Fuzzy n-s-homogeneity and fuzzy weak n-s-homogeneity

Samer Al Ghour[1], Almothana Azaizeh[2]
[1]Department of Mathematics and Statistics, Jordan University of Science and Technology, Jordan
[2]College of Applied Studies and Community Service, Imam Abdurrahman Bin Faisal University, Saudi Arabia

Article Info

Article history:

Received Jan 17, 2019
Revised Jul 16, 2019
Accepted Jul 28, 2019

Keywords:

Homogeneity
n-s-homogeneity

ABSTRACT

Fuzzy n-s-homogeneity and fuzzy weak n-s-homogeneity are introduced in fuzzy bitopological spaces. Several relationships, characterizations and examples related to them are given.

Corresponding Author:

Samer Al Ghour,
Department of Mathematics and Statistics,
Jordan University of Science and Technology,
Irbid 22110, Jordan.
Email: algore@just.edu.jo

1. INTRODUCTION AND PRELIMINARIES

In this paper, we follow the notions and terminologies as appeared in [1]. As defined by Kelly in [2], the triple (X, τ_1, τ_2) where X is a set and τ_1, τ_2 are topologies on X is called a bitopological space. Later on, several authors had studied this notion and other related concepts. Author in [3] studied some ordinary homogeneity concepts in bitopological spaces. By a similar method to that used in defining bitopological spaces, the notion of fuzzy bitopological spaces was defined in [4]. Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be two bitopological spaces. A map$f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called s-homeomorphism if the maps $f: (X, \tau_1) \to (Y, \sigma_1)$ and $f: (X, \tau_2) \to (Y, \sigma_2)$ are homeomorphisms. Let(X, τ_1, τ_2) be a bitopological space. (X, τ_1, τ_2) is s-homogeneous [3] if for any two points $x_1, x_2 \in X$, there is an s-homeomorphism $f: (X, \tau_1, \tau_2) \to (X, \tau_1, \tau_2)$ such that $h(x_1) = x_2$. (X, τ_1, τ_2) is n-s-homogeneous if for any two n-tons $A = \{a_1, a_2, \dots, a_n\}$, $B = \{b_1, b_2, \dots, b_n\}$ in X, there is an s-homeomorphism $h: (X, \tau_1, \tau_2) \to (X, \tau_1, \tau_2)$ such that $h(a_i) = b_i$ for every $i = 1,2, \dots, n$. (X, τ_1, τ_2) is weakly n-s-homogeneous for any two n-tons A and B in X, there is an s-homeomorphism $h: (X, \tau_1, \tau_2) \to (X, \tau_1, \tau_2)$ such that $h(A) = B$ for every $i = 1,2, \dots, n$. Several fuzzy homogeneity concepts were discussed in [5-14]. Let $(X, \Im_1, \Im_2)$ and (Y, Γ_1, Γ_2) be two fuzzy bitopological spaces. A map $f: (X, \Im_1, \Im_2) \to (Y, \Gamma_1, \Gamma_2)$ is called a fuzzy s-homeomorphism if the maps $f: (X, \Im_1) \to (Y, \Gamma_1)$ and $f: (X, \Im_2) \to (Y, \Gamma_2)$ are fuzzy homeomorphisms.

Let (X, τ) be a topological space. The class of all lower semicontinuous mappings from (X, τ) to $[0,1]$ with the usual topology forms a fuzzy topology on X, this fuzzy topology is denoted by $\omega(\tau)$. Also, the family $\{\chi_U : U \in \tau\}$ forms a fuzzytopology on X, this topology is denoted by $X|\tau$. As defined, for fuzzy topological space $(X, \Im)$, the associatedtopological space $\{B^{-1}(a, 1]: B \in \Im\}$ is called the a-cut (level) topological space and denoted by $\Im_a$.

The following three propositions will be used in the sequel:

Proposition 1.1. [15] Let (X,τ_1) and (Y,τ_2) be two topological spaces and $f:(X,\tau_1)\to(Y,\tau_2)$ be a map. Then the following are equivalent:

i. $f:(X,\tau_1)\to(Y,\tau_2)$ is a homeomorphism.
ii. $f:(\omega(X),\tau_1)\to(\omega(Y),\tau_2)$ is a fuzzy homeomorphism.
iii. $f:(X,X|\tau_1)\to(Y,Y|\tau_2)$ a fuzzy homeomorphism.

Proposition 1.2. [1] Let (X,τ) be a bitopological space. Then the following are equivalent:

i. (X,τ_1,τ_2) is n-homogeneous.
ii. $(X,\omega(\tau_1),\omega(\tau_2))$ fuzzy n-homogeneous.
iii. $(X,X|\tau_1,X|\tau_2)$ is fuzzy n-homogeneous.

Proposition 1.3. [11] Let $(X,\Im)$ be a fuzzy topological space and let $f:(X,\Im)\to(X,\Im)$ be a fuzzy continuous (homeomorphism) map. Then $f:(X,\Im_a)\to(X,\Im_a)$ is continuous (homeomorphism) for all $a\in[0,1)$.

2. N-S-HOMOGENEOUS FUZZY BITOPOLOGICAL SPACES

Defintion 2.1. A fuzzy bitopological space $(X,\Im_1,\Im_2)$ is said to be

i. fuzzy s-homogeneous if for any two points $x_1,x_2\in X$, there is a fuzzy s-homeomorphism $h:(X,\Im_1,\Im_2)\to(X,\Im_1,\Im_2)$ such that $h(x_1)=x_2$.
ii. fuzzy n-s-homogeneous if for any two n-tons $A=\{a_1,a_2,\dots,a_n\}, B=\{b_1,b_2,\dots,b_n\}$ in X, there is a fuzzy s-homeomorphism $h:(X,\Im_1,\Im_2)\to(X,\Im_1,\Im_2)$ such that $h(a_i)=b_i$ for every $i=1,2,\dots,n$.
iii. fuzzy weakly n-s-homogeneous if for any two n-tons $A,B\subseteq X$, there is a fuzzy s-homeomorphism $h:(X,\Im_1,\Im_2)\to(X,\Im_1,\Im_2)$ such that $h(A)=B$.

Remark 2.2. Fuzzy s-homogeneous, fuzzy 1-s-homogeneous and fuzzy weakly 1-s-homogeneous are all equivalent.

Theorem 2.3. A fuzzy topological space $(X,\Im)$ is fuzzy n-homogeneous (fuzzy weakly n-homogeneous) iff the fuzzy bitopological space $(X,\Im,\Im)$ is fuzzy n-s-homogeneous (fuzzy weakly n-s-homogeneous).
Proof. Obvious.

Theorem 2.4. Let (X,τ_1,τ_2) be a bitopological space and $h:X\to X$ be a bijective map. Then the following are equivalent:

i. $h:(X,\tau_1,\tau_2)\to(X,\tau_1,\tau_2)$ is an s-homeomorphism.
ii. $h:(X,\omega(\tau_1),\omega(\tau_2))\to(X,\omega(\tau_1),\omega(\tau_2))$ is a fuzzy s-homeomorphism.
iii. $h:(X,X|\tau_1,X|\tau_2)\to(X,X|\tau_1,X|\tau_2)$ is a fuzzy s-homeomorphism.

Proof. (i) ⇒ (ii): Suppose that $h:(X,\tau_1,\tau_2)\to(X,\tau_1,\tau_2)$ is an s-homeomorphism. Then each of the functions $h:(X,\tau_1)\to(X,\tau_1)$ and $h:(X,\tau_2)\to(X,\tau_2)$ is a homeomorphism. By Proposition 1.1, the functions $h:(X,\omega(\tau_1))\to(X,\omega(\tau_1))$ and $h:(X,\omega(\tau_2))\to(X,\omega(\tau_2))$ are fuzzy homeomorphisms. Hence, $h:(X,\omega(\tau_1),\omega(\tau_2))\to(X,\omega(\tau_1),\omega(\tau_2))$ is a fuzzy s-homeomorphism.
The proof of each of (ii) ⇒ (iii) and (iii) ⇒ (i) is similar to the proof of (i) ⇒ (ii).

Theorem 2.5. Let (X,τ_1,τ_2) be a bitopological space. Then the following are equivalent:

i. (X,τ_1,τ_2) is n-s-homogeneous.
ii. $(X,\omega(\tau_1),\omega(\tau_2))$ fuzzy n-s-homogeneous.
iii. $(X,X|\tau_1,X|\tau_2)$ is fuzzy n-s-homogeneous.

Proof. (i) ⇒ (ii): Suppose that (X,τ_1,τ_2) is n-s-homogeneous and let $A=\{a_1,a_2,\dots,a_n\}, B=\{b_1,b_2,\dots,b_n\}$ be any two n-tons in X. Then there is an s-homeomorphism $h:(X,\tau_1,\tau_2)\to(X,\tau_1,\tau_2)$ such that $h(a_i)=b_i$ for every $i=1,2,\dots,n$. By Theorem 2.4, $h:(X,\omega(\tau_1),\omega(\tau_2))\to(X,\omega(\tau_1),\omega(\tau_2))$ is a fuzzy s-homeomorphism. It follows that $(X,\omega(\tau_1),\omega(\tau_2))$ fuzzy n-s-homogeneous.
The proof of each of (ii) ⇒ (iii) and (iii) ⇒ (i) is similar to the proof of (i) ⇒ (ii).

Corollary 2.6. Let (X,τ_1,τ_2) be a bitopological space. Then the following are equivalent:

i. (X,τ_1,τ_2) is s-homogeneous.
ii. $(X,\omega(\tau_1),\omega(\tau_2))$ fuzzy s-homogeneous.
iii. $(X,X|\tau_1,X|\tau_2)$ is fuzzy s-homogeneous.

Theorem 2.7. Let (X, τ_1, τ_2) be a bitopological space. Then the following are equivalent:
i. (X, τ_1, τ_2) is n-s-homogeneous.
ii. $(X, \omega(\tau_1), \omega(\tau_2))$ fuzzy weakly n-s-homogeneous.
iii. $(X, X|\tau_1, X|\tau_2)$ is fuzzy weakly n-s-homogeneous.
Proof. Similar to the proof of Theorem 2.5.

Theorem 2.8. If $(X, \Im_1, \Im_2)$ is a fuzzy n-s-homogeneous fuzzy bitopological space, then $(X, \Im_1)$ and $(X, \Im_2)$ are fuzzy n-homogeneous.
Proof. $A = \{a_1, a_2, \dots, a_n\}, B = \{b_1, b_2, \dots, b_n\}$ be any two n-tons in X. Then there is a fuzzy s-homeomorphism $h: (X, \Im_1, \Im_2) \to (X, \Im_1, \Im_2)$ such that $h(a_i) = b_i$ for every $i = 1,2, \dots, n$. Then $h: (X, \Im_1) \to (X, \Im_1)$ and $h: (X, \Im_2) \to (X, \Im_2)$ are fuzzy homeomorphisms. Hence, $(X, \Im_1)$ and $(X, \Im_2)$ are fuzzy n-homogeneous.

Corollary 2.9. If $(X, \Im_1, \Im_2)$ is a fuzzy s-homogeneous fuzzy bitopological space, then $(X, \Im_1)$ and $(X, \Im_2)$ are fuzzy homogeneous.

The following example shows that the converse of each of Theorem 2.8 and Corollary 2.9 is not true in general:

Example 2.10. Let $X = \{1,2,3,4,5,6\}, \tau_1 = \{\emptyset, X, \{1,2,3\}, \{4,5,6\}\}$ and
$\tau_2 = \{\emptyset, X, \{1,2\}, \{3,4\}, \{5,6\}, \{1,2,3,4\}, \{1,2,5,6\}, \{3,4,5,6\}\}$. Then (X, τ_1) and (X, τ_2) are homogeneous. So, by Proposition 1.2, $(X, X|\tau_1)$ and $(X, X|\tau_2)$ are fuzzy homogeneous. It is not difficult to check that (X, τ_1, τ_2) is not s-homogeneous and by Corollary 2.6, $(X, X|\tau_1, X|\tau_2)$ is not fuzzy s-homogeneous.

Theorem 2.11. If $(X, \Im_1, \Im_2)$ is a fuzzy n-s-homogeneous fuzzy bitopological space, then for each $a \in [0,1), (X, (\Im_1)_a, (\Im_2)_a)$ is n-s-homogeneous.
Proof. Let $a \in [0,1)$ and let $A = \{a_1, a_2, \dots, a_n\}, B = \{b_1, b_2, \dots, b_n\}$ be any two n-tons in X. Then there is a fuzzy s-homeomorphism $h: (X, \Im_1, \Im_2) \to (X, \Im_1, \Im_2)$ such that $h(a_i) = b_i$ for every $i = 1,2, \dots, n$. Thus, $h: (X, \Im_1) \to (X, \Im_1)$ and $h: (X, \Im_2) \to (X, \Im_2)$ are fuzzy homeomorphisms and by Proposition 1.3, $h: (X, (\Im_1)_a) \to (X, (\Im_1)_a)$ and $h: (X, (\Im_2)_a) \to (X, (\Im_2)_a)$ are homeomorphisms.
Hence, $h: (X, (\Im_1)_a, (\Im_2)_a) \to (X, (\Im_1)_a, (\Im_2)_a)$ is an s-homeomorphism.

Corollary 2.12. If $(X, \Im_1, \Im_2)$ is a fuzzy s-homogeneous fuzzy bitopological space, then for each $a \in [0,1), (X, (\Im_1)_a, (\Im_2)_a)$ is s-homogeneous.

The implication in Corollary 2.12 is not reversible in general as it cab be seen from Example 4.7 of [7].

Theorem 2.13. If $(X, \Im_1, \Im_2)$ is a fuzzy n-s-homogeneous fuzzy bitopological space, then $(X, \Im_1, \Im_2)$ is weakly fuzzy n-s-homogeneous.
Proof. Obvious.

The following Example shows that the implication in Theorem 2.13 is not reversible in general:

Example 2.14. The author in [3], showed that $(\mathbb{R}, \tau_{l.r}, \tau_{disc})$ is weakly 2-s-homogeneous but not 2-s-homogeneous. Therefore, by Theorems 2.5 and 2.7 $(\mathbb{R}, \omega(\tau_{l.r}), \omega(\tau_{disc}))$ is fuzzy weakly 2-s-homogeneous but not fuzzy 2-s-homogeneous.

Theorem 2.15. If $(X, \Im_1, \Im_2)$ is a fuzzy n-s-homogeneous fuzzy bitopological space with $|X| \geq n$, then $(X, \Im_1, \Im_2)$ is a fuzzy k-s-homogeneous for every $k \leq n$.
Proof. Let $A = \{a_1, a_2, \dots, a_k\}, B = \{b_1, b_2, \dots, b_k\}$ be any two k-tons in X. Choose distince points $a_{k+1}, a_{k+2}, \dots, a_n$from $X - A$ and choose distinct points $b_{k+1}, b_{k+2}, \dots, b_n$ from $X - B$. Then $\{a_1, a_2, \dots, a_n\}$ and $\{b_1, b_2, \dots, b_n\}$ are two n-tons in $(X, \Im_1, \Im_2)$. Since $(X, \Im_1, \Im_2)$ is fuzzy n-s-homogeneous, then there a fuzzy s-homeomorphism $h: (X, \Im_1, \Im_2) \to (X, \Im_1, \Im_2)$ such that $h(a_i) = b_i$ for every $i = 1,2, \dots, n$. Therefore, we have $h(a_i) = b_i$ for every $i = 1,2, \dots, k$. This ends the proof.

In Theorem 2.15, the cardinality is condition cannot be dropped as the following example shows:

Example 2.16. Let $X = \{1,2\}$ and $\tau = \{\emptyset, X, \{1\}\}$. Then $(X, \omega(\tau), \omega(\tau))$ is fuzzy 3-s-homogeneous but not fuzzy 2-s-homogeneous.

The following example shows that Theorem 2.15 is not true for fuzzy weak n-s-homogeneity:
Example 2.17. Let $X = \{1,2\}, \tau_1 = \{\emptyset, X, \{1\}\}$ and $\tau_2 = \{\emptyset, X\}$. Then $(X, \omega(\tau_1), \omega(\tau_2))$ is fuzzy weakly 2-s-homogeneous but not fuzzy weakly 1-s-homogeneous.

3. CHARACTERIZATIONS

Definition 3.1. [1] A collection of fuzzy points $A_f = \{p_1, p_2, \ldots, p_n\}$ in a set X is said to be fuzzy n-tons(f-n-tons) iffp_i and p_j are distinct for all $i, j \in \{1,2,\ldots,n\}$ with $i \neq j$.

Definition 3.2. [1] Let Ƒ be a collection of fuzzy sets in a set X and f be a map from X to Y. Define $f(Ƒ)$ by $f(Ƒ) = \{f(\lambda): \lambda \in Ƒ\}$.

Definition 3.3. [1] Let A_f be a collection of fuzzy (crisp) points in a set X and $t \in (0,1]$. Then $C(A_f, t)$ will denote $C(A_f, t) = |\{p \in A_f: p(x_p) = t\}|$.

Proposition 3.4. [1] Let $h: X \to Y$ be a bijective map and let p and q be two fuzzy points in X. Then $h(p) = q$ iff $p(x_p) = q(x_q)$ and $h(x_p) = h(x_q)$.
Proposition 3.5. [1] Let $h: X \to Y$ be a bijective map and let $\{\lambda_\alpha: \alpha \in \Delta\}$ be a collection of fuzzy sets in X. Then $h(\bigcup_{\alpha\in\Delta} \lambda_\alpha) = \bigcup_{\alpha\in\Delta} h(\lambda_\alpha)$.

Proposition 3.6. [1] Let X be a non-empty set and $A_f = \{p_\alpha: \alpha \in \Delta\}, B_f = \{q_\alpha : \alpha \in \Omega\}$ be two collections of mutually distinct fuzzy (crisp) points in X. Then $\bigcup_{\alpha\in\Delta} p_\alpha = \bigcup_{\alpha\in\Omega} q_\alpha$iff $A_f = B_f$.

Theorem 3.7. Let $(X, \Im_1, \Im_2)$ be a fuzzy bitopological space.Then the following are equivalent:
i. $(X, \Im_1, \Im_2)$ is fuzzy n-s-homogeneous.
ii. For any two f-n-tons $A_f = \{p_1, p_2, \ldots, p_n\}$ and $B_f = \{q_1, q_2, \ldots, q_n\}$ of X with $p_i(x_{p_i}) = q_i(x_{q_i})$ for every $i = 1,2,\ldots,n$, there exists a fuzzy s-homeomorphism $h: (X, \Im_1, \Im_2) \to (X, \Im_1, \Im_2)$ such that $h(p_i) = q_i$ for every $i = 1,2,\ldots,n$.
iii. For any two f-n-tons $A_f = \{p_1, p_2, \ldots, p_n\}$ and $B_f = \{q_1, q_2, \ldots, q_n\}$of X with $C(A_f, t) = C(B_f, t)$ for all $t \in (0,1]$, there exists a fuzzy s-homeomorphism $h: (X, \Im_1, \Im_2) \to (X, \Im_1, \Im_2)$ such that $h(A_f) = B_f$.
iv. For any two f-n-tons $A_f = \{p_1, p_2, \ldots, p_n\}$ and $B_f = \{q_1, q_2, \ldots, q_n\}$ of X with $C(A_f, t) = C(B_f, t)$ for all $t \in (0,1]$, there exists a fuzzy s-homeomorphism $h: (X, \Im_1, \Im_2) \to (X, \Im_1, \Im_2)$ such that $h(\bigcup_{i=1}^{n} p_i) = \bigcup_{i=1}^{n} q_i$.

Proof. (i) ⇒ (ii): Let $A_f = \{p_1, p_2, \ldots, p_n\}$ and $B_f = \{q_1, q_2, \ldots, q_n\}$ of X with $p_i(x_{p_i}) = q_i(x_{q_i})$ for every $i = 1,2,\ldots,n$. Then $A = \{x_{p_1}, x_{p_2}, \ldots, x_{p_n}\}$ and $B == \{x_{q_1}, x_{q_2}, \ldots, x_{q_n}\}$ are two n-tons. So by (i), there exists a fuzzy s-homeomorphism $h: (X, \Im_1, \Im_2) \to (X, \Im_1, \Im_2)$ such that $h(x_{p_i}\}) = x_{q_i}$ for every $i = 1,2,\ldots,n$. Since by assumption $p_i(x_{p_i}) = q_i(x_{q_i})$ for every $i = 1,2,\ldots,n$, then by Proposition 3.4, $h(p_i) = q_i$ for every $i = 1,2,\ldots,n$.
(ii) ⇒ (iii): Let $A_f = \{p_1, p_2, \ldots, p_n\}$ and $B_f = \{q_1, q_2, \ldots, q_n\}$ be any two f-n-tons of X with $C(A_f, t) = C(B_f, t)$ for all $t \in (0,1]$. Since $C(A_f, t) = C(B_f, t)$ for all $t \in (0,1]$, then we can rewrite A_f and B_f as $A_f = \{p_{11}, p_{21}, \ldots, p_{n1}\}$ and $B_f = \{q_{11}, q_{21}, \ldots, q_{n1}\}$ such that $p_{i1}(x_{p_{i1}}) = q_{i1}(x_{q_{i1}})$ for every $i = 1,2,\ldots,n$. By (ii), there exists a fuzzy s-homeomorphism $h: (X, \Im_1, \Im_2) \to (X, \Im_1, \Im_2)$ such that $h(p_{i1}) = q_{i1}$ for every $i = 1,2,\ldots,n$. Therefore, $h(A_f) = B_f$.
(iii) ⇒ (iv): Let $A_f = \{p_1, p_2, \ldots, p_n\}$ and $B_f = \{q_1, q_2, \ldots, q_n\}$ be any two f-n-tons of X with $C(A_f, t) = C(B_f, t)$ for all $t \in (0,1]$. Then by (iii), there exists a fuzzy s-homeomorphism $h: (X, \Im_1, \Im_2) \to (X, \Im_1, \Im_2)$ such that $h(A_f) = B_f$ and hence $h(\bigcup_{p\in A_f} p) = (\bigcup_{q\in B_f} q)$. Therefore, by Proposition 3.5 it follows that $h(\bigcup_{i=1}^{n} p_i) = \bigcup_{i=1}^{n} q_i$.
(iv) ⇒ (i): Let $A = \{x_1, x_2, \ldots, x_n\}$ and $B = \{y_1, y_2, \ldots, y_n\}$ be any two n-tons in X. Define $A_f = \{p_1, p_2, \ldots, p_n\}$ and $B_f = \{q_1, q_2, \ldots, q_n\}$ by $x_{p_i} = x_i$, $x_{q_i} = y_i$and $p_i(x_{p_i}) = q_i(x_{q_i}) = (1/(1+i))$ for every $i = 1,2,\ldots,n$. Then A_f and B_f are two f-n-tons of X with $C(A_f, t) = C(B_f, t)$ for all $t \in (0,1]$. By (iv), there exists a fuzzy s-homeomorphism $h: (X, \Im_1, \Im_2) \to (X, \Im_1, \Im_2)$ such that $h(\bigcup_{i=1}^{n} p_i) = \bigcup_{i=1}^{n} q_i$. Since h is bijective, then by Proposition 3.5 $\bigcup_{i=1}^{n} h(p_i) = \bigcup_{i=1}^{n} q_i$. Since h is one to one $\{h(p_i): i = 1,2,\ldots,n\}$ is a set of mutually distinct fuzzy (crisp) points. Therefore, by Proposition 3.6 it follows that $\{h(p_i): i = 1,2,\ldots,n\} = \{q_i: i = 1,2,\ldots,n\}$

and thus $h(p_i) = q_i$ for every $i = 1,2,\ldots,n$. This implies that $h(x_i) = y_i$ for every $i = 1,2,\ldots,n$. Therefore, $(X, \Im_1, \Im_2)$ is fuzzy n-s-homogeneous.

Corollary 3.8. A fuzzy bitopological space $(X, \Im_1, \Im_2)$ is fuzzy s-homogeneous iff for any two fuzzy points p, q in the set X with $p(x_p) = q(x_q)$, there exists a fuzzy s-homeomorphism $h: (X, \Im_1, \Im_2) \to (X, \Im_1, \Im_2)$ such that $h(p) = q$.

Theorem 3.9. If $(X, \Im_1)$ and $(X, \Im_2)$ are two fuzzy topological spaces one of which is fuzzy n-homogeneous and the other consists of constant fuzzy sets, then $(X, \Im_1, \Im_2)$ is fuzzy n-s-homogeneous.
Proof. Let $(X, \Im_1)$ be fuzzy n-homogeneous and $(X, \Im_2)$ consists of constant fuzzy sets. Let $A = \{a_1, a_2, \ldots, a_n\}, B = \{b_1, b_2, \ldots, b_n\}$ be two n-tons in X. Since $(X, \Im_1)$ is fuzzy n-homogeneous, there exists a fuzzy homeomorphism $h: (X, \Im_1) \to (X, \Im_1)$ such that $h(a_i) = b_i$ for every $i = 1,2,\ldots,n$. Let $\lambda \in \Im_2$. Then $h^{-1}(\lambda) = \lambda \in \Im_2$. Thus, $h: (X, \Im_2) \to (X, \Im_2)$ is fuzzy continuous. Similarly, $h^{-1}: (X, \Im_2) \to (X, \Im_2)$ is fuzzy continuous. Since h is bijective, then $h: (X, \Im_2) \to (X, \Im_2)$ is a fuzzy homeomorphism. Therefore, we have $h: (X, \Im_1, \Im_2) \to (X, \Im_1, \Im_2)$ is a fuzzy s-homeomorphism with $h(a_i) = b_i$ for every $i = 1,2,\ldots,n$. It follows that $(X, \Im_1, \Im_2)$ is fuzzy n-s-homogeneous.
Corollary 3.10. If $(X, \Im_1)$ and $(X, \Im_2)$ are two fuzzy topological spaces one of which is fuzzy homogeneous and the other consists of constant fuzzy sets, then $(X, \Im_1, \Im_2)$ is fuzzy s-homogeneous.

REFERENCES

[1] A. Fora and S. Al Ghour, "Homogeneity in fuzzy spaces," *Questions and Answers in General Topology*, vol. 19, pp.159-164, 2001.
[2] J. C. Kelly, "Bitopological spaces, " *Proceedings of London Mathematical Society*, vol. 13, pp. 71-89, 1963.
[3] K. Khashan, "Homogeneous bitopological spaces," Master's Thesis, Department of Mathematics, Yarmouk University, Irbid, Jordan,1996.
[4] A. Kandil, "Biproximities and fuzzy bitopological spaces," *Simon Stevin*, vol. 63, pp. 45-66, 1989.
[5] S. Al Ghour, "Homogeneity in fuzzy spaces and their induced spaces," *Questions and Answers in General Topology*, vol. 21, pp. 185-195, 2003.
[6] S. Al Ghour, "SLH fuzzy spaces," *African Diaspora Journal of Mathematics*, vol. 2, pp. 61-67, 2004.
[7] S. Al Ghour and A. Fora, "Minimality and homogeneity in fuzzy spaces," *Journal of Fuzzy Mathematics,* vol. 12, pp. 725-737, 2004.
[8] S. Al Ghour, "Local homogeneity in fuzzy topological spaces," *International Journal of Mathematics and Mathematical Sciences*, vol. 14, 2006.
[9] S. Al Ghour, "Some Generalizations of Minimal Fuzzy Open Sets," *Acta Mathematica UniversitatisComenianae*, vol. 75, pp. 107-117, 2006.
[10] S. Al Ghour and K. Al-Zoubi, "On some ordinary and fuzzy homogeneity types," *Acta Mathematica UniversitatisComenianae*, vol. 77, pp. 199-208, 2008.
[11] S. Al Ghour and A. Fora, "On CDH fuzzy spaces," *Journal of Intelligent & Fuzzy Systems*, vol. 30, pp. 935-941, 2016.
[12] S. Al Ghour and A. Azaizeh, "Fuzzy homogeneous bitopological Spaces," *International Journal of Electrical and Computer Engineering,* vol. 8, pp. 2088-8708, 2018.
[13] S. Al Ghour, "Homogeneous components of a CDH fuzzy space," *International Journal of Electrical and Computer Engineering,* vol. 9, pp. 2691-2694, 2019.
[14] S. Al Ghour, "Densely homogeneous fuzzy spaces," *International Journal of Electrical and Computer Engineering,* vol. 9, pp. 3256-3261, 2019.
[15] A. Fora, "Separation axioms, subspaces and product spaces in fuzzy topology," *The Arab Gulf Journal of Scientific Research*, vol. 8, pp. 1-16, 1990.