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A comparative study of three validities computation methods for multimodel approach

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Article Info	ABSTRACT
	The multimodel approach offers a very satisfactory results in modelling, diagnose and control of complex systems. In the modelling case, this approach passes by three steps: the determination of the model's library, the validities computation and the establishment of the final model. In this context, this paper focuses on the elaboration of a comparative study between three recent methods of validities computation. Thus, it highlight the method that offers the best performances in term of precision. To achieve this goal, we apply, these three methods on two simulation examples in order to compare their performances.

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1.   INTRODUCTION

The establishment of a mathematical model is the first concerns of researchers for application of the advanced techniques of analyses, monitoring, prediction, control and diagnose of complex systems [1, 2]. The multimodel approach has proved a very satisfactory results and a potential benefit in both modelling and identification of complex, nonlinear and/or ill-defined systems, compared to a ″single model approach″. Indeed, the ″single model approach″ consists in determining one model that describes the comportment of the system in all its operating regions. This mission is very difficult and can sometimes be impossible when the system includes set-point changes or/and the co-existence of multiple operating modes [3, 4]. Although, the multimodel approach consists on partitioning the global system's full range operation into multiple smaller ranges. To each range is associated a local model that describes the system behavior in this specific range. The set of the local models forms the called models-library or models-base. A coefficient called validity is associated to each local model of the models-library. Validity estimates each library-model contribution in the reproduction of the real process behavior. Several validities' computation methods have been proposed in the literature [5-13]. These methods depend firstly on the way the library-models were determined and secondly, on the information available on the system. We distinguish two major classes of validities. Firstly the *a priori* validities which can be determined offline by exploiting the *a priori* knowledge available on the system. Secondly, the *a posteriori* validities which must be calculated online by considering the measures carried out at each instant. The present paper is interested by a *postoriori* validity. The majority of methods belonging to the last class are based on the residues computation and are established by measuring omline, at each instant, the distance between the process output and those of the variaous models of the base. These methods are suitable when there is overlapping between data of the different models of

the the models'base [14]. Besides, these methods are limited and cannot be useful with presence of complex and/or ill-defined system. To overcome these problems, Ben Messaoud and all. Have proposed recently, three new methods of computation validities. In fact, the first method is based on a local criterion optimization LCO [15]. The second method uses a hierarchical structuring HS to compute validities [16]. In the third method, validities based on optimal computation OC, are obtained by optimizing a constrained least squares problem [6]. In the present paper, we focus our study to compare the last three methods and to determine the one leads to the best model that can describe perferctly the real process. In order to compare performances assumed by the three proposed methods of validities computation, we consider two numerical simulation examples. In the second part of the paper, we present the general structure of the multimodal approach. The third section discusses the three considered calculation validities methods. Two examples of numerical simulation are given in section four. We finish this work by a conclusion.

2.   GENERAL STRUCTURE OF MULTIMODEL APPROACH
The general structure of the multimodel approach is presented in Figure 1. It is formed by three units: the models library unit, the decision unit and the output unit [12, 17, 18].

2.1. Models library unit
This unit can contain different types of models (either input-output models or state space models). These models are distinguished with respect to four features: Partition strategy, submodel structural identification, transition between models and method of realization [19-21].

2.2. Decision unit
This unit is responsible of computing models validities v_i. It is a fundamental unit as it determines the degrees of contribution of each local model of the library in the composition of the output of the original system. It is assumed that the contribution of all local models' sum is equal to 1 anywhere across the operating space [11, 21].

$$v_i \in [0,1] \; ; \; i = 1,2,\dots,L \tag{1}$$

$$\sum_{i=1}^{L} v_i = 1 \tag{2}$$

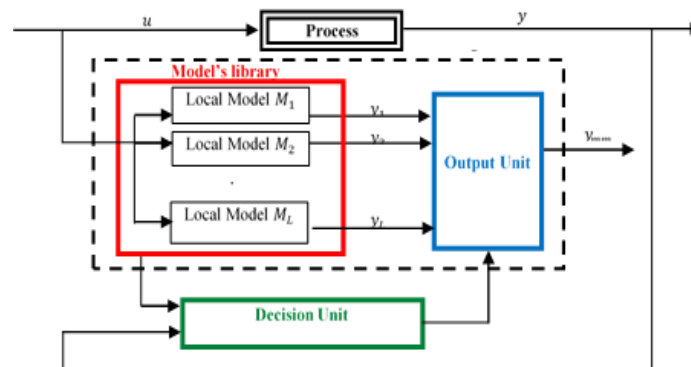Where L represent the number of library' models

Figure 1. General structure of system modelling by multimodel approach

2.3. Output unit
The output unit computes the multimodel output y_{mm} by exploiting outputs obtained by models-library unit and the decision unit as follows:

$$y_{mm}(k) = \sum_{i=1}^{L} v_i(k)\, y_i(k) \tag{3}$$

Where y_i is the output of the model M_i of the models-library and L is the number of library' models.

3. VALIDITIES COMPUTATION

3.1. The first Method: Validities computation by local criterion optimization LCO

This method is presented by Ben Messaoud and al in [15]. In this method, the validity v_i can be estimated, at each instant by minimizing the criterion given by the (4), which represent an equality constrained optimization problem [16]:

$$J = \frac{1}{2}\sum_{i=1}^{L} v_i{}^2(k)\big(y(k) - y_i(k)\big)^2$$
$$\text{subject to: } \sum_{j=1}^{L} v_j(k) = 1 \tag{4}$$

Where: L is the number of library' models.

$y_i(k)$ is the output of ith local model,

$y(k)$ is the measured output of the nonlinear system.

The solution of the optimization criterion above is given by the following expression:

$$v_i(k) = \frac{\frac{1}{r_i{}^2(k)}}{\sum_{j=1}^{L}\frac{1}{r_j{}^2(k)}} \; ; \; i = 1, \dots, L \tag{5}$$

Where $r_i(k)$ represents the residue at the instant k and is based on the online computation of the difference between the process output and those of the various models M_i of the base at the instant k as below:

$$r_i = |y - y_i| \; ; \; i = 1, \dots, L \tag{6}$$

3.2. The Second Method: Validities computation based on an hirarchic structure HS

This method is described by Ben Messaoud and al. in [16] and is summarized as follows:

Let us assume that at instant k, the residues' calculation using the (6) gives ascending values $(r_1 \leq r_2 \leq \cdots \leq r_L)$. The second method for validities computation is based on a quasi-hierarchical structuring whose validities are calculated by the following equation:

$$v_i{}^{simp} = \frac{v_i}{\sum_{j=1}^{L} v_j} \; ; \; i = 1, \dots, L \tag{7}$$

Where:

$$v_i = 1 - \frac{r_i}{\sum_{j=1}^{L} r_j} \; ; \; i = 1, \dots, L \tag{8}$$

In the general case and for each instant k, the base-models' validities are computed by using the following algorithm:

Step 1: Create a table containing all base-models' outputs:

$$Tab_y = [y_1\, y_2 \dots y_L]$$

Step 2: Calculate the corresponding residues by (6).

Step 3: Create a table containing all the obtained residues:

$$Tab_r = [r_1\, r_2 \dots r_L]$$

Step 4: Arrange Tab_r in ascending order into $TabC_r$ and create a table Tab_{ind} for the corresponding indices of Tab_r

Step 5:
    for i =1 to 2
        $r_i' \leftarrow TabC_r[i]$
        Calculate v_i' by (7)
    End

$$y_{pmm_1} = v'_1.Tab_y\big[Tab_{ind}[1]\big] + v'_2.Tab_y\big[Tab_{ind}[2]\big] \tag{9}$$

$j \leftarrow 1$

for i =3 to L

    $r'_i \leftarrow TabC_r[i]$; $r_{pmm_j} = \big|y - y_{pmm_j}\big|$

    Calculate v'_i and v_{pmm_j} by (7)

    if i ==L

        STOP!

    else

$$y_{pmm_{j+1}} = v_{pmm_j}.y_{pmm_j} + v'_i.Tab_y\big[Tab_{ind}[i]\big] \tag{10}$$

    end

    $j \leftarrow j + 1$

end

Step 6: Calculate the validities of the different models from the following loop:

    for i =1 **to** L

        for j=1 **to** L

            if $Tab_{ind}[j]$== i

                if j == 1

$$v_i = v'_j \left(\prod_{k=1}^{L-2} v_{pmm_k} \right) \tag{11}$$

                else if j == L

$$v_i = v'_L$$

                else

$$v_i = v'_j \left(\prod_{k=j-1}^{L-2} v_{pmm_k} \right) \tag{12}$$

                End if

            End if

        End for

    End for

3.3. The Third Method: validities' optimal computation OC

The third proposed method of validities computation v_i is based on the optimization of a constrained least squares problem [6]. The formulation of the quadratic programming is as follows:

$$\min_{v} \frac{1}{2} v^T G v + c^T v$$
$$subject\ to: \begin{cases} Dv = h \\ Fv \geq e \end{cases} \tag{13}$$

Where: $D = (1 \quad 1 \quad \cdots \quad 1) \in 1 \times L$, $h = 1$, $F = \begin{pmatrix} -I_{L \times L} \\ I_{L \times L} \end{pmatrix}$ and $e = \begin{pmatrix} -1 \\ \vdots \\ -1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \in 2L \times 1$;

$$G = A^T A \text{ and } c = -A^T b. \tag{14}$$

The active-set method is chosen for solving the problem above [17]. The details of validities' computation by this method are given by the following algorithm:

Step 1   Determine an initial feasible point v^0 by solving the linear programming problem below (16), and by using the simplex method:

$$\min_{(v,z)} w^T z$$

$$subject\ to: \begin{cases} d_i^{\ T} v + \gamma_i z_i = h_i\ ;\ i = 1 \\ f_j^{\ T} v + \gamma_{j+1} z_{j+1} \geq e_j\ ;\ j = 1, \dots, 2L \\ z \geq 0 \end{cases} \tag{15}$$

where: $w = (1 \quad 1 \quad \cdots \quad 1)^T \in (2L + 1) \times 1$,

$\qquad \gamma_i = -sign(d_i^{\ T} \tilde{v} - h_i)$ for $i = 1$

and $\gamma_{j+1} = 1$ for $j = 1, \dots, 2L$ with $\tilde{v}$ is an initial estimate of the vector v.

Step 2   Determine the working set W^0 which contains the indexes representing the equality and active inequality constraints:

$$W^0 = \{i | d_i^{\ T} v^0 = h_i\} \cup \{j | f_j^{\ T} v^0 = e_j\} \tag{16}$$

for $l = 0,1,2, \dots$ **do**

Find the search direction p^l and the Lagrange multipliers λ^l and μ^l by solving KKT-equations (Karush-Kuhn-Tucker) of the quadratic subproblem:

$$\min_p \frac{1}{2} p^T G p + (G v^l - c)^T p$$

$$subject\ to: \begin{cases} d_i^{\ T} p = 0\ ;\ i \in W^l \\ f_j^{\ T} p = 0\ ;\ j \in W^l \end{cases} \tag{17}$$

if $p^l = 0$ **then**

    if $\mu^l \geq 0$ **then**

        STOP! With $v^* = v^l$ is the optimal solution;

    else

        Find the most negative component of μ^l:

$$\mu^l_{\ j_0} = min \{\mu^l_{\ j} | \mu^l_{\ j} < 0, j \in W^l\} \tag{18}$$
$$v^{l+1} = v^l;$$

        Remove constraint corresponding to the most negative component of μ^l from the working set: $W^{l+1} \leftarrow W^l \backslash \{j_0\}$;

    end if

else $\{p^l \neq 0\}$

    Compute the step-length α^l that guarantees the satisfaction of all constraints from:

$$\alpha^l = min \left(1, \frac{e_j - f_j^{\ T} v^l}{f_j^{\ T} p^l} | j \notin W^l \text{ and } f_j^{\ T} p^l < 0\right) \tag{19}$$

    Update: $v^{l+1} \leftarrow v^l + \alpha^l p^l$;

    Update W^l:

        if $\alpha^l = 1$ **then**

$$W^{l+1} \leftarrow W^l;$$

        else

          Add one constraint to the working set: $W^{l+1} \leftarrow W^l \cup \{j_0\}$

          where

$$\alpha^l = \frac{e_{j_0} - f_{j_0}^{\ T} v^l}{f_{j_0}^{\ T} p^l} \text{ for } f_{j_0}^{\ T} p^l < 0\ ;$$

        end if

    end if

end for

4.   SIMULATION EXEMPLES

To compare the performances offered by the three considered computation validities methods, we propose two numical simulation examples.

4.1. Exemple 1

This first example is a discrete system with time varying parameters $a_1(k), a_2(k), b_1(k)$ and $b_2(k)$ described by the (21) [11, 22, 23]. The variation laws of the process parameters are given by Figure 2.

$$y(k) = -a_1(k)y(k-1) - a_2(k)y(k-2) + b_1(k)u(k-1) + b_2(k)u(k-2) \qquad (20)$$

By applying the multimodel approach, Talmoudi et al. [18] has established the following models-library transfer functions:

$$H_1(z^{-1}) = \frac{0.18104z^{-1}+0.071832z^{-2}}{1-1.1657z^{-1}+0.2073z^{-2}} \qquad (21)$$

$$H_2(z^{-1}) = \frac{0.10423z^{-1}+0.1325z^{-2}}{1-1.2806z^{-1}+0.3258z^{-2}} \qquad (22)$$

$$H_3(z^{-1}) = \frac{0.018301z^{-1}+0.20512z^{-2}}{1-1.3801z^{-1}+0.42936z^{-2}} \qquad (23)$$
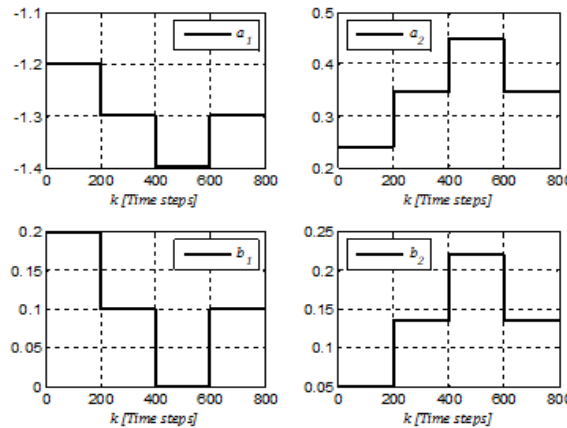

Figure 2. Evolution of parametres variation laws

After determining the library-models, we compute the validities degrees of each model of the library by using the three methods described above. The evolutions of validities degrees obtained by using LCO, HS and OC methods are presented, respectively, in Figures 3, 4 and 5. These Figures show that OC method gives the best results in term of precision. Indeed, it is clear, from the considered system (20), that at each range of time, there is only one transfer function that is totally valid. Or, Only the OC method demonstrates this feature in Figure 5. LCO and HS methods give for each range of time one model having a validity degree near to 1 but not equal to 1.

The multimodel output is, then, obtained by the fusion of the different models outputs $y_1(k)$, $y_2(k)$ and $y_3(k)$ weighted by their respective validity degrees computed by the different proposed methods:

$$y_{mmLCO}(k) = v_{1LCO}(k)y_1(k) + v_{2LCO}(k)y_2(k) + v_{3LCO}(k)y_3(k) \qquad (24)$$

$$y_{mmHS}(k) = v_{1HS}(k)y_1(k) + v_{2HS}(k)y_2(k) + v_{3HS}(k)y_3(k) \qquad (25)$$

$$y_{mmOC}(k) = v_{1OC}(k)y_1(k) + v_{2OC}(k)y_2(k) + v_{3OC}(k)y_3(k) \qquad (26)$$

Where:
- $y_{mmLCO}, y_{mmHS}, y_{mmOC}$ are multimodel ouputs using respectively LCO, HS and OC methods.
- $v_{1LCO}(k)$, $v_{2LCO}(k)$ and $v_{3LCO}(k)$ are validities for LCO method.
- $v_{1HS}(k)$, $v_{2HS}(k)$ and $v_{3HS}(k)$ are validities for HS method.
- $v_{1OC}(k)$, $v_{2OC}(k)$ and $v_{3OC}(k)$ are validities for OC method.

Figure 6 shows the evolutions of the relative errors obtained when we apply the three considered validities computation methods. It is clear that OC method offers the best precision by comparison to the two other methods.
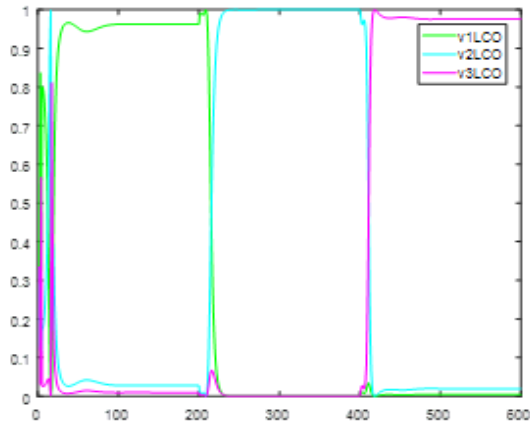

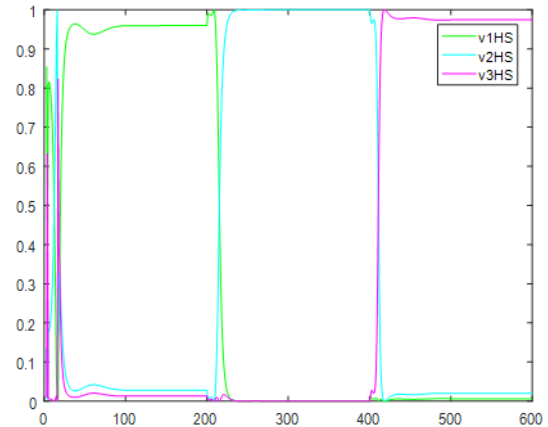Figure 3. Validities evolution of LCO method
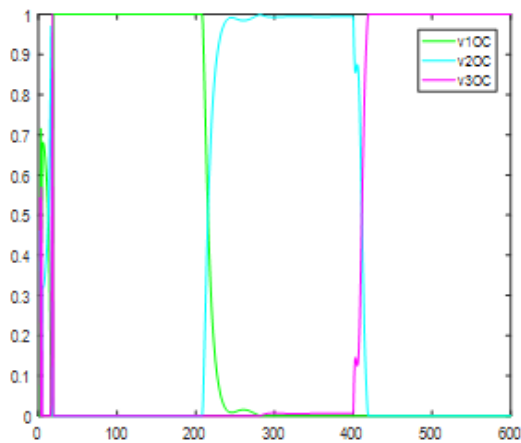

Figure 4. Validities evolution of HS method

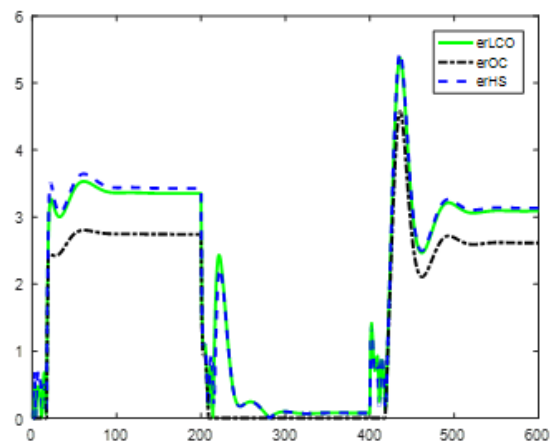Figure 5. Validities evolution of OC method

Figure 6. Relative errors evolutions

4.2. Exemple 2

The second considered example is an uncertain complex system represented by the following equation [6, 20, 18]:

$$x(t) + \tau(x)\dot{x}(t) = k(x)u(t) ; \ x \in [0,1] \tag{27}$$

Where: $k(x) = 36x(t)(x(t) - 1) + 10$
and $\tau(x) = 15 - 10x(t)$.

It is clear that the considered system is nonlinear. Indeed, the gain and the time constant vary with x. This nonlinearity appears more in Figures 7 and 8. The last Figures shows the evolution of the time constant and the gain variations.

The minimal and maximal values of respectively the gain and the time constant are:

$$k_{min} = 1; \ k_{max} = 10; \ \tau_{min} = 5; \ \tau_{max} = 15; \tag{28}$$

By using the Kharitonov's algebraic approach [9], we obtain the following models' library:

$$F_1(s) = \frac{1}{1+15s}; \quad F_2(s) = \frac{1}{1+5s}$$
$$F_3(s) = \frac{10}{1+15s}; \quad F_4(s) = \frac{10}{1+5s}$$

(29)

The system (27) and the four transfer functions (29) are implanted in the Matlab Simulink environment. Matlab discretize these equations by using the Bogacki-Shampine method with sampling time T_s equal to $0.1\ s$.
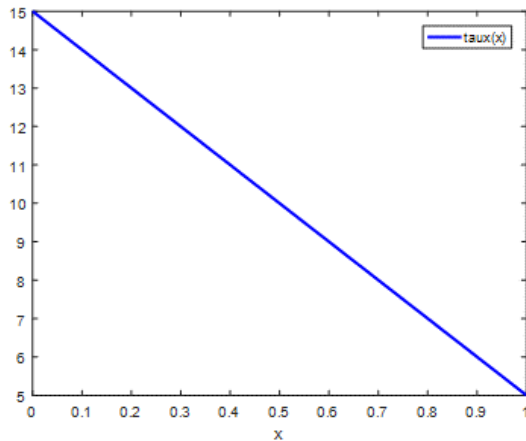
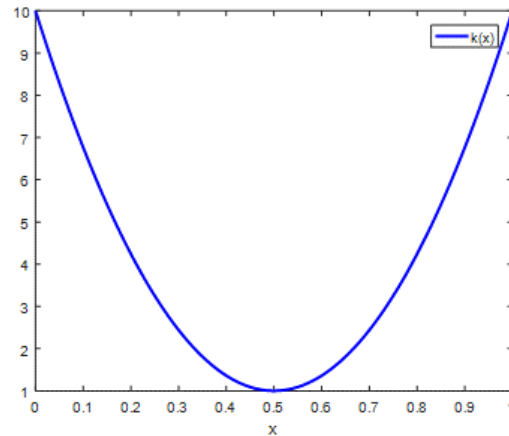Figure 7. The constant time variation

Figure 8. The gain variation

The Figure 9 shows the evolutions of the process and the multimodel states obtained by using the three computation validities methods and by applying a unit step at the input. It is clear the multimodel state for OC method coincides perfectly with the process state. Although the two other states relative to LCO and HS methods follows the process state but with an error relatively important. These errors are represented in Figure 10 which shows that the relative error correspondent to OC method is equal to 0 at every instant. The other relative errors are relatively important.
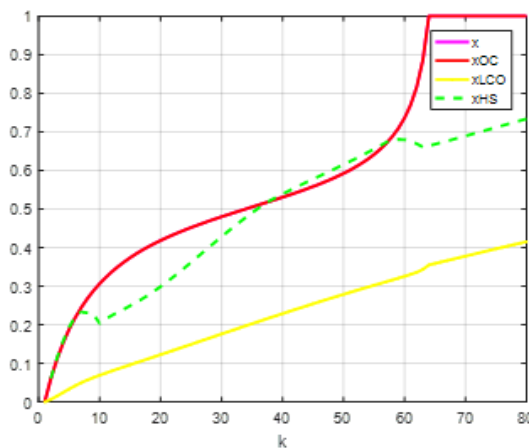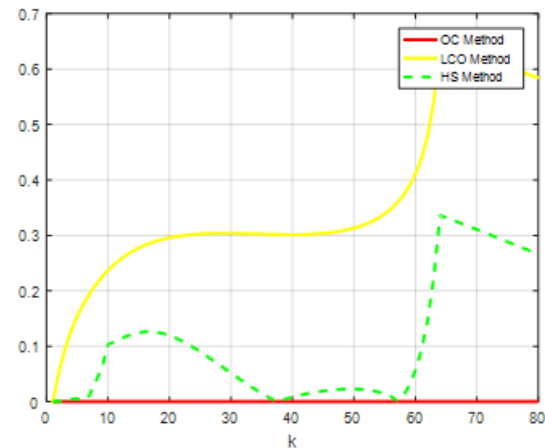
Figure 9. Process and multimodel states

Figure 10. Relative errors evolutions

Where: x represents the process state
x_{LCO} is the multimodel state obtained by using LCO method,
x_{HS} is the multimodel state obtained by using HS method,
x_{OC} is the multimodel state obtained by using OC method.

5.   CONCLUSION

On our research, we are interested by exploiting the multimodel approach for modelling complex, non-linear or/and uncertain systems. This paper presents a large comparative study of three validities computation methods. These methods are recently proposed by Ben Messaoud and al. We are succeeded to prove that the method based on resolving an optimization problem by using the active set method i.e OC method, gives the best results of modelling with a good precision.

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Abdennacer Ben Messaoud received his Engineering Diploma in Electric and Automatic Engineering, in 2002, and Master degree in Automatic Control and Intelligent Techniques, in 2009, from National School of Engineers of Gabes (ENIG), Tunisia. He has over 13 years of industrial experience. Currently, he is pursuing his PhD thesis at Laboratory of Analysis Conception and Control Systems (LACS) at National Engineering School of Tunis (ENIT), Tunisia. His research interests include multimodel approach and modelling of fractional-order systems.

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