

## HOPX Crossover Operator for the Fixed Charge Logistic Model with Priority Based Encoding

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### ABSTRACT

In this paper, we are interested to an important Logistic problem modelised us optimization problem. It is the fixed charge transportation problem (FCTP) where the aim is to find the optimal solution which minimizes the objective function containing two costs, variable costs proportional to the amount shipped and fixed cost regardless of the quantity transported. To solve this kind of problem, metaheuristics and evolutionary methods should be applied. Genetic algorithms (GAs) seem to be one of such hopeful approaches which is based both on probability operators (Crossover and mutation) responsible for widen the solution space. The different characteristics of those operators influence on the performance and the quality of the genetic algorithm. In order to improve the performance of the GA to solve the FCTP, we propose a new adapted crossover operator called HOPX with the priority-based encoding by hybridizing the characteristics of the two most performant operators, the Order Crossover (OX) and Position-based crossover (PX). Numerical results are presented and discussed for several instances showing the performance of the developed approach to obtain optimal solution in reduced time in comparison to GAs with other crossover operators.

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## 1. INTRODUCTION

Logistic model are the most important problem that need to be optimized for the smooth operation of the entire supply chain [1]. It determines the number and type of plants, warehouses and distribution centers (DCs) to be used. It also establishes distribution channels and the quantity of products to be shipped from suppliers for each customer. Logistic model covers a wide range of formulations from linear deterministic models to nonlinear stochastic complexes ones.

For the first formulation called linear logistic model problem or liner transportation problem. It is a network optimization problem introduced by Hitchcock [2] which consists to minimize the total cost in order to transport homogeneous products from several sources to several deposits satisfying the limits of supply and demand. This model can be find in industry, planning, communication network, scheduling, transportation and attribution. Several search attacks the linear logistics models which can be solved by the simplex method introduced by George Dantzig in 1947 [3]. Also, it can be solved by approximation methods such us the method of Russell and the method of Vogel [4], [5].

The second formulation, which is the objective of this study, is the fixed charge logistic model. The literature around this first case extension is very rich. Hirsch and Danzig in [6] was the first to formulate the FCTP us extension of the liner logistic model. Many practical transportation and distribution problems, such as the minimum cost network flow (transshipment) problem with a fixed-charge for logistics, can be

formulated as fixed-charge logistic model. For instance, a fixed cost may be incurred for each shipment between a given plant and a given customer and a facility of a plant may result in a fixed amount on investment. The FCTP takes these fixedcharge into account, so that the TP can be considered as an FCTP with equal fixed costs of zero for all routs.

Balainski in 1961 modified the FCTP to a linear integer problem [7]; he observed that there is an optimal solution for the modified version of FCTP. Adlakha in [8] proposed a method which consists of two parts; it gets the best initial solution in the first part and uses techniques to improve this solution and to check its optimality.

The genetic algorithm (GA) is an evolutionary algorithm that represents a famous metaheuristic proposed by Jhon Holland in 1975 [9] which is inspired by biological mechanisms such as Mendel's laws and the theory of evolution proposed by Charles Darwin. It uses the same vocabulary as in biology and classical genetics; so we speak of gene, chromosome, population [10]-[13].

The genetic algorithm has been used to solve many combinatorial problems including FCTP [14]. Its main advantage is that it allows a good combination between the exploitation of solutions and the exploration of the research space. This is established as a function of the GA parameters respectively. However, its disadvantage lies in two points; a computational time large enough to be able to converge towards the optimal solution and the convergence that is a big problem for GAs. In addition, the different characteristics of the genetic operators influence on the performance and the quality of the GA. For this reason and in order to improve the performance of the GA to solve the FCTP, we propose a new adapted crossover operator called HOPX with the priority-based encoding by hybridizing the characteristic of the two most performment operators, the Order Crossover (OX) and Position-based crossover (PX).

This paper is organized as follows: The second section gives an overview of the formulation of the fixed charge logistic model. The third part presents the GA as a method of resolution and these parameters which have influenced the quality of the results. In addition to the proposed hybrid crossover operator called HOPX. The fourth section deals with the development and implementation of the GA with the new operator where several numerical results are presented showing his performance in comparison with variants operators.

## 2. LOGISTICS MODEL DESCRIPTION AND FORMULATION

Logistic is a profound factor on the added value of each business. Examines each of this activity is done at several levels: logistic engineering, technical publications, procurement... Suggests to each element a separate responsibility towards the product arrived at the customer or the consumer in the complex network [15].

Considering the following graph  $G = (N, A)$ , which consists of a finite set of nodes  $N = \{1, 2, \dots, n\}$  and a set of directed arcs  $A = \{(i, j), (k, l), \dots, (s, t)\}$ , which joins pairs of nodes in  $N$ . It is graphically illustrated in Figure 1.

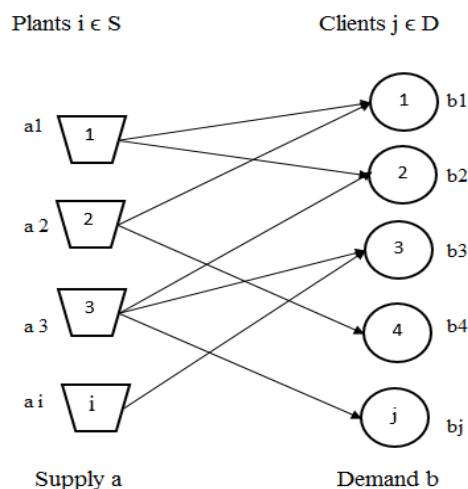


Figure 1. Transportation plan

### 2.1. Fixed-charge Logistic Model

The FCTP is much more difficult to solve due to the presence of fixed costs, which cause discontinuities in the objective function. In the fixed-charge logistic model, two types of costs are considered simultaneously when the best course of action is selected: (1) variable costs proportional to the activity level; and (2) fixed costs. The FCTP seeks the determination of a minimum cost transportation plan for a homogeneous commodity from a number of plants to a number of customers. It requires the specification of the level of supply at each plant, the amount of demand at each customer, and the transportation cost and fixed cost from each plant to each customer. The goal is to allocate the supply available at each plant so as to optimize a criterion while satisfying the demand at each customer. The usual objective function is to minimize the total variable cost and fixed costs from the allocation. It is one of the simplest combinatorial problems involving constraints. The fixedcharge logistic model with  $I$  plants and  $J$  customers can be formulated as follows:

$$\text{Min } Z = \sum_{i=1}^m \sum_{j=1}^n (C_{ij}x_{ij} + f_{ij}y_{ij}) \quad (1)$$

$$\sum_{j=1}^n x_{ij} = S \quad i = 1, 2, \dots, m \quad (2)$$

$$\sum_{i=1}^m x_{ij} = D_j \quad i = 1, 2, \dots, n \quad (3)$$

$$x_{ij} \geq 0 \quad i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n$$

$$y_{ij} = 0 \quad \text{if } x_{ij} = 0$$

$$y_{ij} = 1 \quad \text{if } x_{ij} > 0$$

$x_{ij}$  : Unknown quantity to be transported from source  $i$  to destination  $j$ ;

$c_{ij}$  : Variable transportation cost from source  $i$  to destination  $j$ ;

$f_{ij}$  : Fixed transportation cost associated with road  $(i,j)$ ;

$y_{ij}$  : A binary variable which is 1 if  $x_{ij} > 0$  and 0 if  $x_{ij} = 0$ ;

$S_i$  : Amount of supply at source  $i$ ;

$D_j$  : Amount of demand at destination  $j$ .

## 3. PRIORITY-BASED GENETIC ALGORITHM FOR FIXED CHARGE LOGISTIC MODEL

In order to apply the genetic algorithm (GA), it is necessary to choose and adaptate the representation method for the solution of the problem. There are several methods of representation that are used to solve logistics models using GA, there is the matrix representation [15], representation by the number of präfer [16], and there is also the priority based representation which is a new and more adapted for encoding and decoding the logistics models. It is first used by Gen and Altiparmak [17] to code and decode a two-stage transport problem. Then, the GA consists to several steps: Initialization processes (Generation of a population of solutions), evaluation, selection, crossover and mutation operators to create a new population of solutions.

### 3.1. Encoding chromosome

For the priority based representation, the solution (chromosome) is represented by an integral chain of length equaling the number of sources plus the number of clients. Each gene in this chromosome indicates the identification of a node (number).

In this representation, a gene in a chromosome contains two types of information:

- The position of a gene to represent the nodes (source / destination)
- The value of a gene which represents the priority of the node for the construction of a transport tree.

A chromosome consists of the priorities of sources and deposits to obtain a transport tree, its length is equal to the total number of sources  $m$  and deposits  $n$ , ie.  $(m+n)$ . Each chromosome can be reconstructed in a random way and there is no need for a correction algorithm after the generation of a population [18].

### 3.2. Selection and evaluation method

Selection and evaluation are genetic processes to evaluate product solutions and compare them with existing chromosomes in order to choose the best to preserve them in memory. Therefore; the evaluation

function will select or refuse an individual to retain only those individuals with the best cost according to the current population. In practice there are several types of selection applied in genetic algorithms, especially we employ the elitist mechanism; In fact, the best solution of the current population is selected and preserved in memory [19].

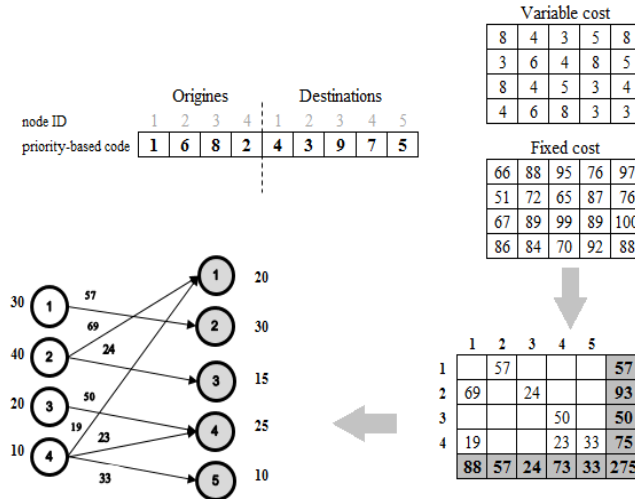


Figure 2. Priority-based chromosomes and transportation trees

### 3.3. Adapted mutation

Mutation operator operates by exchanging information within a chromosome. However, instead of using this operator between two parents, we use between two segments of a parent. We have chosen a swap mutation operator that is proposed by (Michalewicz, 1992) which permutes the values of two randomly selected positions for each chromosome to reduce the risk of reproducing a chromosome with the same solution [20], [21]. The procedure of swap mutation illustrated below works as follows:

#### Procedure of the SWAP mutation

**Input** : One parent  
**Step 1** : Select two element at random  
**Step 2** : Swap the element on these positions  
**Output** : One offspring

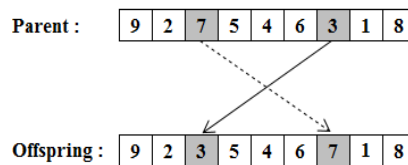


Figure 3. Example of the SWAP mutation operator

### 3.4. Proposed crossover operator for FCTP

The crossover operators are genetic processes. They work on two different parents (chromosomes) by combining the characteristics of the two chromosomes, they consist in applying procedures with a certain probability called the crossover rate  $[P_c \in (0, 1)]$  on the individuals selected to give birth to one or more (usually two) offspring. In this context, in order to ameliorate the performance of the GA to solve the fixed charge logistic model, we have chosen the two more performante crossover operators (OX and PX) following a comparative study carried out for several ones [18]. Then, we proposed an hybridization of this two

operators by highlighting the characteristics of each of them. The procedure of the proposed operator is presented bellow and the Figure 4. illustrate the procedure as an example.

**Procedure of the HOPX operator :**

**Input:** Tow parents;  
**Step 1 :**Select 1/3 substring from one parent at random;  
**Step 2 :**Select 1/4 substring and 1/4 of set of positions from same parent at random.  
**Step 3 :** Produce a proto-child by copying the nodes on these positions into the corresponding positions of it;  
**Step 4 :** Delete the nodes which are already selected from the second parent;  
 The resulted sequence of nodes contains the nodes of the proto-child needs;  
**Step 5 :** Place the nodes into the unfixed positions of the proto-child from left to right according to the order of the sequence to produce one offspring;  
**Output:** Tow offspring.

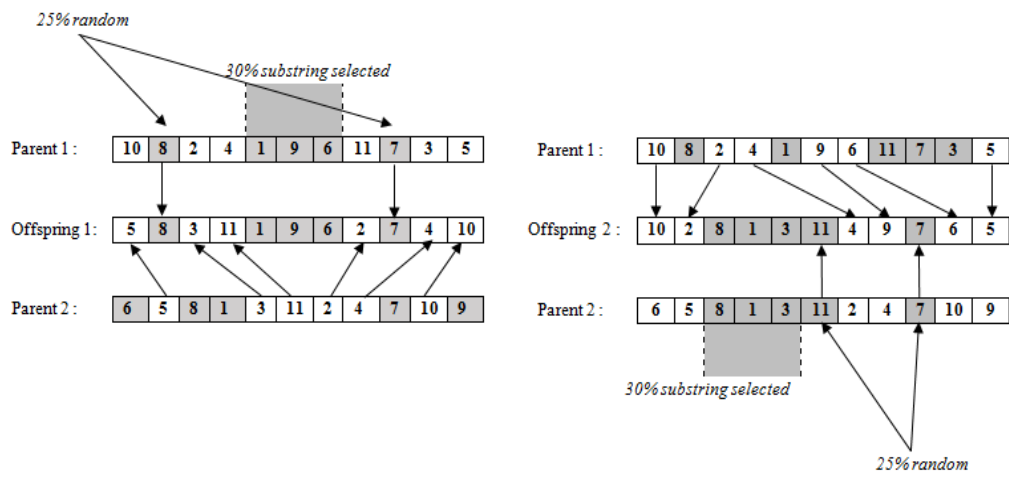


Figure 4. Example of the HOPX operator

**4. COMPUTATIONAL RESULTS**

The main objectives of this work are to improve the behavior of the genetic algorithm with respect to the crossover operators because they have an influence on the quality and performance of these algorithms and to establish to what extent the proposed algorithm solves the different types of logistic problems compared to the already used methods.

In this section, we have applied the developed HOPX operator to Logistic model with fixed cost in order to compare the results with those obtained using other adapted operators namely OPEX, OX, PX. In this study, we have chosen six instances 4x5, 5x10, 10x10, 10x20, 20x30, 30x50 already used in several articles [22], knowing that the optimal solution is known for small instances. Therefore, we have changed the genetic algorithm parameters, such as the number of iterations to see the influence of the latter on the results and on the quality of the solutions. This algorithm is coded in JAVA language in the NetBeans 8.0.2 IDE.

Table 1. presents the simulation results obtained from 20 times by the genetic algorithm based on each operator used to solve the logistic problem with fixed cost. For the results displayed in this table, we notice that the genetic algorithm with the HOPX operator allowed obtaining the optimal solution known for small instances as GAs with other crossover operators. However, for larger instances a better optimal solution is achieved with an improvement in computation time. Indeed, for small instances, 4\*5 and 5\*10, the obtained results show that GA with all crossover operators used allow to obtain the best chromosome satisfying the optimal solution. However, for problems with large size, with the proposed crossover operator HOPX, we are optimizing the objective function with a slight precision in terms of optimal solution. Then, the latter is more advantageous to the level of number of iterations with a reduced execution time. This means that HOPX is more appropriate than other operators that have already used to solve the fixed charge logistic model see Figure 5. and Figure 6.

It should be noted that the genetic algorithm with the developed crossover operator is also applied to the linear logistic model which also shows its performance with respect to the GAs with the other mentioned operators.

Table 1. Best and average results by different operator for the test problems of Fixed-charge Logistic Model

Problem Size m x n	Parameter		OPEX		OX		PX		HOPX	
	Popsize	Maxgen	Best	Avrg	Best	Avrg	Best	Avrg	Best	Avrg
4x5	20	300	9291	9291	9291	9291	9291	9291	9291	9291
	30	500	9291	9291	9291	9291	9291	9291	9291	9291
5x10	20	300	12718	12751	12718	12751	12718	12751	12718	12718
	30	500	12718	12734	12718	12734	12718	12734	12718	12718
10x10	20	500	13987	14074	13987	14139	14065	14133	13987	14047
	30	700	13934	14074	13987	14074	13934	14065	13934	13987
10x20	20	500	22258	22484	22376	22531	22198	22834	22150	22258
	30	700	22095	22484	22095	22198	22095	22532	22095	22198
20x30	20	500	32840	34119	33192	34581	32683	34414	32492	33142
	30	700	32526	33917	33917	33236	32492	33234	32471	32936
30x50	20	700	55611	56399	56399	56705	55450	56007	55269	55450
	30	1000	55143	55912	55912	55912	55106	55407	54114	55106

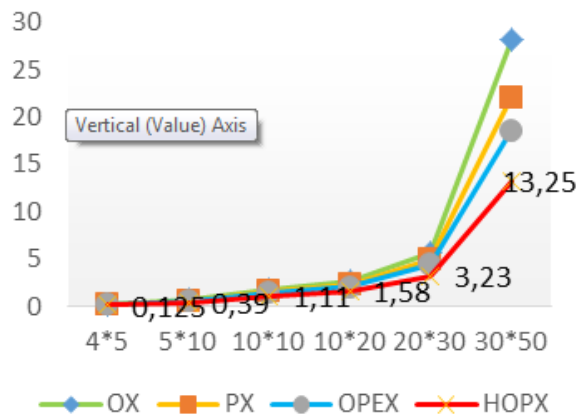


Figure 5. Average computation time for different operators for fixed charge Logistic Models

Table 2. gives the best chromosome for the different instances which represent the solution for the fixed charge logistic problem.

Table 2. Best chromosome for the test problems

Instance	Best chromosome
4x5	1 6 8 2 4 3 9 7 5
5x10	11 1 7 3 13 14 2 5 6 12 4 10 9 15 8
10x10	8 5 14 12 19 2 1 6 15 20 9 10 7 11 3 16 4 13 18 17
10x20	24 5 21 10 9 17 13 19 12 6 29 1 28 3 18 20 15 4 7 11 25 23 26 27 16 30 22 2 8 14
20x30	5 42 13 31 33 30 23 41 48 9 27 50 11 29 40 16 20 25 3 22 28 43 32 44 38 6 26 37 49 2 24 36 7 18 35 14 8 1 46 34 17 10 15 4 12 21 45 39 47 19
30x50	59 50 66 47 8 29 2 51 74 56 35 65 27 5 28 68 25 21 19 40 38 76 53 46 71 42 14 55 61 7 41 22 6 13 72 36 15 11 33 79 57 4 49 30 43 73 60 52 44 58 26 48 64 23 9 75 20 67 24 70 32 3 62 39 10 45 54 12 17 78 16 1 69 63 77 18 34 37 80 31



Figure 6. Optimum solution with different crossover operators OPEX, OX, PX, HOPX with different instances for fixed charge Logistic Models

**5. CONCLUSION**

In this work, we are interested in solving logistics models which are classified as NP-complete combinatorial problem called fixed charge transportation problem (FCTP). Its resolution by exact methods is very difficult, consequently the metaheuristics can be exploited such as genetic algorithms. Then, we are very interested in improving the performance of GAs through the crossover operator that has a great effect. After presenting a comparison with three crossover operators (OX, OPEX, and PX) adapted to our problem with priority based encoding, we proposed a new hybrid operator with the two operators OX and PX that we called (HOPX). Numerical results were developed supporting the conclusion that the proposed HOPX operator is more efficient than the other presented operators either for the optimal solution level or the execution time, especially for larger instances.

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