New robust bounded control for uncertain nonlinear system using mixed backstepping and lyapunov redesign

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ABSTRACT

This paper presents a new robust bounded control law to stabilize uncertain nonlinear system with time varying disturbance. The design idea comes from the advantages of backstepping with Lyapunov redesign, which avoid the needs of fast switching of discontinuous control law offered by its counterpart - a variable structure control. We reduce the conservatism in the design process where the control law can be flexibly chosen from Lyapunov function, hence avoiding the use of convex optimization via linear matrix inequality (LMI) in which the feasibility is rather hard to be obtained. For this work, we design two type control algorithms namely normal control and bounded control. As such, our contribution is the introduction of a new bounded control law that can avoid excessive control energy, high magnitude chattering in control signal and small oscillation in stabilized states. Computation of total energy for both control laws confirmed that the bounded control law can stabilize with less enegry consumption. We also use Euler's approximation to compute average power for both control laws. The robustness of the proposed controller is achieved via saturation-like function in Lyapunov redesign, and hence guaranting asymptotic stability of the closed-loop system.

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1. INTRODUCTION

Stabilizing nonlinear unstable system requires massive control effort. Therefore, a challenge to control system engineers is the requirement to bound the control effort. If the switching type or discontinuous stabilizing function is applied, the law tends to produce high-magnitude of chattering in the control signal. As such, the designed control law must be bounded in respect the admissible set of input to the system. For instance, it is easy to stabilize unstable systems by forcing their poles to the left-hand-side of the S-plane so that the closed-loop system stable. Theoretically, placing the closed-loop poles near to ∞ result in fast convergence rate but require high energy as a trade-off. In some industrial cases are DC drive systems where the constraints are due to the physical limitation of the motor drive such as converter protection, magnetic saturation and motor overheating that make the current command is limited to an admissible set of input [1].

For another case such as electric vehicles where the controlled variable is a speed, the motor torque or voltage may be bounded. To date, many approaches have been proposed to design the bounded control law such as variable structure approach as explained in [2]-[4]. Some reviews show that variable structure control has been an established method for nonlinear control since early 1960s. Since then, variable structure control has been applied for robust regulation of fuzzy systems [5], tracking systems [6], optimal control [7], output feedback control systems [8], [9] and many more. There also immerge many methods to search sliding surface for variable structure control. Based on literature, convex optimization via LMI has been used widely to search sliding surface [4], [10], [11] and observer-based control [12] and output feedback sliding mode control in [13]. However in [10], [4], the approach require nonsingular transformation to obtain the LMI such that sliding surface exists, hence involving generalized inverse to reach main results.

Another nonlinear control approach namely Backstepping technique was developed in early 1990s. As such, progress in backstepping controllers for nonlinear uncertain systems with bounded control is rather slow and still at infancy level. Only in 1990s the research in backstepping scheme for bounded control problem is getting blossom, but does not consider uncertainties and exogenous perturbations. For instance, progress in backstepping for nonlinear system with bounded control in [14], backstepping with bounded feedbacks for nonlinear system in [15] and integrator backstepping for 4^{th} order integrator cascade system with state constraint in [16]. Approach in [16] is a recursive-interlacing procedure for tracking, under assumption that the system is linear and no uncertainty is considered. More recently, authors in [17] devised robust adaptive control algorithm for uncertain nonlinear system using backstepping with consideration of input saturation and external disturbance.

This paper propose robust backstepping with Lyapunov redesign to stabilize uncertain system. The design begins with the stabilization of unperturbed subsystem via judicious selection of control Lyapunov function. Afterward, the control Lyapunov function for nominal subsystem is utilized again for system with uncertainties such that the effect of uncertainties is monitored for robustness. The challenge might be in searching for robust Lyapunov functions and some advanced mathematics to structure the system dynamics equation to suit control approach, that is a strict feedback form. However, the remaining tasks of selecting a control law is straightforward such that the necessity of Lyapunov stability criterion is fulfilled. Despite offering flexible design, other advantages of backstepping with Lyapunov redesign is the elimination of useful nonlinear terms can be avoided, leading to reduce control complexity. Early work on Lyapunov redesign has been established in 1979 and can be reviewed in [18] and [19]. However to date, the use of Lyapunov redesign in backstepping has been at a new stage. Latest work by [20] augment nonlinear damping functions to the unperturbed stabilizing function in redesign phase. Whereas [21] uses saturation-type control as an augmentation to the stabilizing function in redesign phase. Both approaches proved robust stability and robust performance. In this article, we use backstepping with Lyapunov redesign approach to design normal control and bounded control in order to stabilize system with matched uncertainties. We prove the stability robustness upon perturbation in initial states as well as lumped matched uncertainties via Lyapunov sufficiency and necessity criterion. The numerical systems in [10] is used as test bed and the system structure is rearranged to ease of design where the disturbance-like function is separated from model uncertainties.

As the fact that stabilizing unstable system require massive energy, one of the performance index in this work is to obtain control signal with less average power and energy. As such, in bounded control design, we relax the transient performance by allowing the closed-loop-system to be slightly sluggish in order to respect the admissible set of inputs to the system. Therefore, our bounded control law satisfies small control property. Our main concern also to avoid excessive control energy, reduce high magnitude chattering control signal and eliminate small oscillation in the stabilizing state without having catastrophic effect to the closed-loop system.

The content of this paper is as follows. Section 2 discuses about theoretical background. Section 3 and section 4 discus about normal controller design. Section 5 discuses about bounded controller design. Section 6 tabulates performance indices. Lastly, we concluded the results in Section 7.

2. PROBLEM FORMULATION

Let the system to be controlled be represented by the following differential equation [10]:

$$\dot{x} = Ax + B(u + \rho(x, u, t)) \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input and $\rho(\cdot): \mathbb{R}^n \text{ x } \mathbb{R}^m \text{ x } \mathbb{R}^+ \rightarrow \mathbb{R}^m$ is the lumped uncertainty which matched to the control input. $A \in \mathbb{R}^{nxn}$ and $B \in \mathbb{R}^{nxm}$ is the system characteristic and input matrix with full rank $m < n$.

2.1. Normal backstepping

We represented the system (1) in strict feedback form (i.e $m = 1$) to facilitate recursive backstepping and admits robust control Lyapunov function. Thus our approach restricted to strict feedback form:

$$\dot{x}_i = F_i(X_i) + G_i(X_i)x_{i+1}, \; i = 1,2,\dots,n-1 \tag{2a}$$

$$\dot{x}_n = F_n(X) + G_n(X)[x_{1+1} + \rho(x,u,t)] \tag{2b}$$

where $A \triangleq F(x) = \begin{bmatrix} a_{11} & a_{12} & 0 & \cdots & 0 \\ a_{21} & a_{22} & a_{23} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n-1,1} & a_{n-1,2} & a_{n-1,3} & \cdots & a_{n-1,n} \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix}$ and $B \triangleq G(x) = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b_{n,n+1} \end{bmatrix}$.

Alternatively, the system can be represented as

$$\dot{x} = F(x) + G(x)u + H(x)\rho(\cdot) \tag{3}$$

If there exist some continuous function $E(x)$ which satisfies structural condition $H(x) = G(x)E(x)$, then the matching condition can be applied to the class of system in (3). Control problem now is to stabilize $\dot{x}$ with control $u(t)$. The design begins with seeking a control Lyapunov function $V(x)$ for nominal system $\dot{x} = F(x) + G(x)u$. The function should be smooth, positive definite, radially unbounded such that.

$$\nabla V(x) \cdot G(x) = 0 \Rightarrow \nabla V(x) \cdot F(x) < -\mu(x) \tag{4}$$

for all $x \neq 0$ and some function $\mu \in \mathbb{R}^+$. For system with order ≥ 2 (that is $i = 2,\dots,n-1$, the stabilizing function for x_i-subsystem is designed such that the derivative of $V(x)$ negative definite. The stabilizing function behaves as a desired state for subsystem x_{i+1}. We therefore define error dynamics between the desired state and actual state in subsystem x_{i+1}. For that, we make the error dynamics perished after $t \to \infty$ to facilitates control design for next subsystem x_{i+1}. After the stabilizing function for nominal subsystem is successfully designed, the control Lyapunov function for nominal subsystem is re-used for redesign phase. In redesign phase, the stabilizing function is augmented with robust control or robust function such as nonlinear damping function [20], saturation-type function [21] or others to combat with uncertainties. In redesign, overall control for particular subsystem is obtained using the priori Lyapunov function such that when $\nabla V(x) \cdot G(x) = 0$, implies.

$$\nabla V(x) \cdot F(x) + \|\nabla V(x) \cdot H(x)\| \leq 0 \tag{5}$$

Then, the approach is back-step and being repeated recursively such that the actual control $u(t)$ is finally obtained.

2.2. Bounded control

Consider the unperturbed version of system in Equation (3):

$$\dot{x} = F(x) + G(x)u \tag{6}$$

where controls take place in the open unit ball $\mathcal{B}_m = \{u \in \mathbb{R}^m < -u_{lower}\}$. With Lyapunov theorem, there exists a positive definite, proper and smooth function as before, namely V(x). There also exists operators $\mathcal{F}(x) \triangleq \nabla V(x) \cdot F(x)$ and $\mathcal{G}(x) \triangleq \nabla V(x) \cdot G(x)$ that contribute to the existence of a continuous and regular feedback law $\alpha(x): \mathbb{R}^n \longrightarrow \mathcal{B}_m$ where:

$$\alpha(x) = \begin{cases} -\frac{\mathcal{F}(x)+\sqrt{\mathcal{F}(x)^2+\mathcal{G}(x)^4}}{b\left(1+\sqrt{1+\mathcal{G}(x)^2}\right)}, & if\ \mathcal{G}(x) \neq 0 \\ 0, & if\ \mathcal{G}(x) = 0 \end{cases} \tag{7}$$

If $\alpha(x)$ continuous at origin, then $\alpha(x)$ satisfies small control property with respect to system in Equation (6) as in definition 1.

Definition 1: Small Control Property
For system in equation (6) satisfy small control property, there is known control Lyapunov function V(x), for every $\epsilon > 0$, there exists a $\delta(\epsilon) > 0$ so that for all $x \neq 0$ and $\|x\| < \delta$, there is control u with $\|u\| < \epsilon$ such that $\mathcal{F}(x) + \mathcal{G}(x)u < 0$.

With definition 1, we use Lemma 1 to reach main results.
Lemma 1: Assume that $\mathcal{F}(\cdot)$, $\mathcal{G}(\cdot)$ and ϵ are real numbers such that $\mathcal{F}(\cdot) < \epsilon|\mathcal{G}(\cdot)|$ and $0 < \epsilon \leq \sigma$ for $\sigma \in \mathbb{R}$, then there exists a stabilizing function $\alpha(\mathcal{F},\mathcal{G})$ with property.

$$|\alpha(\mathcal{F},\mathcal{G})| < min\{2\epsilon + |\mathcal{G}|, \sigma\} \tag{8}$$

Proof:
Without loss of generality, we set $\mathcal{G}(\cdot) > 0$; thus $\mathcal{F}(\cdot) < \epsilon\mathcal{G}(\cdot)$. If $\mathcal{F}(\cdot) \leq 0$, condition $\mathcal{F}(\cdot) < \epsilon|\mathcal{G}(\cdot)|$ is not valid. Therefore,

$$|\alpha(\mathcal{F},\mathcal{G})| = \frac{\mathcal{G}}{1+\sqrt{1+\mathcal{G}^2}} < min\left\{\frac{\mathcal{G}}{2}, 1\right\} \tag{9}$$

When $\mathcal{F}(\cdot) > 0$, we can see that $|\alpha(\mathcal{F},\mathcal{G})|$ bounded by σ as $\epsilon \leq \sigma$, and also bounded by its numerator. This yields:

$$|\alpha(\mathcal{F},\mathcal{G})| = \frac{\epsilon\mathcal{G}+\sqrt{(\epsilon\mathcal{G})^2+\mathcal{G}^4}}{\mathcal{G}\left(1+\sqrt{1+\mathcal{G}^2}\right)} = \frac{\epsilon+\sqrt{\epsilon^2+\mathcal{G}^2}}{1+\sqrt{1+\mathcal{G}^2}} < min\{2\epsilon + |\mathcal{G}|, \sigma\} \tag{10}$$

3. BACKSTEPPING CONTROL

Consider numerical values for system in (1) be $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ and $\rho(x,u,t) = 0.5\sin(120\pi t) + 0.1\cos(x_1)x_2$. Respectively, the nonlinear system can be represented in strict feedback form:

$$\dot{x}_1 = x_2 \tag{11a}$$

$$\dot{x}_2 = x_1 + \sin(120\pi t) + 0.2x_2\cos(x_1) + 2u \tag{11b}$$

It is easy to see that for any $K_1 \in \mathbb{R}^+ > 0$, $x_2 \triangleq \vartheta_1(x_1) = -K_1x_1$ be a desired state that stabilized x_1-subsystem in equation (11a). This can be confirmed by a Lyapunov function $V_1(x_1) = {x_1}^2/2$, where the derivative about x_1-subsystem is negative definite:

$$\dot{V}_1(x_1) = -K_1{x_1}^2 \leq 0 \tag{12}$$

We define error dynamics, z between the desired state ϑ_1 and actual state x_2:

$$z = x_2 - \vartheta_1 \tag{13}$$

Our control problem now is to design control law to weaken z. Therefore, we represent actual system in Equations (11a) and (11b) in new coordinate $[x_1\ z]$:

$$\dot{x}_1 = -K_1x_1 \tag{14a}$$

$$\dot{x}_2 = x_1 + \sin(120\pi t) + 0.2x_2\cos(x_1) + 2u \tag{14b}$$

Cosine terms in system parameters can be lumped together and regarded $\xi(t) = \sin(120\pi t)$ as exogenous time varying disturbance. Hence, we design the stabilizing function for unperturbed z-subsystem in Equation (14b). With Lyapunov function $V_2(x_1, z) = ({x_1}^2 + z^2)/2 = V_1(x_1) + z^2/2$, the stabilizing function is obtained as

$$\vartheta_2(x_1, z) = -\frac{1}{2}(K_1 + K_2z) - x_1 - 0.1z\cos(x_1) - 0.1\vartheta_1\cos(x_1)\ , K_2 \in \mathbb{R}^+ > 0 \tag{15}$$

which asymptotically stabilized unperturbed system in Equation (14). This can be proved by inserting the stabilizing function $\vartheta_2(x_1, z)$ in the derivative of $V_2(x_1, z)$ about system in Equation (14), which yields:

$$\dot{V}_2(x_1, z) = -K_1 {x_1}^2 - K_2 z^2 \leq 0 \qquad (16)$$

4. LYAPUNOV REDESIGN

Consider system (14b) in the presence of $\xi(t)$. We augment the stabilizing function $\vartheta_2(x_1, z)$ with robust control $u_1(x_1, z, t)$. Our control problem now is to design $u_1(x_1, z, t)$ such that overall system is stabilized and robust toward $\xi(t)$. Therefore, our chosen control law is:

$$u = \vartheta_2(x_1, z) + u_1(x_1, z, t) \qquad (17)$$

4.1. Preliminary results - normal control law

In Lyapunov redesign, one may easily found that a saturation-like function

$$u_1(x_1, z, t) = -\xi_g \frac{2z\xi_g}{2z\xi_g + \tau e^{-at}}, t > 0, a > 0 \qquad (18)$$

with $\xi_g = \frac{\sin(120\pi t) - x_1}{2}$, when constitute to final control law

$$u = -\frac{1}{2}(K_1 + K_2 z) - x_1 - 0.1z\cos(x_1) - 0.1\vartheta_1\cos(x_1) - \xi_g \frac{2z\xi_g}{2z\xi_g + \tau e^{-at}} \qquad (19)$$

will stabilize overall system in Equation (11).

4.2. Proof of stability

With saturation-like function in (18), re-consider Lyapunov function $V_2(x_1, z)$. The derivative about $[x_1\ z]$ yields:

$$\begin{aligned}
\dot{V}_2(x_1, x_2) &= -K_1 {x_1}^2 + z[x_1 + 0.2z\cos(x_1) - 0.2\vartheta_1\cos(x_1) + K_1 + 2[\vartheta_2 + u_1] + \Delta(t)] \\
&= -K_1 {x_1}^2 + z[2x_1 + 0.2z\cos(x_1) - 0.2\vartheta_1\cos(x_1) + K_1 + 2\vartheta_2] + z[2u_1 + \Delta(t) - x_1] \\
&= -K_1 {x_1}^2 - K_2 z^2 + 2z[-\xi_g \frac{2z\xi_g}{2z\xi_g + \tau e^{-at}} + \xi_g] \\
&= -K_1 {x_1}^2 - K_2 z^2 - \frac{\|2z\xi_g\|^2}{\|2z\xi_g\| + \tau e^{-at}} + 2z\xi_g \\
&= -K_1 {x_1}^2 - K_2 z^2 + \frac{\|2z\xi_g\|\tau e^{-at}}{\|2z\xi_g\| + \tau e^{-at}} \\
&< -K_1 {x_1}^2 - K_2 z^2 + \tau e^{-at}
\end{aligned} \qquad (20)$$

If $x_2 \triangleq \vartheta_1$ were the actual control and $z \equiv 0$, it is shown that x_1 preserve $\dot{V}_2(x_1, x_2)$ negative outside the residual compact set β where:

$$\beta = \left\{ x_1 : |x_1| \leq \sqrt{\frac{\tau e^{-at}}{K_1}} \right\} \qquad (21)$$

Hence, the control law in (19) guarantees asymptotic stability of $x_1(t)$ as shown in Equation (22):

$$\|x_1\|_\infty = \left\{ |x_1(0)|, \sqrt{\frac{\tau e^{-at}}{K_1}} \right\} \qquad (22)$$

Figure 1 shows stabilized state x_1 upon perturbation in initial states $X(0) = [1\ 1]$ for both backstepping and variable structure control. Figure 4 confirm asymptotic stability condition in Equation (22). Figure 2 and Figure 3 show control signal for backstepping control law and variable structure control law in [17] respectively. Figure 4 shows overall system trajectories for both backstepping and variable structure control.

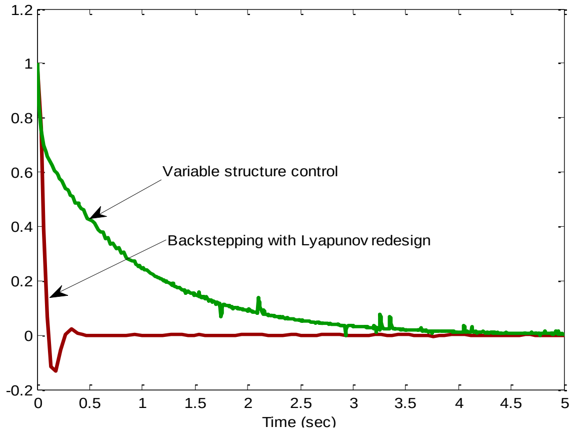

Figure 1. Stabilizing x1- comparison between bacstepping with Lyapunov redesign and variable structure control

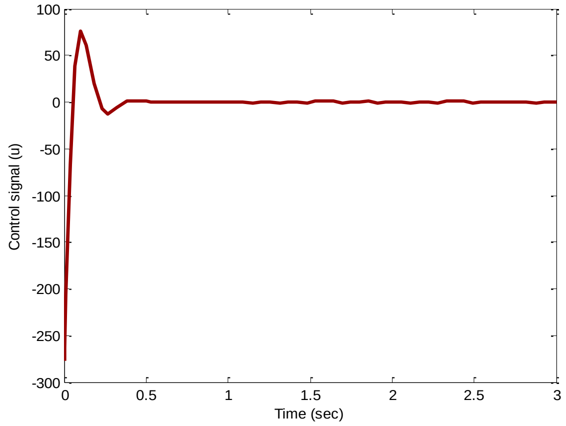

Figure 2. Control signal - backstepping with Lyapunov redesign

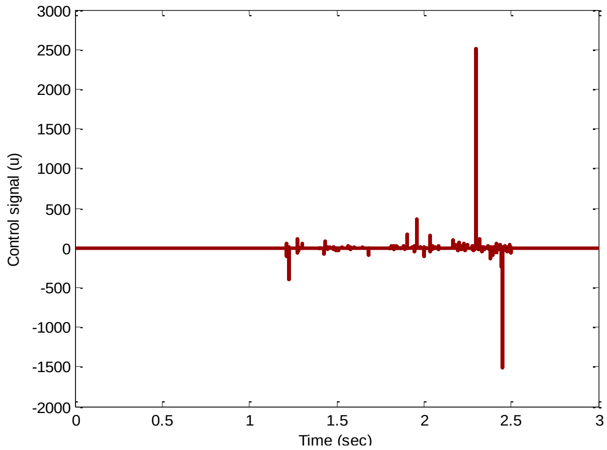

Figure 3. Control signal for variable structure control

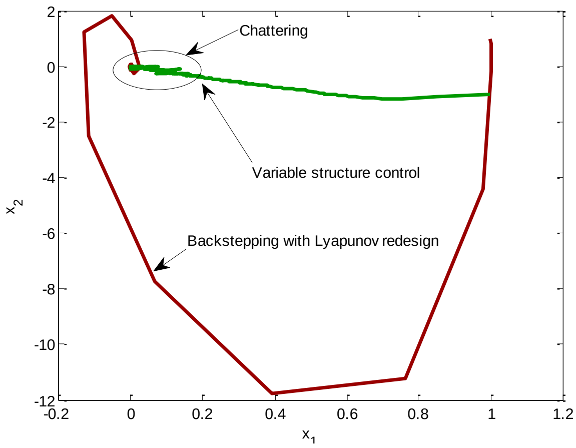

Figure 4. Overall system trajectories

5. MAIN RESULTS – BOUNDED CONTROL

In this section, we extend our result by designing a bounded control law which robust toward $\xi(t)$ and perturbation in initial states. It is observed that backstepping control eliminates high frequency chattering in system trajectories (Figure 1 and Figure 4) and offers smooth control signal as shown in Figure 2. However, the stabilizing state by backstepping still having low frequency oscillation during steady state. Moreover, stabilizing using backstepping require massive energy during the transient part (Figure 2). As such, it is desirable to bound the control law without having catastrophic effect to the closed-loop system. The rest of this paper improve the control law in equation (19) such that the control law satisfies small control property and bounded within admissible set of inputs. To assist our design, we exploit *definition 1* and *Lemma 1*.

5.1. Bounded backstepping

With $V_1(x_1) = x_1{}^2/2$ and from Equation (7), we can easily obtain the stabilizing function for x_1-subsystem in equation (11a) be:

$$\alpha(x_1) = -\frac{x_1}{1+\sqrt{1+x_1{}^2}} \tag{23}$$

and with $V_2(x_1, z) = (x_1{}^2 + z^2)/2 = V_1(x_1) + z^2/2$, the stabilizing function for unperturbed system in

Equation (11) is defined as

$$\pi_2(x_1,z) \triangleq -x_1 - \frac{1}{2}\left[K_2 z + \left(0.2Cos(x_1) - \frac{\partial\alpha}{\partial x}\right)\left(z - \alpha(x_1)\right)\right] \quad (24)$$

Via backstepping and Lyapunov redesign, we reach the bounded control law:

$$u_{bound} = -\frac{1}{2}K_2\left[x_2 + \frac{x_1}{1+\sqrt{1+x_1{}^2}}\right] - x_1 - \frac{1}{2}\left[\left(0.2Cos(x_1) - \frac{\partial\alpha}{\partial x}\right)x_2\right] - \frac{\xi(t)}{2}\frac{2z\xi(t)}{z\xi(t)+\tau e^{-at}} \quad (25)$$

5.2. Proof of stability

With saturation-like function in Equation (18), consider again the derivative of Lyapunov function $V_2(x_1,z)$ be:

$$\begin{aligned}\dot{V}_2(x_1,x_2) &= -\frac{x_1{}^2}{1+\sqrt{1+x_1{}^2}} + z\left[2x_1 + \left(0.2\cos(x_1) - \frac{\partial\alpha}{\partial x}\right)x_2 + 2[\pi_2 + u_1] + \xi(t)\right] \\ &= --\frac{x_1{}^2}{1+\sqrt{1+x_1{}^2}} - K_2z^2 + 2z[u_1 + \frac{\xi t)}{2}] \\ &= -\frac{x_1{}^2}{1+\sqrt{1+x_1{}^2}} - K_2z^2 + 2z[\left(\frac{\xi(t)}{2}\right)\frac{2z\frac{\xi(t)}{2}}{2z\frac{\xi(t)}{2}+\tau e^{-at}} + \frac{\xi(t)}{2}] \\ &= -\frac{x_1{}^2}{1+\sqrt{1+x_1{}^2}} - K_2z^2 - \frac{\left\|2z\frac{\xi(t)}{2}\right\|^2}{\left\|2z\frac{\xi(t)}{2}\right\|+\tau e^{-at}} + 2z\frac{\xi(t)}{2} \\ &< -\frac{x_1{}^2}{1+\sqrt{1+x_1{}^2}} - K_2z^2 + \tau e^{-at}\end{aligned} \quad (26)$$

Hence, completes the proof. The advantage of this control is that the number of control parameters has been reduced as tabulated in Table 1:

Table 1. Control Parameters

	K_1	K_2	τ	a
Normal Control in (19)	2	23	0.05	0.05
Bounded Control in (25)	Not required	23	0.05	0.05

For perturbation in $X(0) = [1\ 1]$, Figure 5 shows smooth bounded control signal where the amount of control energy is bounded within $-18 < u_{\text{bound}} < 0$ unit as compared with the unbounded or normal control of $-275 < u < 75$ (comparison chart is shown in Figure 6). Figure 7 shows that bounded control law eliminates small frequency oscillation in the stabilizing state, which is the drawback of the normal backstepping control law in (19). Overall system trajectories in Figure 8 shows both control laws guarantee asymptotic stability upon perturbation in initial states. However, normal control does not satisfy small control property as bounded control does. Another advantages of the resulting bounded control law is that the control signal is fairly smooth although the solution convergence rate slightly sluggish.

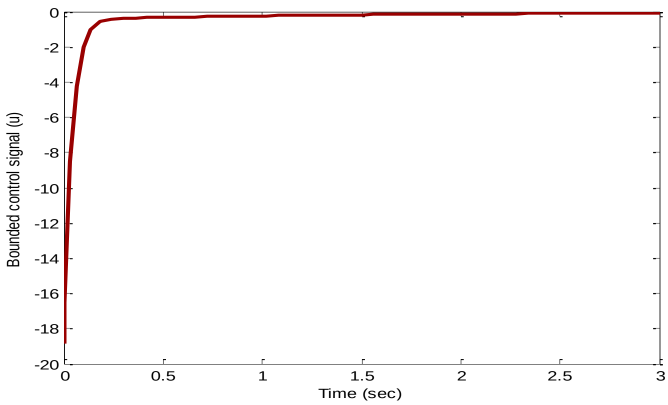

Figure 5. Bounded control signal in Equation (25)

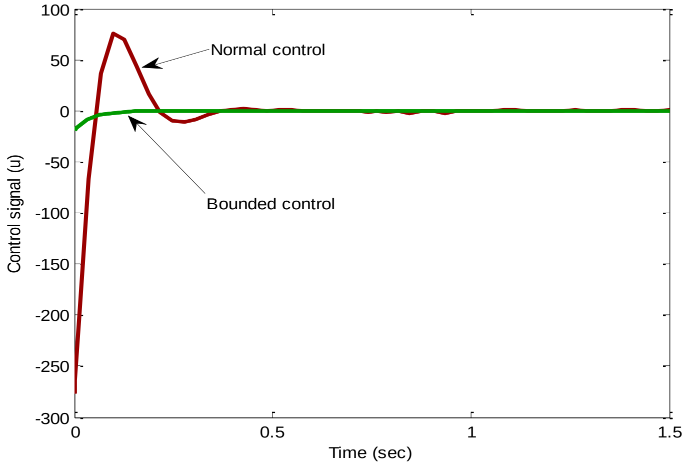

Figure 6. Control signal - comparison between normal control in Equation (19) and bounded control in Equation (25)

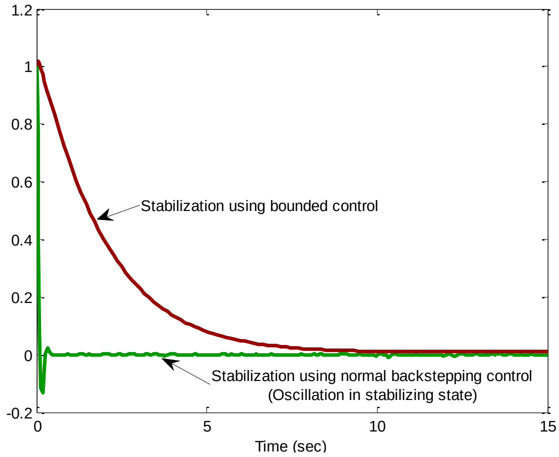

Figure 7. Stabilizing x1 - comparison between normal control in Equation (19) and bounded control in Equation (25)

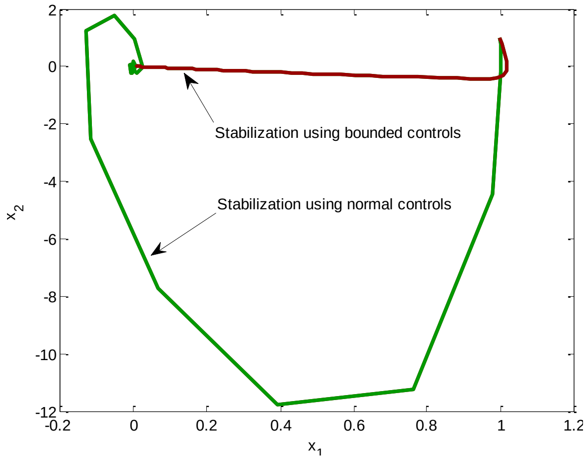

Figure 8. System trajectories - comparison between normal control in Equation (19) and bounded control in Equation (25)

6. PERFORMANCE INDICES

Steering perturbed states toward origin require massive energy. As such, it is important to observe the amount of energy and average power produced by both normal control law in (19) and bounded control law in (25). The energy of both control signals are computed by the area under the squared signals as

$$E_u(t) = \int_{-\infty}^{\infty} \|u(t)\|^2 dt \tag{27}$$

and

$$E_{u_{bound}}(t) = \int_{-\infty}^{\infty} \|u_{bound}(t)\|^2 dt \tag{28}$$

After the stabilization takes place, control signal (19) and (25) decay in the steady state and approaching zero. The time average of energy (energy per unit time) is considered as signal power of both control signal in (19) and (25). In this case, Euler's approximation can be exploited to compute the average power of the control signal. Therefore, average power for both control signals are

$$\int_a^b u(t)\, dt \cong \sum_{n=0}^{N-1} u(a + n\Delta t)\Delta t \tag{29}$$

and

$$\int_a^b u_{bound}(t)\, dt \cong \sum_{n=0}^{N-1} u_{bound}(a + n\Delta t)\Delta t \quad (30)$$

where N is the number of integral part in Euler's approximation, $b - a$ is the control signal duration and $\Delta t = (b - a)/N$ is the duration (or interval) for each integral part in seconds. With $X(0) = [1\ 1]$ and signal interval $\Delta t = 0.005$ seconds, Table 2 tabulates the energy, power and the initial control magnitude for both controllers within 20 seconds of stabilization process.

Table 2. Performance indices for controllers (19) and (25)

	Total Energy (Joule)	Average Power (watt)	Initial control magnitude (unit)
Normal Control in (19)	280660	8.6663 x 10^{-4}	$\lvert u \rvert = 275$
Bounded Control in (25)	623.955	4.2029 x 10^{-4}	$\lvert u_{bound} \rvert = 17.5$

7. DISCUSSION

This article presented didactic approach to stabilize uncertain nonlinear numerical system. The controller has been developed using backstepping technique and Lyapunov redesign. Therefore, the control law is continuous and does not require high frequency switching which normally not suitable for some application that require continuous control signals. In some application, the control signal has to be bounded within admissible set of inputs. As such, the improvement has been made to produce robust control law that is allowed to be bounded without catastrophic effects to system performance and stability. For bounded control law, the number of control parameters that need to be defined have been reduced. Bounded controller consumes less energy as well as less average power for stabilization process. To stabilize the system with $X(0) = [1\ 1]$, bounded controller requires only 623.955 joule energy with average power 4.2029 x 10-4 watt as compared to normal controller which require 280660 joule and 8.6663 x 10-4 watt respectively. Bounded controller produces initial control around $\lvert u_{bound} \rvert = 17.5$ units which is lower than the signal produced by normal controller, i.e $\lvert u \rvert = 275$ units.

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