

## A hybrid bacterial foraging and modified particle swarm optimization for model order reduction

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### ABSTRACT

This paper study the model reduction procedures used for the reduction of large-scale dynamic models into a smaller one through some sort of differential and algebraic equations. A confirmed relevance between these two models exists, and it shows same characteristics under study. These reduction procedures are generally utilized for mitigating computational complexity, facilitating system analysis, and thence reducing time and costs. This paper comes out with a study showing the impact of the consolidation between the Bacterial-Foraging (BF) and Modified particle swarm optimization (MPSO) for the reduced order model (ROM). The proposed hybrid algorithm (BF-MPSO) is comprehensively compared with the BF and MPSO algorithms; a comparison is also made with selected existing techniques.

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## 1. INTRODUCTION

Scientists and engineers are often challenged with the analysis, design, and synthesis of real-life problems due to the regularly increasing size of system models showing up by the present technology and societal and environmental processes. In such studies, the initial step is the refinement of a mathematical model which can be an alternative to the real problem. Modelling and controlling of complex-dynamic-systems (CDS) is the farthest essential areas of study in many engineering fields and sciences [1]-[3]. In various cases and engineering applications, the dynamic system model under the study can be complicated to some extent and pose challenges when used. Where there is high and complex mathematical model show exactly the problem at hand, but it is not suitable for the numerical simulation. To overcoming this problem, model order reduction (MOR) approach is used, which aims to convert a system model from higher order to a lower order to facilitate the computational complexity of such problem and has lately been intensively sophisticated for use with piecemeal more CDS inclusive both optimization and control [4]-[5].

Various popular MOR methods for linear and nonlinear large-scale dynamical systems, are available in the researches for MOR [6]-[8]. The need for new innovative and advanced approaches is justified. Theories of evolutionary computation are proposed [9] and mathematically formulated as a new way to model and control of CDS [10]-[12].

All the above-mentioned methods didn't take into account the merge between the BF and PSO for the reduced order model. This paper comes out with a study showing the impact of the consolidation between the BF and PSO for the reduced order model. As well as, the results are counterweight with the original BF and with the proposed MPSO.

## 2. PROBLEM FORMULATION

A straight time-invariant single-input single-output framework can be described by the following function:

$$G_n(s) = \frac{N_n(s)}{D_n(s)} = \frac{a_0 + a_1s + a_2s^2 + \dots + a_{n-1}s^{n-1}}{b_0 + b_1s + b_2s^2 + \dots + b_n s^n} \quad (1)$$

Where

$$\left. \begin{array}{l} a_i : 0 \leq i \leq n-1 \\ b_i : 0 \leq i \leq n \end{array} \right\} \text{ are known as scalar constants.}$$

To get the  $r^{\text{th}}$  ( $r < n$ ) ROM ( $G_r(s)$ ) which is represented in the form:

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{e_0 + e_1s + e_2s^2 + \dots + e_{r-1}s^{r-1}}{f_0 + f_1s + f_2s^2 + \dots + b_r s^r} \quad (2)$$

Where  $e_i : 0 \leq i \leq r-1$ ,  $f_i : 0 \leq i \leq r$ . The integral-square-error (ISE) between  $G_r(s)$  and  $G_n(s)$  models is calculated to gauge the quality of the ROM. ISE is known by:

$$ISE = \sum_{i=0}^n [y(t_i) - y_r(t_i)]^2 \quad (3)$$

Where  $y(t_i)$  and  $y_r(t_i)$  are the unit step responses of the original and ROM, correspondingly. The transfer function matrix of the multi\_input multi\_output system can be symbolized in the formula:

$$G_n(s) = \frac{1}{D_n(s)} \begin{bmatrix} a_{11}(s) & a_{12}(s) & \dots & a_{1p}(s) \\ a_{21}(s) & a_{22}(s) & \dots & a_{2p}(s) \\ \dots & \dots & \dots & \dots \\ a_{m1}(s) & a_{m2}(s) & \dots & a_{mp}(s) \end{bmatrix} \quad (4)$$

where  $p = \text{no. of input}$  and  $m = \text{no. of output}$ .

$$g_{ij}(s) = \frac{a_{ij}(s)}{D_n(s)} \quad (5)$$

where  $i = 1, 2, \dots, p$ ,  $j = 1, 2, \dots, m$

To get ( $r^{\text{th}} < n$ ) ROM represented in the form of:

$$G_r(s) = \frac{1}{D_r(s)} \begin{bmatrix} e_{11}(s) & e_{12}(s) & \dots & e_{1p}(s) \\ e_{21}(s) & e_{22}(s) & \dots & e_{2p}(s) \\ \dots & \dots & \dots & \dots \\ e_{m1}(s) & e_{m2}(s) & \dots & e_{mp}(s) \end{bmatrix} \quad (6)$$

The overall form of  $R_{ij}(s)$  from  $G_r(s)$  is reserved as:

$$R_{ij}(s) = \frac{e_{ij}(s)}{D_r(s)} \quad (7)$$

To obtain the  $r^{th}$  order reduced transfer matrix  $G_r(s)$ , the factors of the communal denominator  $D_r(s)$  and the numerator  $e_{ij}(s)$  of the  $G_r(s)$  are designed by decreasing the ISE between the  $g_{ij}(s)$  and  $R_{ij}(s)$  order models.

**2.1. BF algorithm**

Recently, BF has become increasingly suitable as a global optimization technique in science and engineering subjects [13], [14]. The basic structural details of the BF algorithm are depicted in Figure 1.

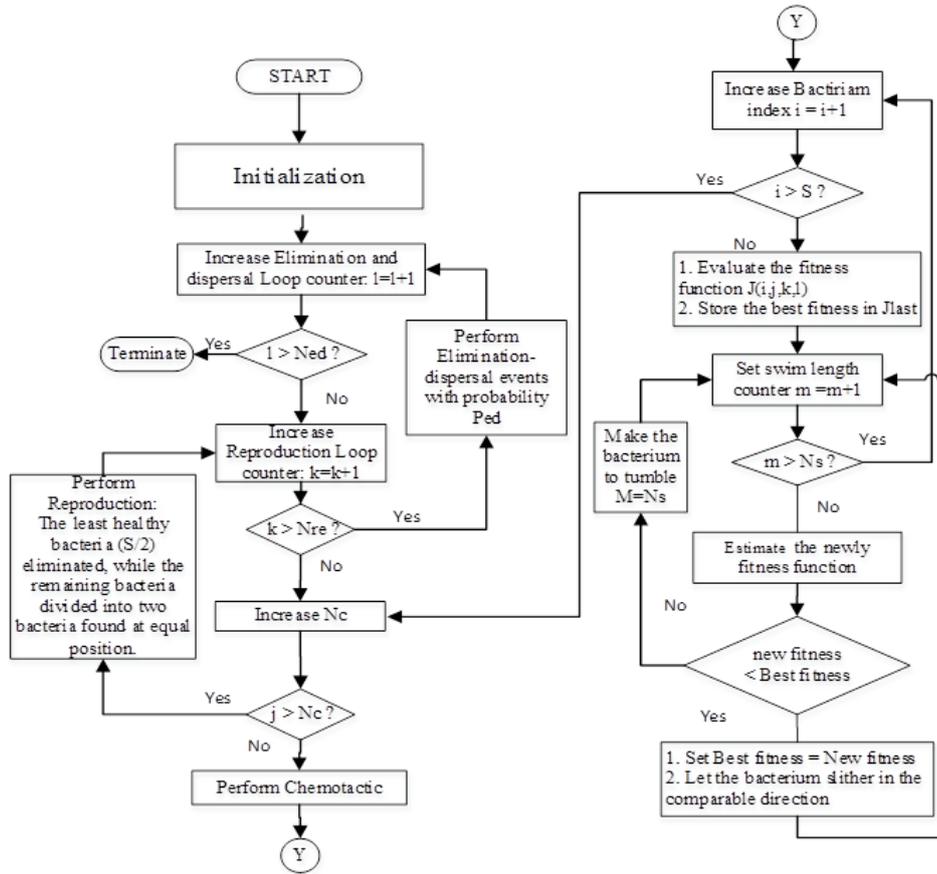


Figure 1. Basic structural of the BF algorithm

**2.2. MPSO algorithm**

PSO has seen many changes since it's introduced by Eberhart and Kennedy [15]. Whenever researchers learn about this technology, new versions are discovered, incorporated into new applications. PSO is a populace grounded streamlining tool in which the framework is set through various arbitrary-possibility elements famous as particles. Every particle takes a position ( $X_t^i$ ) and speed ( $V_t^i$ ), which are refreshed by the accompanying equations:

$$V_{t+1}^i = w \cdot V_t^i + c_1 r_1 (P_{Lbest}^i - X_t^i) + c_2 r_2 (P_{Gbest}^i - X_t^i) \tag{8}$$

$$X_{t+1}^i = X_t^i + V_{t+1}^i \tag{9}$$

Where  $w =$  is the inertia\_weight factor,

$c_1-c_2 = \text{the cognitive\_social acceleration factors}, r_1 - r_2 = \text{randomlly } [0, 1],$

$P_{Lbest}^i = \text{the best result achieved by } i \text{ particle}$

$P_{Gbest}^i = \text{the best result achieved by all particle}$

Different approaches are beneficial in texts for adjusting [16]-[19]. The proposed MPSO described as follows:

Step 1: Identify the factors of PSO.

Step 2: Create an initial populace with M particles.

Step 3: estimate the fitness of each particle.

Step 4: Update according to one of the strategies proposed by us in [19].

Step 5: Update  $X_i^i$  and  $V_i^i$  for each particle by using Equations (8) and (9).

Step 6: Check the termination conditions.

### 2.3. BF-MPSO algorithm

To improve the performance; recent methods combine the PSO and BF algorithms together [20]-[22]. Here we propose a hybrid algorithm (combining the features of BF and the proposed MPSO (BF-MPSO) to acquire a ROM as shown in Figure 2.

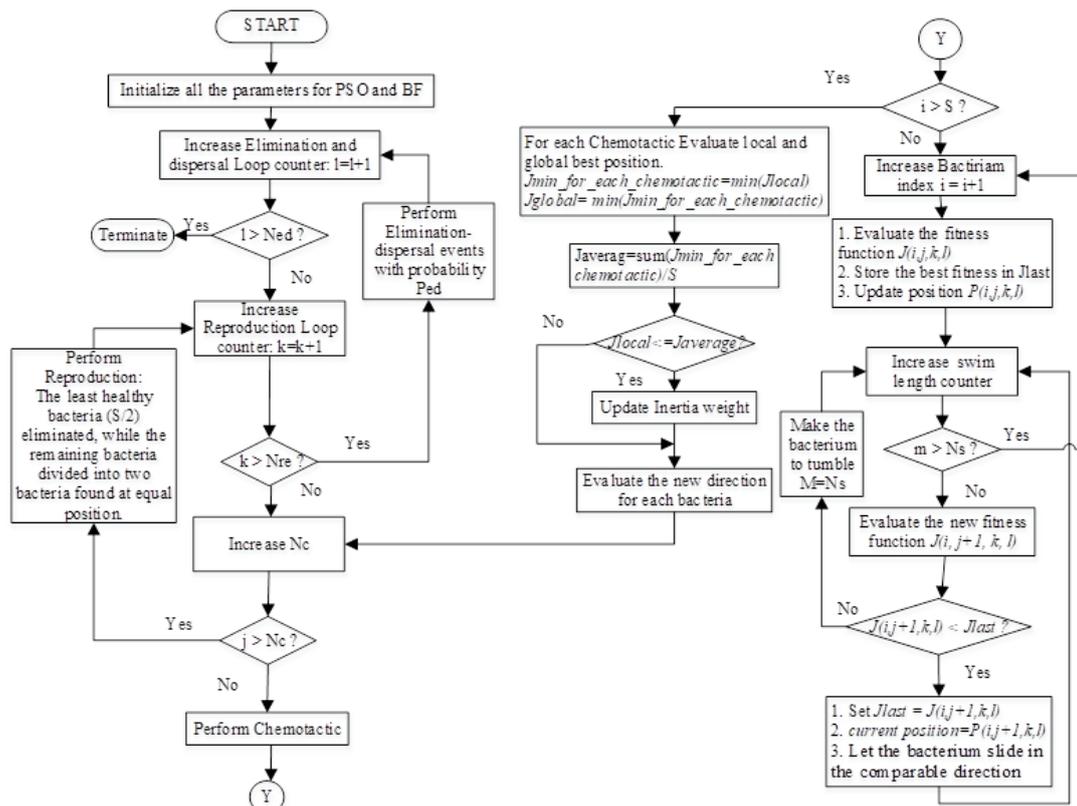


Figure 2. Basic structural of the proposed BF-MPSO algorithm

### 3. RESULTS AND ANALYSIS

In this segment, the proposed advancement strategies are attempting to limit the ISE as indicated in (3). The proposed techniques were actualized on a Pentium IV 3-GHz PC in the MATLAB 2010 condition. The exhibitions of the BF, MPSO, and BF-MPSO calculations were assessed utilizing consistent estimations of the underlying elements proclaimed in Table 1.

Table 1. Factors Used for Modified PSO, BF and BF\_MPSO Algorithms

Parameters	Value
Swarm size	50
Maximum-number of generations	50
Cognitive-social acceleration factors ( $c_1, c_2$ )	1.2, 0.8
Inertia-weight ( $w_{\min} - w_{\max}$ )	0.4–0.9
Number-of-bacteria ( $S$ )	20
Chemotactic-steps ( $N_c$ )	200
Maximum swim length ( $N_s$ )	2
Dispersal-number-of-bacteria ( $N_{ed}$ )	2
Reproduction number ( $N_{re}$ )	2
Dispersal probability ( $P_{ed}$ )	0.05

Example 1: Consider the 4th order system given in [4]:

$$G_4(s) = \frac{14s^3 + 249s^2 + 900s + 1200}{s^4 + 19s^3 + 102s^2 + 190s + 120}$$

The step-responses of the full and ROMs are displayed in Figure 3(a). Likewise, to assess the feature of the model in the frequency space Figure 3(b) shows the frequency-amplitude attributes of the full and ROMs. Keeping in mind the ISE and mean square error were computed, to compare the proposed method with different ROMs, as appeared in Table 2. A comparison for the conjunction of the fitness function with the number of generations for the two MPSO schemes is presented in Figure 4. Likewise, Figure 5 displays the variant of the minimum fitness with the number of chemotactic steps.

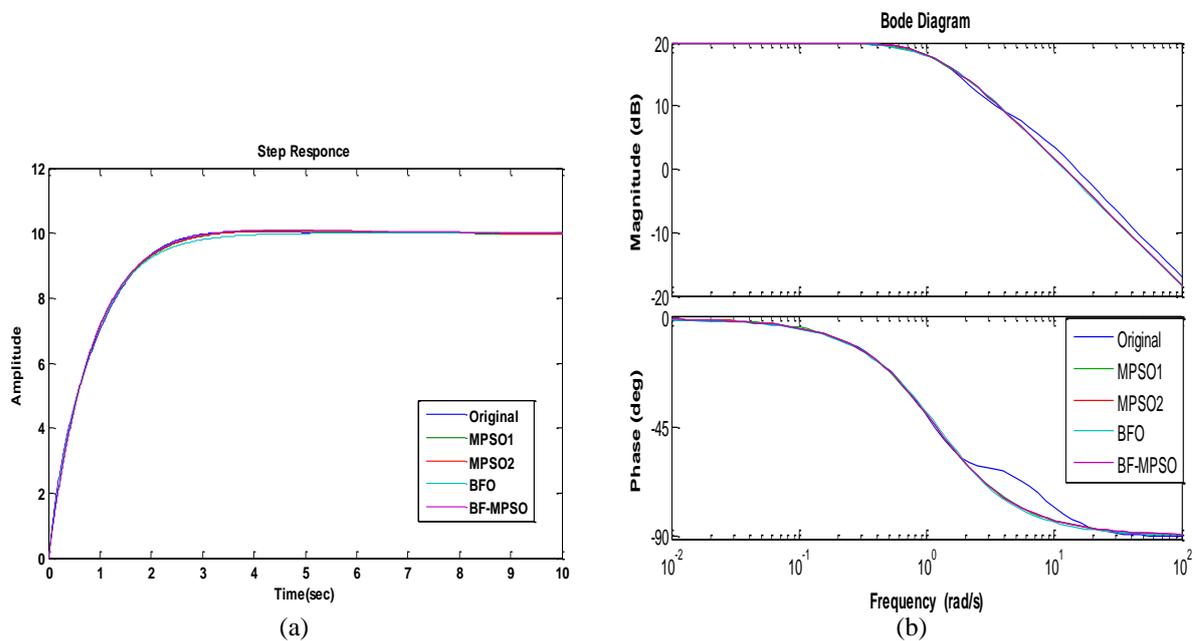


Figure 3. Original and reduced models responses for example 1

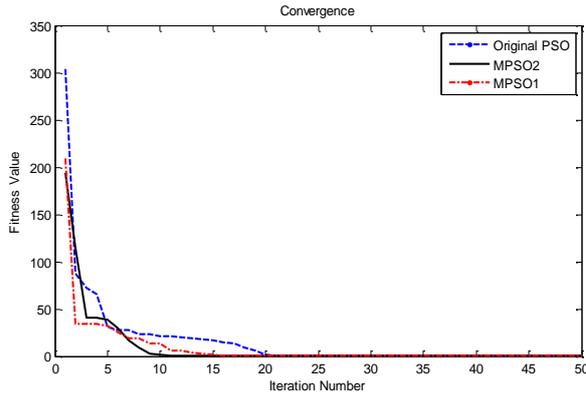


Figure 4. Evolution processes of the MPSO strategies for example 1

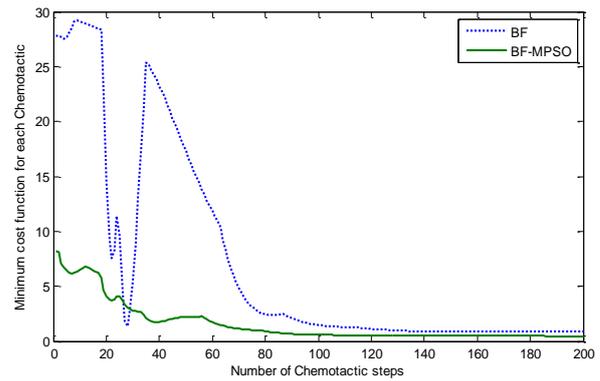


Figure 5. Evolution processes of BF and BF-MPSO methods for example 1

Example 2: Consider the 8<sup>th</sup> system model presented in [5]:

$$\frac{40320+185760s+222088s^2+122664s^3+36380s^4+5982s^5+514s^6+18s^7}{40320+109584s+118124s^2+67284s^3+22449s^4+4536s^5+546s^6+36s^7+s^8}$$

The step responses of the full and ROMs are presented in Figure 6(a). Also, Figure 6(b) displays the frequency-amplitude attributes of the full and ROMs. Also, the ISE and mean square error were computed, to compare the proposed method with different ROMs, as appeared in Table 3. A comparison for the conjunction of the fitness function with the number of generations for the two MPSO schemes is presented in Figure 7. Figure 8 displays a plot of the variation of the minimum fitness with the number of chemotactic steps.

Table 2. Evaluation of Error Index Values for Example 1

Method	ROM	RMS-Error	ISE
Proposed MPSO1	$\frac{154.6s + 46.31}{12.82s^2 + 18.74s + 4.66}$	0.0631	0.4020
Proposed MPSO2	$\frac{836.5s + 399.1}{69.39s^2 + 112.9s + 40.03}$	0.0615	0.3820
Proposed BF	$\frac{148.3s + 466.4}{12.5s^2 + 51.73s + 46.64}$	0.0917	0.8504
Proposed BFMPSO	$\frac{53.46s + 25.5}{4.421s^2 + 7.196s + 2.55}$	0.0677	0.4638
Ref. [4]	$\frac{30s + 40}{3s^2 + 6s + 4}$	0.1428	2.0609
Ref. [22]	$\frac{12.0166s + 12.0226}{1.016s^2 + 2.1155s + 1.202}$	0.0665	0.4472
Ref. [7]	$\frac{2.8863s + 51.4892}{s^2 + 4.15032s + 5.1489}$	0.3146	9.9998
Ref. [8]	$\frac{14s + 12.53}{s^2 + 2.138s + 1.253}$	0.3016	9.1872
Ref. [23]	$\frac{70.588}{s^2 + 5.2941s + 7.0588}$	0.3301	11.003

Table 3. Evaluation of Error Index Values for Example 2

Method	ROM	RMS Error	ISE
Proposed MPSO1	$\frac{340.4s + 104.3}{19.97s^2 + 137.7s + 104.3}$	0.0095	0.0092
Proposed MPSO2	$\frac{682s + 208.8}{40.13s^2 + 277.4s + 209.8}$	0.0075	0.0057
Proposed BF	$\frac{40.89s + 14.96}{1.908s^2 + 17.15s + 14.65}$	0.0569	0.3281
Proposed BFMPSO	$\frac{90.75s + 28.23}{5.316s^2 + 36.5s + 28.12}$	0.0086	0.0076
Ref. [4]	$\frac{1.99s + 0.43184}{s^2 + 1.17368s + 0.43184}$	0.4371	19.296
Ref. [5]	$\frac{6.7786s + 2}{s^2 + 3s + 2}$	0.1651	2.7825
Ref. [22]	$\frac{88.04s + 26.48}{4.021s^2 + 28.59s + 2.648}$	4.6467	$2.37 \times 10^2$

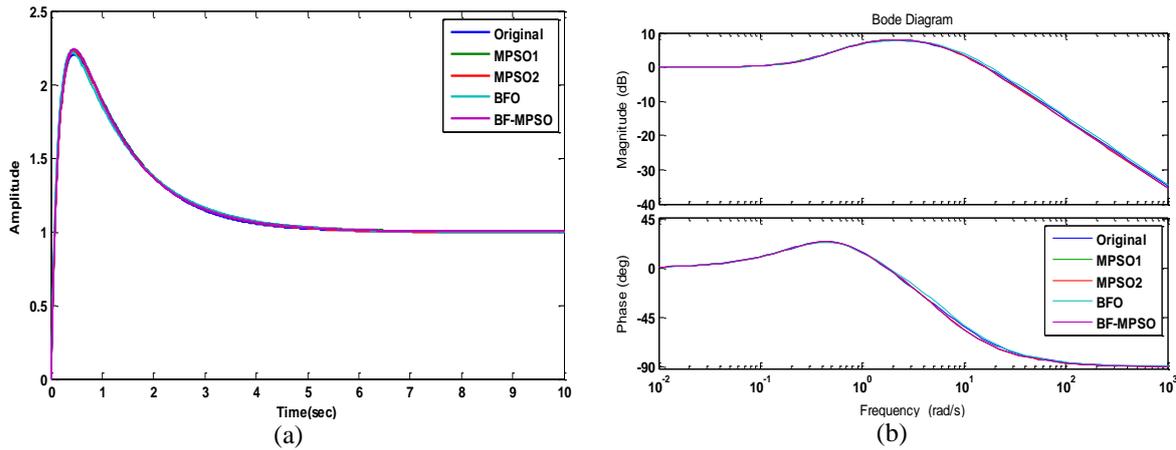


Figure 6. Original and reduced model responses for example 2: (a) step responses; (b) frequency responses

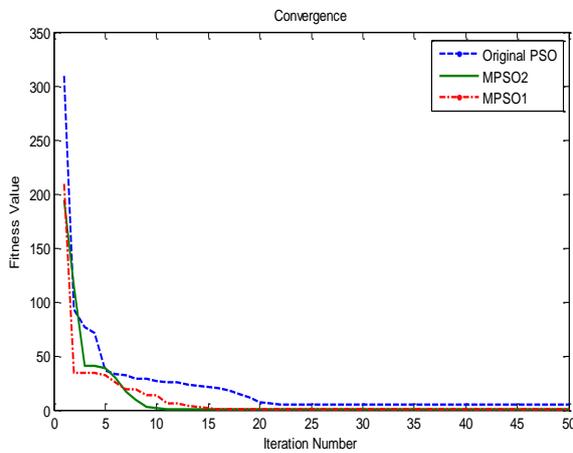


Figure 7. Evolution processes of two MPSO methods for Example 2

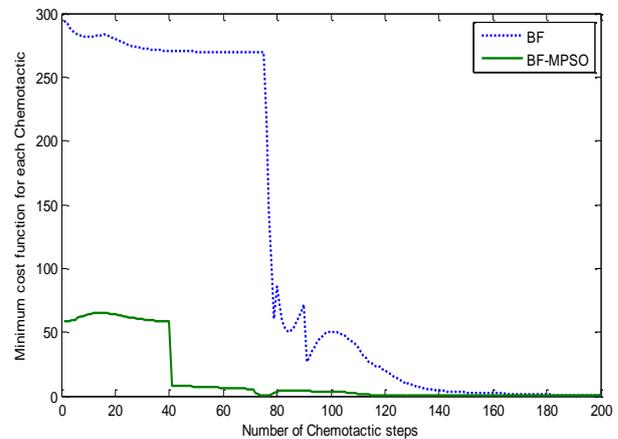


Figure 8. Evolution processes of BF and BF-MPSO methods for example 2

Example 3: A system model specified in [9], which is 6th order 2-input 2-output:

$$G_6(s) = \begin{bmatrix} \frac{2(5+s)}{(1+s)(10+s)} & \frac{(4+s)}{(2+s)(5+s)} \\ \frac{(10+s)}{(1+s)(20+s)} & \frac{(6+s)}{(2+s)(3+s)} \end{bmatrix}, G_6(s) = \frac{1}{D(s)} \begin{bmatrix} a_{11}(s) & a_{12}(s) \\ a_{21}(s) & a_{22}(s) \end{bmatrix}$$

Where:

$$D(s) = (1+s)(2+s)(3+s)(5+s)(10+s)(20+s) = 6000 + 13100s + 10060s^2 + 3491s^3 + 571s^4 + 41s^5 + s^6$$

$$a_{11}(s) = 6000 + 7700s + 3610s^2 + 762s^3 + 70s^4 + 2s^5$$

$$a_{12}(s) = 2400 + 4160s + 2182s^2 + 459s^3 + 38s^4 + s^5$$

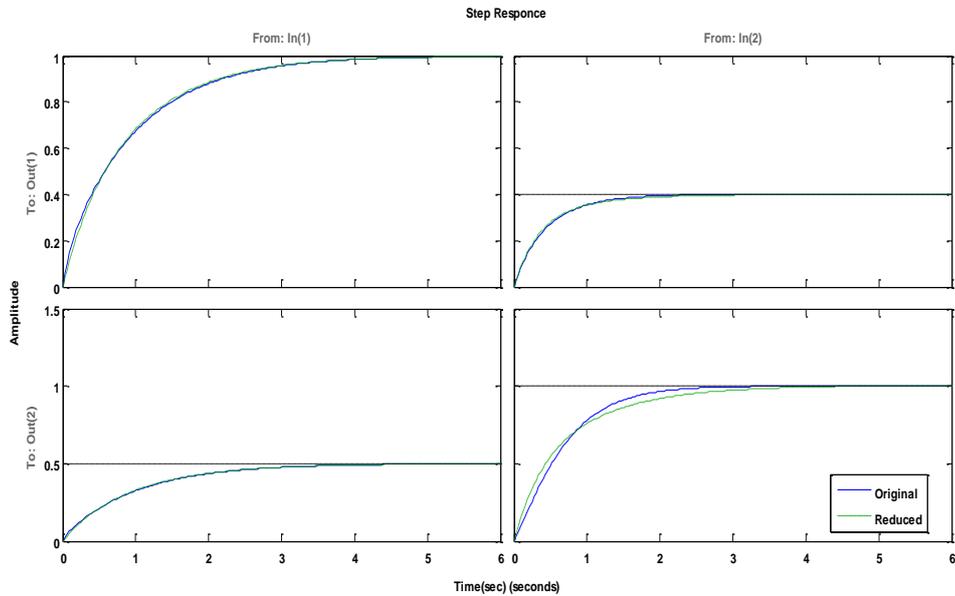
$$a_{21}(s) = 3000 + 3700s + 1650s^2 + 331s^3 + 30s^4 + s^5$$

$$a_{22}(s) = 6000 + 7700s + 3610s^2 + 601s^3 + 42s^4 + s^5$$

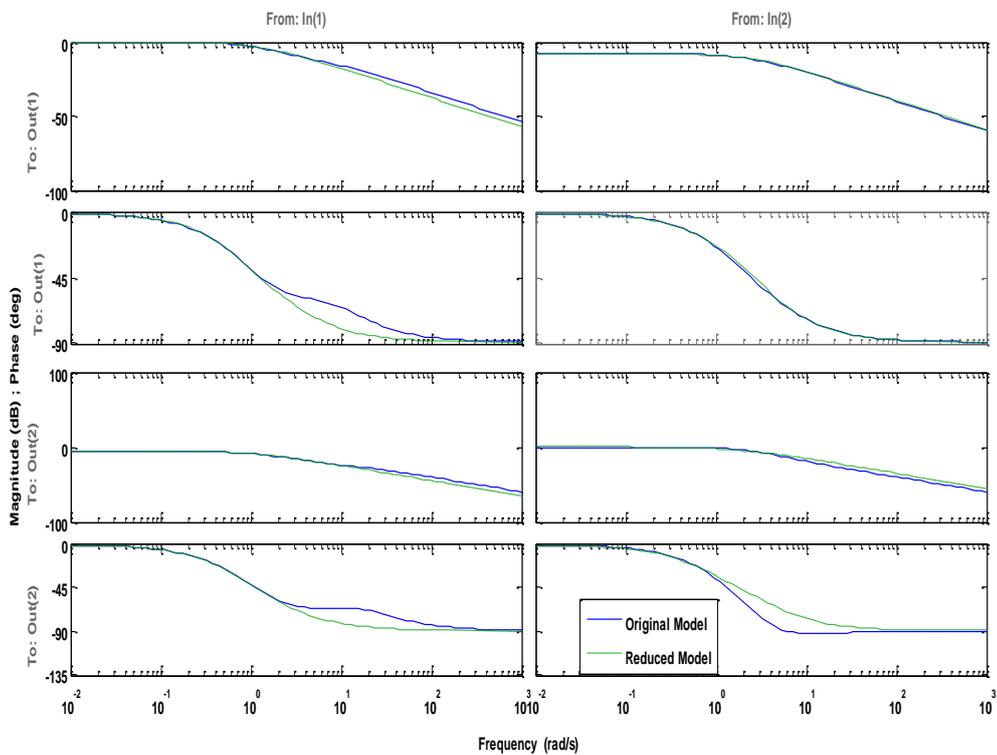
By utilizing the second procedure of the MPSO calculation, the ROM system  $G_2(s)$  was:

$$G_2(s) = \frac{\begin{bmatrix} (1.317s + 2.998) & (1.031s + 1.202) \\ (0.5782s + 1.499) & (1.781s + 3.014) \end{bmatrix}}{(s+3)(s+1)}$$

The step responses of the full and ROMs are displayed in Figure 9(a). Likewise, Figure 9(b) displays the frequency-amplitude features of the full and ROMs. To compare the proposed method with different ROMs, the ISE and mean square error were computed, as shown in Table 4.



(a)



(b)

Figure 9. Original and reduced model responses for example 3

Table 4. Evaluation of Error Index Values for Example

Method	Reduced Model	R11	R12	R21	R22
Proposed MPSO2	$r_{11} = 1.317s + 2.998$				
	$r_{12} = 1.031s + 1.202$	<i>RMS Error</i> =	<i>RMS Error</i> =	<i>RMS Error</i> =	<i>RMS Error</i> =
	$r_{21} = 0.5782s + 1.499$	0.004504	0.001927	0.001475	0.017657
	$r_{22} = 1.781s + 3.014$	ISE =	ISE =	ISE =	ISE =
	$D_r = (s^2 + 4s + 3)$	0.004077	$7.465 \times 10^{-4}$	$4. \times 10^{-4}$	0.062667
Ref. [9]	$r_{11} = 1.328s + 3.104$				
	$r_{12} = 1.034s + 1.241$	<i>RMS Error</i> =	<i>RMS Error</i> =	<i>RMS Error</i> =	<i>RMS Error</i> =
	$r_{21} = 0.5818s + 1.552$	0.004332	0.002527	0.001423	0.018692
	$r_{22} = 1.824s + 3.104$	ISE =	ISE =	ISE =	ISE =
	$D_r = (s^2 + 4.109s + 3.104)$	0.003772	0.001283	$4.068 \times 10^{-4}$	0.070228
Ref. [23]	$r_{11} = -1.541s + 8.946$				
	$r_{12} = 0.9941s + 3.579$	<i>RMS Error</i> =	<i>RMS Error</i> =	<i>RMS Error</i> =	<i>RMS Error</i> =
	$r_{21} = -0.994s + 4.473$	0.047831	$6.634 \times 10^{-5}$	0.025207	0.004436
	$r_{22} = 0.5468s + 8.946$	ISE =	ISE =	ISE = 0.127719	ISE =
	$D_r = (s^2 + 6.511s + 8.946)$	0.459849	$8.845 \times 10^{-7}$		0.003955
Ref. [24]	$r_{11} = 1.079s + 0.7091$				
	$r_{12} = 0.9031s + 0.2837$	<i>RMS Error</i> =	<i>RMS Error</i> =	<i>RMS Error</i> =	<i>RMS Error</i> =
	$r_{21} = 0.1955s + 0.3546$	0.027311	0.059892	0.039632	0.069859
	$r_{22} = 0.6895s + 80.7091$	ISE =	ISE =	ISE =	ISE =
	$D_r = (s^2 + 1.548s + 0.7091)$	0.149925	0.720996	0.315713	0.980948

#### 4. CONCLUSION

In this paper, we presented a comparative study of three algorithms for ROM optimization problems, namely: MPSO, BFO, and MPSO\_BF. From Figures 3, 6, and 9, unmistakably observed that the suggested techniques keep up steady state value and stability in the ROMs, while Figures 4 and 7 delineate that the convergence speed of the second MPSO strategy is the fastest among the two strategies. Figures 5 and 8 illustrate that the speed of convergence and additionally the precision of the proposed BF-MPSO is better than that of BF. In addition, these algorithms can use a smaller number of chemotactic steps, which makes them faster. At long last, the information showed in Tables 2, 3, and 4 exhibits that the proposed calculation performs well in contrast with other accessible procedures.

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