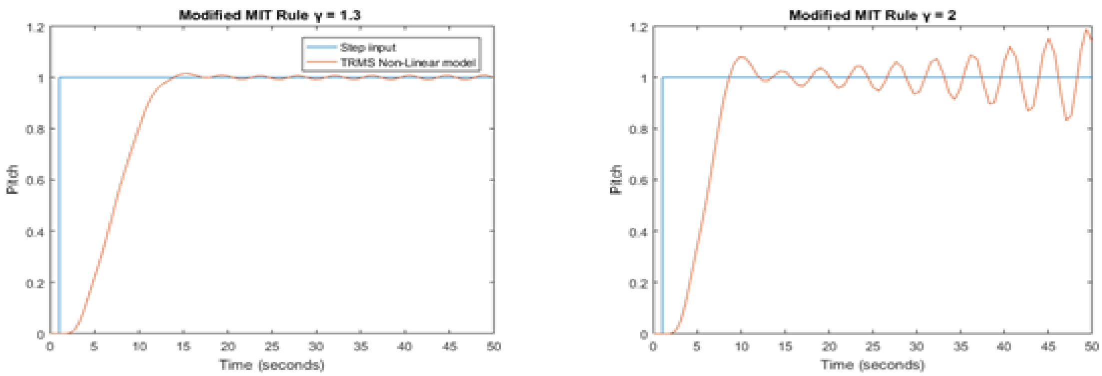
Figure 17. (a) Stable response, (b) unstable step response

2) Modified MIT Rule

The choice of value of α is arbitrary as it is only a dividing factor. The system in this case is stable for $-1.3 \leq \gamma \leq 1.3$ as in Table 4, changing the value of α will only change the region for which γ gives a stable response i.e. if we decrease α then γ will also have to be decreased to give a stable response. Figure 18 shows stable step response for $\gamma = 1.3$ and unstable step response for $\gamma = 2$.

Table 4. Variation in γ and its effect on the system for a fixed α ($\alpha = 2.3$)

γ	System
1.3	Stable
2	Unstable
-1.3	Stable with an inverse response



(a) (b)

Figure 18. (a) Stable response, (b) Unstable response

b) Reference Tracking

The superiority of MIT and Modified MIT rule can be seen from Figures 19 and 20 where perfect tracking is obtained with zero steady state errors.

1) MIT Rule
2) Modified MIT Rule
3) Comparison of controller performance with different reference model for tracking input (Persistence Excited Signal)

The results obtained in Figure 21 shows that $\dot{K}_c$ converges when the reference model chosen is G(s) and does not converge when 1/(s+1) is chosen as the reference model. This proves the validity and importance of using **Theorem 1** in our application and using G(s) as a reference for tracking inputs.

4) Comparison between MIT and modified MIT rule

A comparison between MIT and Modified MIT was made with respect to the rate of convergence of $\dot{K}_c$ and it was found that in Modified MIT $\dot{K}_c$ converges faster than in MIT as can be seen from Figure 22.

This behavior can be attributed to the normalized gradient algorithm which Modified MIT uses. The gradient does not become zero at saddle points thus convergence is faster.

5) Comparison of robustness for PID and modified MIT rule

In Figure 23 the PID response and Modified MIT rule response was compared and the value of deviations from the reference signal during tracking when an input pulse was given at 200 seconds is shown in Table 5. It shows the high degree of robustness provided by Modified MIT rule, the deviation from the reference is very less and the time taken for the system to resume tracking when compared with PID is also less. Modified MIT rule gives better robustness and tracking performance at the same time than the PID.

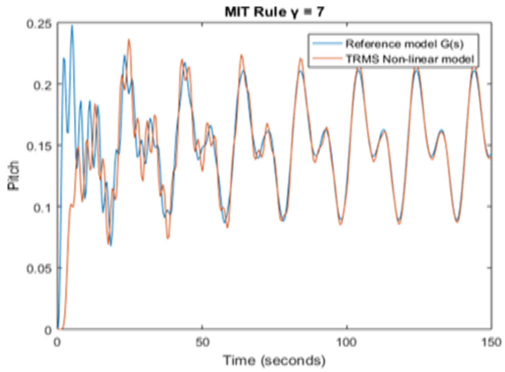


Figure 19. Reference tracking of main rotor with MIT rule for γ=7, showing zero steady state with superior performance

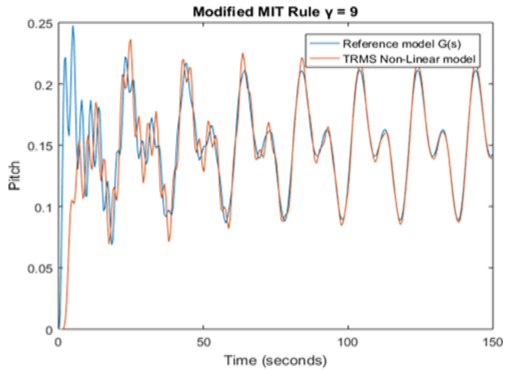


Figure 20. Reference tracking of main rotor with Modified MIT rule for γ=9, showing zero steady state with superior performance

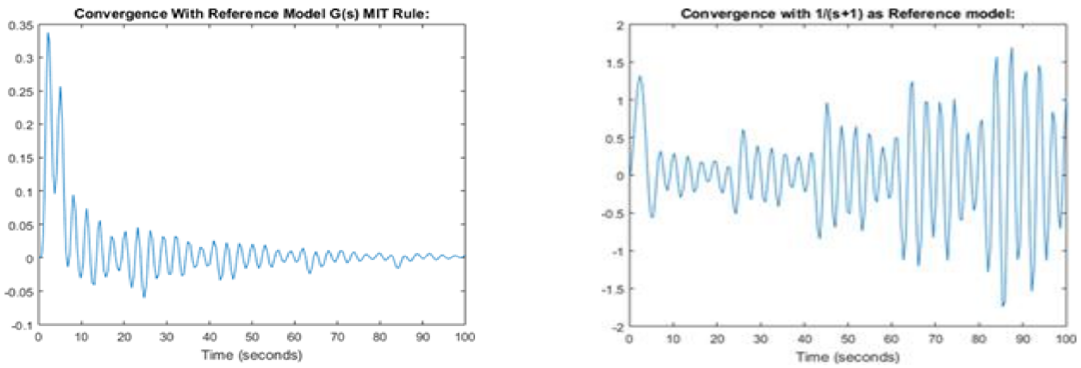


Figure 21. Shows convergence of $\dot{K}_c$ when the reference model chosen for MIT rule is (a) G(s) and (b) 1/(s+1)

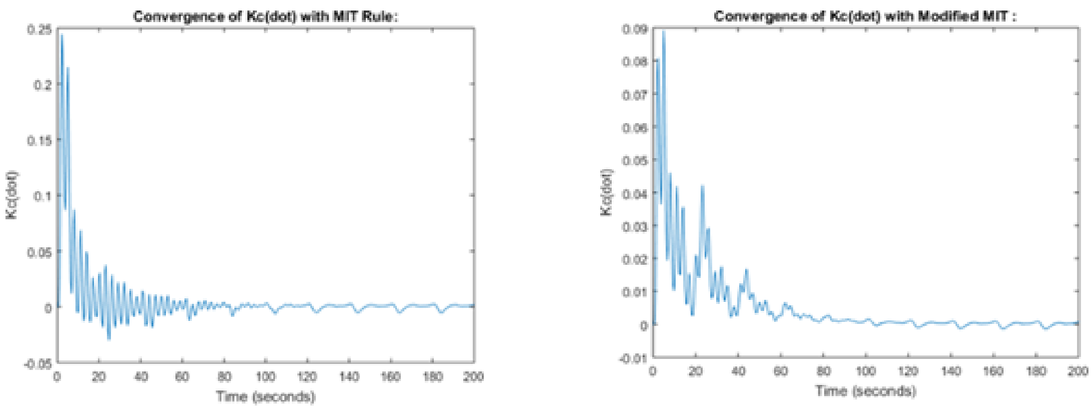
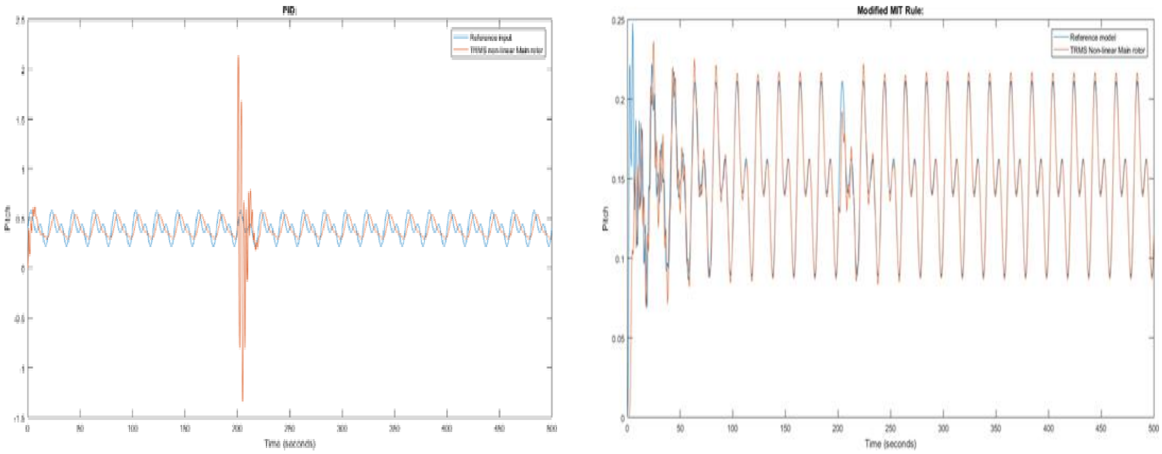


Figure 22. Shows convergence of $\dot{K}_c$ in (a) using MIT rule and (b) using modified MIT rule



(a) (b)

Figure 23. (a) Shows the PID response (b) Response with Modified MIT Rule with the input pulse at 200 seconds while tracking

Table 5. Comparison of robustness between the controllers

	Modified MIT Rule	PID
Peak deviation from reference pitch	0.0193	1.847
Time taken to stabilize (seconds)	13	24

3.3. Important guidelines for implementing the controllers for TRMS and TRMS alike systems
3.3.1. MIT and Modified MIT Rule
a) Performance

In terms of high performance it is desired to achieve zero steady state errors with absolute convergence, but when does this happen. By observation and according to **Theorem 1** absolute convergence will only be attained when the frequency response of chosen reference model is similar to that of actual plant and the frequency range should be around but not limited to frequencies of the input signal used, as shown in Figure 24. It is not known in advance which inputs are going to be applied to the system therefore it is advised to first select the operating frequency range of the system and then choose the reference model, whose output will be exactly followed by the plant. In case when this is not followed the system still tracks the reference model but with steady state errors, see Figure 25.

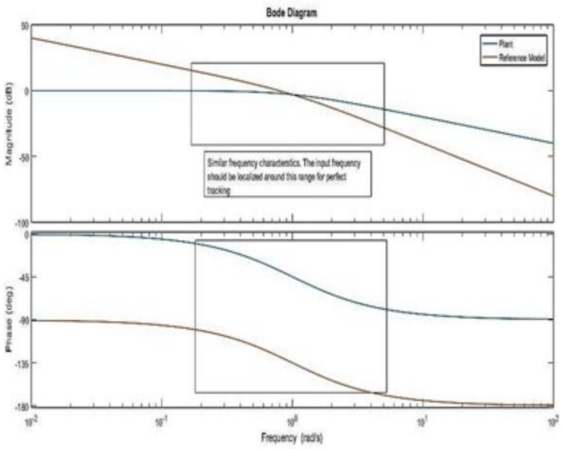


Figure 24. Bode plot for plant and chosen reference model

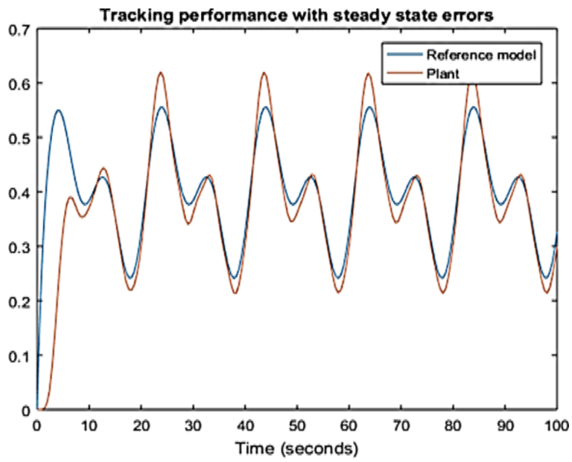


Figure 25. Imperfect Tracking with arbitrary reference model

b) Choosing the right adaptive gain for any input signal

It is well known that configuration of MIT rule depends highly on inputs and is different for different inputs. The gamma value needs to be tuned for different inputs this creates a problem for the user hence we propose a solution. Many input signals of wide range of frequencies were given to the TRMS main rotor and as a rule of thumb adaptive gain should be chosen as **γ=8**. As long as the input is PE (Persistence Excited) this value of gain will give satisfactory tracking performance for any input signal. This result does not hold for step changes as they are not PE signals.

c) Unexpected problem with high adaptive gain

Increasing the adaptive gain to a very high value decrease the rise time and tracking of input is faster but results in distortion of the output from the plant. The system never becomes unstable and steady state is reached after a long time, Figure 26. By experimentation this result holds good for large number of input signals and it was found that the fulfillment of **Theorem 2** is a sufficient but not a necessary condition. Thus the user can be assured that stability for very high adaptive gains will hold good for most input PE signals.

d) Stability

It is a misconception that all adaptive schemes can always stabilize an unstable plant, here MIT rule cannot stabilize an unstable open loop plant. $\dot{K}_c$ does not converge and output grows without bound. Hence MIT rule can only be used for open loop stable plants.

e) Robustness

The user do not have to worry about the degree of robustness as it remains the same for all inputs and all values of adaptive gain.

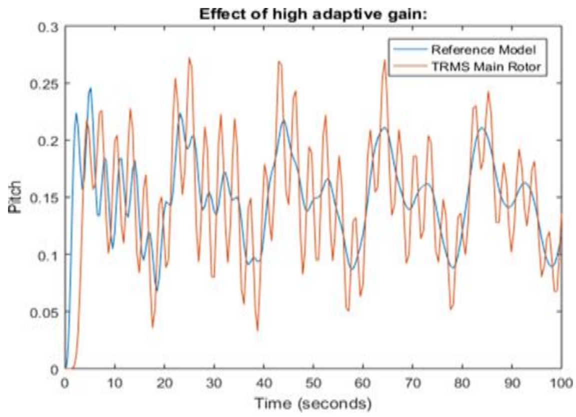


Figure 26. Tracking with large adaptive gain

3.3.2. Adaptive PID and Approximate Adaptive PID

a) Performance

There cannot be any generalization made about the adaptive gain in this case, the user has to manually tune the controller for each new input signal.

b) Choice of weighing factor α_1 and α_2

Both of them are chosen as zero but if they are present then Ki and Kd do not converge and become oscillatory. Oscillations increase further if we increase value of weighing factor, see figure 27. Thus the user should chose weighing factor as zero for all cases.

c) Choice of transfer function model of the plant

The controller will not function well if the transfer function of the plant is not a close approximation of the real plant. Thus if there is uncertainty in the transfer function then Approximate Adaptive PID should be used where no plant knowledge is required.

d) Capability of stabilizing open loop unstable plant, the issue of cross-coupling

A very powerful use of the Approximate Adaptive PID is that it can stabilize open loop unstable plants. Cross coupling is a destabilizing factor in TRMS if the user wants to control the main rotor not separately but with the tail rotor as well, there is no need for designing decouplers which is done in [11-15] TRMS when operating with main and tail rotor becomes an open loop unstable plant. The approximate PID can stabilize it with satisfactory performance. In accordance with **Theorem 4** adaptive laws for Approximate PID are independent of the estimated transfer function, thus it can be used for open loop unstable systems.

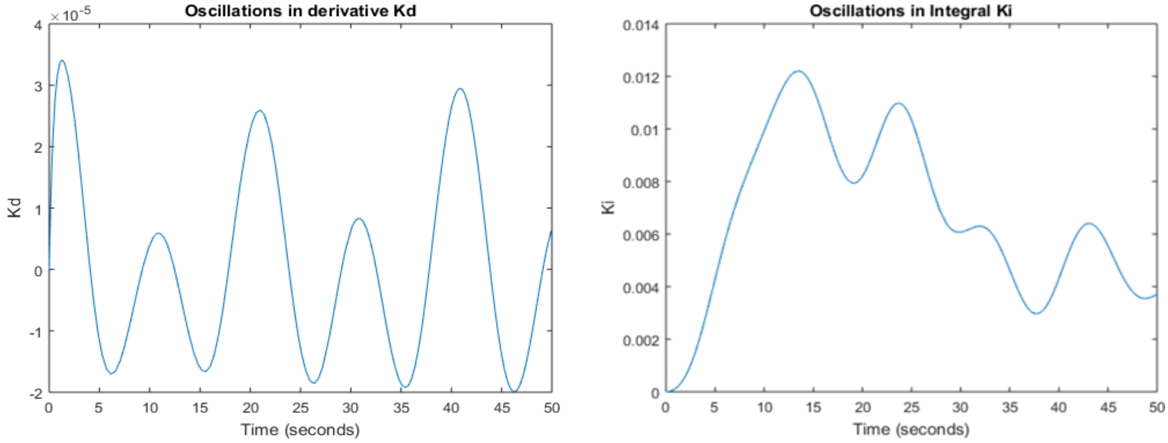
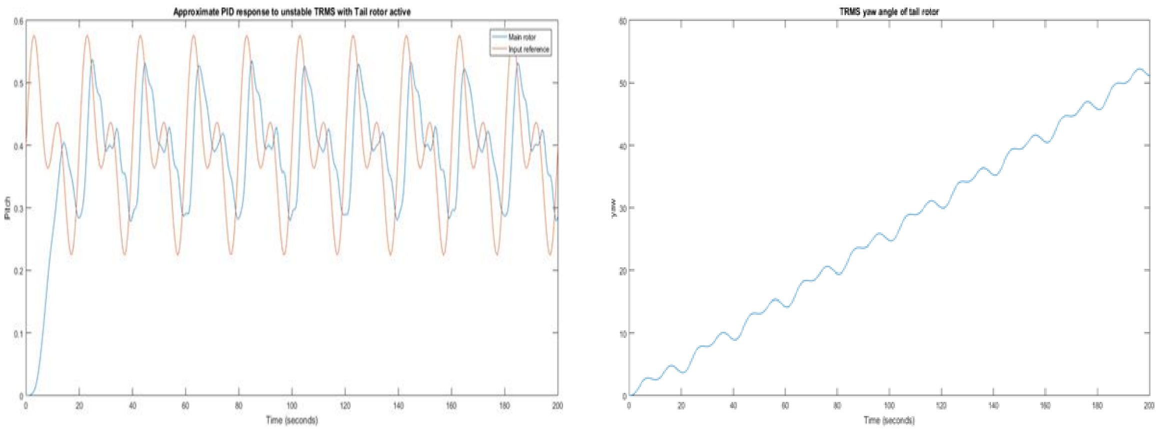


Figure 27. Kd and Ki for non zero value of α_1 and α_2



(a) (b)

Figure 28. (a) Unstable main rotor tracking under cross-coupling (b) Response from tail rotor
In this paper analysis of tail rotor and cross coupling has not been made but if the user wishes, a single approximate PID can be used to control the main rotor when tail is also moving

3.3.3. Comparing all the control schemes according to the performance specifications

Table 6 shows the performance specifications, it can be seen that the Adaptive controllers give superior performance in comparison to that of the PID. Also the performance of Approximate MIT Rule was better than other adaptive schemes having lower rise time and settling time with a overshoot which was negligible in comparison with that of PID but a little more than other adaptive schemes thus showing a more aggressive controller.

Table 6. Comparison of Performance of the adaptive controllers and the PID

	MIT Rule	Modified MIT rule	Adaptive PID	Approximate Adaptive PID	PID
Rise Time(s)	8.473	7.142	9.93	10.636	5.58
Overshoot (%)	0.8	1.6	0.9	0.8	37.1
Settling Time(s)	15.77	13.84	22.2	23.83	38.3

4. CONCLUSION

The adaptive controllers satisfied all the three requirements of maintaining global stability, high degree of robustness and perfect tracking performance all at the same time which had not been fulfilled by others previously. Modified MIT gave better performance when compared to other adaptive controllers. Thus the choice for controlling the open loop stable main rotor is the Modified MIT Rule. In addition to this Approximate Adaptive PID was able to stabilize the TRMS main rotor when the tail was simultaneously working. These schemes are easy to implement and the user can take advantage of the important guidelines section under results and analysis to quickly implement them for TRMS and other similar higher order non-linear systems.

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