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Decomposition-Coordinating Method for Parallel Solution of a Multi Area Combined Economic Emission Dispatch Problem

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ABSTRACT

Multi-area Combined Economic Emission Dispatch (MACEED) problem is an optimization task in power system operation for allocating the amount of generation to the committed units within the system areas. Its objective is to minimize the fuel cost and the quantity of emissions subject to the power balance, generator limits, transmission line and tie-line constraints. The solutions of the MACEED problem in the conditions of deregulation are difficult, due to the model size, nonlinearities, and the big number of interconnections, and require intensive computations in real-time. High-Performance Computing (HPC) gives possibilities for the reduction of the problem complexity and the time for calculation by the use of parallel processing techniques for running advanced application programs efficiently, reliably and quickly. These applications are considered as very new in the power system control centers because there are not available optimization methods and software based on them that can solve the MACEED problem in parallel, paying attention to the existence of the power system areas and the tie-lines between them. A decomposition-coordinating method based on Lagrange's function is developed in this paper. Investigations of the performance of the method are done using IEEE benchmark power system models.

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1. INTRODUCTION

Deregulation of the electricity industry has been deployed in many countries to improve the economic efficiency of power system operation [1]. Electric utility systems are interconnected to achieve high operating efficiency and to produce cheap electricity with minimum production cost, maximum reliability, and better operating conditions [2]. The term multi-area power system stands for the interconnected power system. In a multi-area power system, generation and loads are coordinated with each other through the tie-lines among the areas. A load change in any one of the areas is taken care of by all generators in all areas. Similarly, if a generator is lost in one control area, governing action from generators in all connected areas will increase generation outputs to make-up the mismatch [3].

The objective of Multi-Area Combined Economic Emission Dispatch (MACEED) problem is to determine the amount of the power generated by each generator in a system and the power transfer between the areas so as to minimize the total generating cost without violating the limitations on the power produced by the generators and on the amount of tie line power transfer. In a multi-area power system, the actual local generation may not be balanced with the local demand due to the import and export of power in various areas. Areas of individual power systems are interconnected to operate with maximum reliability, reserve sharing, improved stability and less production cost than when they operate as isolated areas. Consideration of the transmission capacity among the areas in the multi-area power system is one of the important problems in the operation of the power system, while solving the MACEED problem. Tie line power transfer limits and power demand between area's are considered as additional constraints in the MACEED problem. Hence the MACEED is considered as a large scale optimisation problem. MACEED is a complex problem with many different formulations and it has been considered following the development of the competitive electricity markets and the smart grid technologies. New formulation of MACEED problem is proposed in this paper.

The recent methods to solve the multi-area power system problems are based on decomposition of the overall power system problem into subproblems corresponding to the power system areas and a coordinator which are solved in an iterative way. Currently available decomposition techniques assume that the models and control objectives of the areas are formulated to be non-overlapping as it was mentioned in [4], i.e., the border of one area is at the same time also the border of a neighboring area. This type of multi-area system model is considered in the paper.

Multiarea Economic Dispatch (MAED) problem is reviewed in this section according to the methods used for its solution. They are broadly classified into classical and heuristics methods.

The papers [5]-[9] used classical methods to solve the MAED problem. The paper [5], [6] and [10] used Lagrange's algorithm for MAED problem solution. The papers [5] and [6], proposed an approach to incorporate power contracts into multi-area unit commitment and solve the economic dispatch solutions using an adaptive Lagrangian method. Combined Economic Emission Dispatch (CEED) problem for interconnected grids is investigated in [8]. Emissions and costs of generation are objective functions, and the interconnected grid is divided into several sub-grids. The calculation of each subarea problem is performed in parallel. A strategy is proposed that each sub-area sets different multi-objective function according to different conditions/constraints using a Linear Weighted Sum Method (LWSM). LWSM is used to change the multi-objective problem into single objective problem and it gives a preference degree of decision makers to the objective functions. The direct search method is extended to facilitate economic sharing of generation and reserve across areas and minimize the total generation cost in the multi-area reserve constrained economic dispatch in [11]. Jacobian based algorithm is used to calculate penalty factors for an area in a multi-area power system in [7]. The paper [9] developed a multi-area generation scheduling scheme that can provide proper unit commitment in each area, and effectively preserve the tie-line constraints using an improved dynamic programming method.

The research papers [9], [12]–[19] used heuristic methods to solve the MAED problem. The paper [12], presents a decomposition approach to multiarea generation scheduling problem using expert system. It proposes a large-scale mixed integer-nonlinear optimization process to solve the problem using a two-layer decomposition technique. In the first layer of decomposition, the problem is divided into several sub-problems during the considered period. The information that the problem sends to each sub-problem is the load demands of all areas at the corresponding time period and the output of the sub-problem is the system operation cost at that time. The coordination factor of this layer of decomposition is the operation cost of the system in the given period, which should be minimum. The second layer of decomposition divides the previous sub-problems further according to the control areas in the power pool. The sub-problem for each area receives system Lagrange multiplier λ and returns the area multiplier λ. The coordination at this level uses the difference between the system multiplier λ and the area multiplier λ which should be zero except for areas that reach their generation limits.

A Particle Swarm Optimisation (PSO) algorithm to solve various types of economic dispatch (ED) problems in power systems such as, multi-area ED with tie line limits, ED with multiple fuel options, combined environmental economic dispatch, and the ED of generators with prohibited operating zones using both the PSO method and the Classical Evolutionary Programming (CEP) approach is described in [13]. In the PSO method, there is only one population in each iteration that moves towards the global optimal point. This is unlike the CEP method, which has to deal with two populations, the parents and the children, in each iteration. This makes the PSO method computationally faster in comparison to the CEP method.

In [14] and [15], MACEED problem is investigated to address the environmental issue of the economic dispatch problem. The MACEED problem is first formulated and then an Improved Multi-objective Particle Swarm Optimization (MOPSO) algorithm is developed to derive a set of Pareto-optimal solutions. Each Pareto- optimal solution is a tradeoff between operational cost and pollutant emissions. The paper [16] reviews and compares some evolutionary techniques for Multi Area Economic Dispatch (MAED) problem solutions using i) Classical Differential Evolution (DE), ii) Classical Particle Swarm Optimization (PSO), and iii) An improved PSO with a parameter automation strategy having Time Varying Acceleration Coefficients (PSO_TVAC).

Fuzzified Particle Swarm Optimization (FPSO) algorithm for solving the security-constrained multi-area economic dispatch problem for an interconnected power system is investigated in [17] and [18]. The FPSO algorithm is based on the combined application of fuzzy logic strategy incorporated in the particle swarm optimization algorithm.

MACEED and SACEED problem is solved using PSO method in [19] and [20] respectively. SACEED problem with valve point is considered in [21]. The gravational search method is used in [22] and [23] to solve the economic dispatch problem with multiple generator systems with considering the prohibited operating zones.

In deregulated power system environment [24] and [25] it is essential to formulate and solve the economic dispatch problem for multi-area cases with tie line constraints. The new structure of the power system requires new optimisation methods based on the specifics of this structure and providing fast, reliable and relevant to the requirements of the power system as a whole and to its elements solutions. These solutions can give deeper insight into the dynamics of the system. Methods using various types of decomposition and coordination approaches in solution of the power dispatch problems are capable to respond to the above requirements.

The existing situation is that most of the conventional gradient and heuristic methods are time consuming and still use a sequential method to solve the MACEED problem [14]-[17], [26], and [27]. The traditional heuristic methods for MACEED problem do not always guarantee global best solutions; they often achieve a fast and near global optimal solution [1],[2],[14]-[19]. Researches have constantly observed that all these methods very quickly find a good local solution but get stuck there for a number of iterations without further improvement, sometimes causing premature convergence.

Therefore, this paper develops an efficient algorithm in dealing with a large-scale MACEED problem using the decomposition approach. In comparison to lambda, direct, dynamic, Jacobian and heuristic search techniques, the Lagrange's gradient method is more powerful tool for calculation of the complex optimisation problems. This method generally begins with an initial feasible solution and refines the solution repeatedly until the optimal solution is found.

Lagrange's decomposition-coordinating method and algorithm are developed for multi-area economic dispatch problem solution in Parallel MATLAB environment using a Cluster of Computers. The function of Lagrange is decomposed in a number of sub-functions of Lagrange according to the number of areas by using the values of the Lagrange variables as coordinating ones. Then the initial problem is decomposed in areas sub-problems and a coordinating sub-problem. The optimal solution of the coordinating sub-problem determines the optimal solutions of the areas sub-problems and the optimal solution for the initial multi-area problem. A four area three generator and a four area four generator IEEE bench mark models are used to test and validate the results obtained by the developed software in the MATLAB Cluster of Computers.

This paper formulates the MACEED problem in section 2, Lagrange's decomposition-coordinating method and algorithm for solution of the MACEED problem are described in section 3 and 4 respectively, single area and multi-area CEED problem solutions for 4 area 10 generator system and 4 area 12 generator systems are presented in section 5. The conclusion is given in section 6.

2. MATHEMATICAL FORMULATION OF THE MACEED PROBLEM

A schematic diagram of a multiarea power system is shown in Figure 1. The system has M areas. Every area has its own set of generators $N_m, m = \overline{1,M}$. The areas are interconnected by tie-lines as shown in Figure 1.

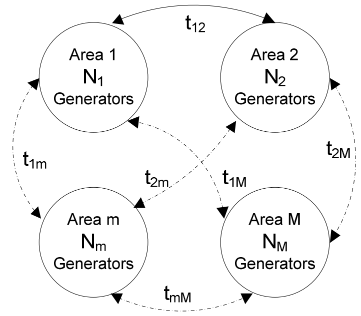


Figure 1. Model of a Multi-Area power system with tie-line power transfer

Single criterion or multi-criteria dispatch problems can be formulated for every area. Multi-criteria combined dispatch problem is considered in the paper for every area: Two types of criteria are considered:

Type one: Operational cost

The total operational cost is the sum of the generation cost and the cost of transmission of the power between the areas, as follows: The operational cost for power production in all areas

$$F_C(P) = \sum_{m=1}^{M}\sum_{n=1}^{N_m}\left(a_{mn}P_{mn}^2 + b_{mn}P_{mn} + c_{mn}\right) \tag{1}$$

where

M is the number of the interconnected areas

N_m is the number of generators belonging to area $m = \overline{1,M}$ and committed to the power production in this area

P_{mn} is the real power produced by the n^{th} generator in the m^{th} area

$P_m = \left[P_{m1}\ P_{m2}\ \ P_{m3}\ ......P_{mN_m}\right]^T$ is the vector of the real power produced in the m^{th} area, and $m = \overline{1,M}$

$P = \left[P_1\ P_2\ \ P_3\ ......P_M\right]^T$ is the vector of the real power produced in the whole power system

a_{mn}, b_{mn}, c_{mn} are the cost coefficients for the power produced by the n^{th} generator in the m^{th} area.

The operational cost for transmission of the real power through the tie-lines is given as:

$$F_T(P_T) = \sum_{m=1}^{M}\sum_{\substack{j=1\\ j\neq m}}^{M}\left(q_{mj}P_{Tmj} + q_{jm}P_{Tjm}\right) \tag{2}$$

Where

P_{Tmj} is the real power flow from the area m to the area j and

P_{Tjm} is the real power flow from the area j to the area m

$P_{Tm} = \left[P_{Tm,m+1}\ P_{Tm,m+2}\ \ P_{Tm,m+3}\ ......P_{Tm,M}\right]^T, m = \overline{1,M}$ is the vector of the real power transmission between the m^{th} area and all other area.

$P_T = \left[P_{T1}\ P_{T2}\ \ P_{T3}\ ......P_{T,M-1}\right]^T$ is the vector of the real power transmission between all areas.

Type two: Emission quantity

An additional criterion expressing minimisation of the pollutant emission is considered. This criterion refers only to the power generation. The tie-lines are not considered in this case because the transmission of the power does not create chemical pollution. Quantity of the pollutant emission caused by the power production is expressed as:

$$F_E(P) = \sum_{m=1}^{M}\sum_{n=1}^{N_m}\left(d_{mn}P_{mn}^2 + e_{mn}P_{mn} + f_{mn}\right) \tag{3}$$

Where:

d_{mn}, e_{mn}, f_{mn} are the emission coefficients of the n^{th} generator in the m^{th} area.

The combined solution of the real power dispatch problem for the multi-area system determines the optimal power to be produced by the generators in every area and the optimal values of the power transferred between the areas through the tie-lines.

The mathematical formulation of the MACEED problem criterion is based on introduction of a price penalty factors to convert the emission values of the criterion (3) into cost values and the dispatch problem to be described by a single criterion $F(P)$.

Various types of price penalty factors are proposed in [28]. Derivations in the paper are based on the min/max penalty factor, as follows:

$$h_{mn} = \sum_{m=1}^{M}\sum_{n=1}^{N_m}\frac{a_{mn}P_{mn,min}^2 + b_{mn}P_{mn,min} + c_{mn}}{d_{mn}P_{mn,max}^2 + e_{mn}P_{mn,max} + f_{mn}}\ [\$ / kg]$$

Where

$h_m = \left[h_{m1} \;\; h_{m2} \; .... \; h_{mN_m} \right]$ is the vector of the penalty factors for the m^{th} area, $m = \overline{1, M}$

$h = \left[h_1 \;\; h_2 \; .... \; h_M \right]$ is the vector of the penalty factors for the whole power system.

$P_{mn,\min}$ and $P_{mn,\max}$ are the minimum and the maximum real power that can be produced by the n^{th} generator in the m^{th} area.

Then the MACEED problem is formulated in the following way: Find the vector of the real power P and the vector of the power transmitted through the tie-lines P_T such that the criterion (4) is minimised under the following constraints for

$$F(P) = \sum_{m=1}^{M} \left[\begin{array}{l} \left[\sum_{n=1}^{N_m} \left(a_{mn} P_{mn}^2 + b_{mn} P_{mn} + c_{mn} \right) + \sum_{\substack{j=1 \\ j \neq m}}^{M} \left(q_{mj} P_{Tmj} + q_{jm} P_{Tjm} \right) \right] \\ + \sum_{n=1}^{N_m} h_{mn} \left(d_{mn} P_{mn}^2 + e_{mn} P_{mn} + f_{mn} \right) \end{array} \right] \quad (4)$$

i. Minimum and maximum real power produced by every generator

$$P_{mn,\min} \leq P_{mn} \leq P_{mn,\max}, \; n = \overline{1, N_m}, \; m = \overline{1, M} \quad (5)$$

ii. Minimum and maximum active power sent through the tie-lines

$$P_{Tm,\min} \leq P_{Tm} \leq P_{Tm,\max}, \; m = \overline{1, M} \quad (6)$$

These limits are valid for the two directions of the power flow and can be written as

$$\begin{array}{ll} P_{Tmj,\min} \leq P_{Tmj} \leq P_{Tmj,\max} & j = \overline{1, M}, \; j \neq m, \; m = \overline{1, M} \\ P_{Tjm,\min} \leq P_{Tjm} \leq P_{Tjm,\max} & j = \overline{1, M}, \; j \neq m, \; m = \overline{1, M} \end{array} \quad (7)$$

iii. Power balance

The balance between the power production and the power demand for the m^{th} area and for the whole system is given by Equation (8),

$$\sum_{n=1}^{N_m} P_{mn} = P_{Dm} + P_{Lm} + \sum_{\substack{j=1 \\ j \neq m}}^{M} \left[P_{Tmj} - \left(1 - \rho_{jm} \right) P_{Tjm} \right], \quad m = \overline{1, M} \quad (8)$$

Where

$$P_{Lm} = \sum_{n=1}^{N_m} \sum_{r=1}^{N_m} \left(P_{mn} B_{mnr} P_{mr} \right) + \sum_{n=1}^{N_m} B_{m0n} P_{mn} + B_{m00}, \quad m = \overline{1, M} \quad (9)$$

Where

$B_{mnr}, B_{m0n}, B_{m00}$ are the transmission loss coefficients of the interconnected power system

ρ_{jm} is the fractional loss rate from the area j to the area m

P_{Lm} is the real power loss of the n^{th} transmission line in the m^{th} area.

P_{Dm} is the real power demand in the m^{th} area.

P_{Tmj} is the real power flow from the area m to the area j

P_{Tjm} is the real power flow from the area j to the area m

Then equation (8) can be written as follows:

$$\sum_{n=1}^{N_m} P_{mn} = P_{Dm} + \sum_{n=1}^{N_m} \sum_{r=1}^{N_m} \left(P_{mn} B_{mnr} P_{mr} \right) + \sum_{n=1}^{N_m} B_{m0n} P_{mn} + B_{m00} + \sum_{\substack{j=1 \\ j \neq m}}^{M} \left[P_{Tmj} - \left(1 - \rho_{jm} \right) P_{Tjm} \right], \; m = \overline{1, M} \quad (10)$$

The formulated problem given by Equations (1 to 10) is characterised as:
- A combined optimisation dispatch problem for every area.
- The price penalty factors are introduced for every generator separately.
- The areas are interconnected by tie-lines with two direction of real power transfer.
- The optimisation dispatch problem for the interconnected power system is multicriterial one.
- Dimension of the interconnected problem depends on the number of areas, number of the generators and the tie-lines.

Calculation of the solution of the multicriterial interconnected problem is difficult and time consuming. If the solution has to be done often and in real-time, then better computational approaches are needed. One of them is a parallel computing of the area's sub-problems and coordination of the obtained areas solutions using a High Performance Parallel Computing (HPPC) environment (Cluster of computers). Figure 2 shows the structure of the Cluster of Computers working in MATLAB parallel computing software environment used to implement the optimisation algorithms. Parallelization of the solution is done through decomposition of the MAEED problem according to the power system interconnected areas and coordination of the obtained solutions for every area by a coordinator.

This approach requires:

i. A decomposition-coordinating method to be developed in order to obtain an algorithm for parallel calculation.
ii. Software development based on the above algorithm allowing allocation of the separate sub-problems to the structure of the HPPC environment – A Cluster of Computers.
iii. Implementation of the developed software in the HPPC environment, investigations of the capabilities of the developed algorithm.
iv. Evaluation and verification of the obtained solution by comparison of the results obtained by sequential and parallel ones.

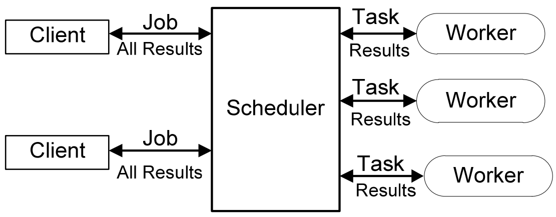
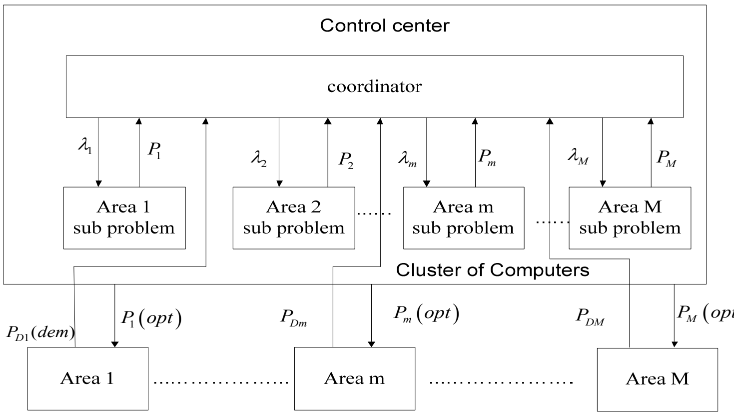


Figure 2. Structure of the MATLAB Parallel Computing Toolbox

The proposed method is based on classical Lagrange’s optimisation [28]-[31]. The literature review found that the classical Lagrange's method has been used till now only for sequential solution of the MACEED problem in [5],[8],[13]-[18], and [20]. The Lagranges method is introduced in early 1972 for the power system spinning reserve determination in a multi system configuration [32], an interim multi-area economic dispatch [33], and two area power systems economic dispatch problem in [34].

The paper introduces a parallel solution of the MAEED problem by considering two-level calculation structure, Figure 3, where the initial optimisation problem is decomposed into sub-problems (for every area one), solved on the first level, and the obtained solutions are coordinated by a coordinating sub-problem on the second level - in order to obtain the global solution of the whole problem.

Implementation of the MAEED problem in real-time is done in the following way, Figure 3: All data from the separate areas are sent using the communication system to the main control center where the Cluster of computers is located. The problem is solved and the optimal solutions are sent to the areas to be used for control of the generators power production. This scenario can be repeated through some selected periods of time, for example every hour.



where : λ_1 – coordinating variable; (opt) - optimal; (dem) - demand

Figure 3. Real-time implementation structure of the MAEED problem solution

3. DEVELOPMENT OF LAGRANGE'S DECOMPOSITION-COORDINATING METHOD FOR SOLUTION OF THE MACEED PROBLEM

Development of the method is based on construction of a function of Lagrange for the multi-area dispatch problem, as follows:

$$L = \begin{bmatrix} \sum_{m=1}^{M}\sum_{n=1}^{N_m}\left(a_{mn}P_{mn}^2 + b_{mn}P_{mn} + C_{mn}\right) + \sum_{m=1}^{M}\sum_{n=1}^{N_m} h_{mn}\left(d_{mn}P_{mn}^2 + e_{mn}P_{mn} + f_{mn}\right) + \sum_{m=1}^{M}\sum_{\substack{j=1\\ j\neq m}}^{M}\left(q_{mj}P_{Tmj} + q_{jm}P_{Tjm}\right) + \\ + \sum_{m=1}^{M}\left\{\lambda_m\left\{-\sum_{n=1}^{N_m}P_{mn} + P_{Dm} + \sum_{n=1}^{N_m}\sum_{r=1}^{N_m}P_{mn}B_{mnr}P_{mr} + \sum_{n=1}^{N_m}B_{m0n}P_{mn} + B_{m00} + \sum_{\substack{j=1\\ j\neq m}}^{M}\left[P_{Tmj} - \left(1-\rho_{jm}\right)P_{Tjm}\right]\right\}\right\} \end{bmatrix} \quad (11)$$

Where:

$\lambda = \left(\lambda_1 \quad \lambda_2 \quad \ldots.\lambda_m \ldots.\lambda_M\right)$ is the vector of the Lagrange's multipliers.

It is necessary to find the values of P_{mn}, P_{Tmj}, P_{Tjm} and λ_m, $m = \overline{1,M}$ such that the function of Lagrange has an optimum solution which is minimum according to P_{mn}, P_{Tmj}, P_{Tjm}, and maximum according to λ_m, under the constraints.

$$P_{mn,\min} \leq P_{mn} \leq P_{mn,\max}, m = \overline{1,M}, n = \overline{1,N_m} \quad (12)$$

$$P_{Tmj,\min} \leq P_{Tmj} \leq P_{Tmj,\max}, m = \overline{1,M}, j = \overline{1,M}, j \neq m \quad (13)$$

$$P_{Tjm,\min} \leq P_{Tjm} \leq P_{Tjm,\max}, m = \overline{1,M}, j = \overline{1,M}, j \neq m \quad (14)$$

The optimal solution is found on the basis of the necessary conditions for optimality according to the real powers and according to the Lagrange's multipliers as follows:

$$\frac{\partial L}{\partial P_{mn}} = 2a_{mn}P_{mn} + b_{mn} + 2h_{mn}d_{mn}P_{mn} + h_{mn}e_{mn} + \lambda_m\left\{-1 + 2\sum_{r=1}^{N_m}B_{mnr}P_{mr} + B_{m0n}\right\} = 0 \quad (15)$$

Where $n = \overline{1,N_m}$, and $m = \overline{1,M}$

$$\frac{\partial L}{\partial P_{Tmj}} = q_{mj} + \lambda_m = e_{pTmj} = 0,\ m = \overline{1,M},\ j = \overline{1,M},\ j \neq m \tag{16}$$

$$\frac{\partial L}{\partial P_{Tjm}} = q_{jm} - \lambda_m\left(1-\rho_{jm}\right) = e_{pTjm} = 0,\ m = \overline{1,M},\ j = \overline{1,M},\ j \neq m \tag{17}$$

Where e_{pTmj} , and e_{pTjm} are the values of the corresponding derivatives.

$$\frac{\partial L}{\partial \lambda_m} = -\sum_{n=1}^{N_m} P_{mn} + P_{Dm} + \sum_{n=1}^{N_m}\sum_{r=1}^{N_m} P_{mn}B_{mnr}P_{mr} + \sum_{n=1}^{N_m} B_{m0n}P_{mn} + B_{m00} + \sum_{\substack{j=1\\ j\neq m}}^{M}\left[P_{Tmj} - \left(1-\rho_{jm}\right)P_{Tjm}\right] = e_{\lambda m} = 0 \tag{18}$$

where $e_{\lambda m}$ is the value of the derivative, $m = \overline{1,M}, j = \overline{1,M}, j \neq m$

Solution of the system of Equations (15) – (18) can be achieved in a hierarchical calculation structure using decomposition approach based on selection of coordinating variables [28]. These are selected to be $\lambda_m, m = \overline{1,M}$

It is supposed that the values of the coordinating variables are given by the coordinator in the two level hierarchical structure of calculations, Figure 3 as follows:

$$\lambda_m = \lambda_m^l, m = \overline{1,M} \tag{19}$$

Where *l* is the index of the iterations on the coordinating level

When λ_m is known the Equation (15) can be solved in a decentralized way on a first level of the calculation structure and the Equations (16) to (18) can be solved on the second level for the obtained on the first level variables P_{mn}

Derivation of the solution of equation (15) is as follows: Equation (15) is transformed to express explicitly all terms of P_{mn} :

$$\frac{\partial L}{\partial P_{mn}} = 2a_{mn}P_{mn} + 2\lambda_m B_{mnn}P_{mn} + b_{mn} + 2h_{mn}d_{mn}P_{mn} + h_{mn}e_{mn} + \lambda_m\left(2\sum_{\substack{r=1\\ r\neq n}}^{N_m} B_{mnr}P_{mr} + B_{m0n} - 1\right) = 0 \tag{20}$$

If $\lambda_m \neq 0$ and is given by the coordinator Equation (20) can be written as:

$$\frac{\partial L}{\partial P_{mn}} = \left(\frac{a_{mn}}{\lambda_m} + \frac{h_{mn}d_{mn}}{\lambda_m} + B_{mnn}\right)P_{mn} + \sum_{\substack{m=1\\ m\neq n}}^{N_m} B_{mnr}P_{mr} + \frac{1}{2}\left(\frac{b_{mn}}{\lambda_m} + \frac{h_{mn}e_{mn}}{\lambda_m} + B_{m0n} - 1\right) = 0 \tag{21}$$

From here

$$\left(\frac{a_{mn} + h_{mn}d_{mn}}{\lambda_m} + B_{mnn}\right)P_{mn} + \sum_{\substack{r=1\\ r\neq n}}^{N_m} B_{mnr}P_{mr} = \frac{1}{2}\left(1 - \frac{b_{mn} + h_{mn}e_{mn}}{\lambda_m} - B_{m0n}\right), \begin{array}{l} m = \overline{1,M} \\ n = \overline{1,N_m} \end{array} \tag{22}$$

Equation (22) can be written in a vector matrix form in Equation (23)

$$\begin{bmatrix} \frac{a_{m1}+h_{m1}d_{m1}}{\lambda_m}+B_{m11} & B_{m12} & .. & B_{m1N_m} \\ B_{m21} & \frac{a_{m2}+h_{m2}d_{m2}}{\lambda_m}+B_{m22} & ... & B_{m2N_m} \\ : & : & : & : \\ B_{mN_m1} & B_{mN_m2} & .. & \frac{a_{mN_m}+h_m d_m}{\lambda_m}+B_{nN_mN_m} \end{bmatrix} \begin{bmatrix} P_{m1} \\ P_{m2} \\ : \\ P_{mN_m} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1-\frac{b_{m1}+h_{m1}e_{m1}}{\lambda_m}-B_{m01} \\ 1-\frac{b_{m2}+h_{m2}e_{m2}}{\lambda_m}-B_{m02} \\ : \\ 1-\frac{b_{mN_m}+h_m e_m}{\lambda_m}-B_{n0N_m} \end{bmatrix}, m=\overline{1,M} \quad (23)$$

In a short form Equation (23) can be written as

$$E_m P_m = D_m, m=\overline{1,M} \quad (24)$$

If the value of $\lambda_m^l, m=\overline{1,M}$ is known, then the solution for the active power of the m^{th} area for the lth iteration is

$$P_m^l = E_m^l \setminus D_m^l, m=\overline{1,M} \quad (25)$$

Solution of the equations (16) and (17) can be done by gradient procedures as follows:

a) Initial values of $P_{Tmj}^{q_m}$ *and* $P_{Tjm}^{q_m}$ are guessed

$$P_{Tmj} = P_{Tmj}^{s_m} \text{ and } P_{Tjm} = P_{Tjm}^{g_m}, s_m = 1, g_m = 1$$

Where $s_m = \overline{1,k_m}$ and $g_m = \overline{1,k_m}$ are the indexes of the gradient procedures in the m^{th} area for the tie-line powers $P_{Tmj}^{s_m}$ and $P_{Tjm}^{g_m}$ respectively, $k_{m,}$ $m=\overline{1,M}$ are the maximum number of iteration for the m^{th} area. The value of k_m can be different for each of the areas.

b) The improved values of the tie-lines real power are

$$P_{Tmj}^{s_m+1} = P_{Tmj}^{s_m} - \alpha_{Tmj}\, e_{PTmj}^{s_m} \quad (26)$$

$$P_{Tjm}^{g_m+1} = P_{Tjm}^{g_m} - \alpha_{Tjm}\, e_{PTjm}^{g_m} \quad (27)$$

Where α_{Tmj} and α_{Tjm} are the steps of the gradient procedures.

$e_{PTmj}^{s_m}$ and $e_{PTjm}^{g_m}$ are given by Equations (16) and (17), $m=\overline{1,M}, j=\overline{1,m}, j \neq m$

The calculation of the values of the $P_{Tmj}^{s_m+1}$ *and* $P_{Tjm}^{g_m+1}$ stops when

$$e_{PTmj}^{s_m} \leq \varepsilon_1 \text{ and } e_{PTjm}^{g_m} \leq \varepsilon_2 \quad (28)$$

Where: ε_1 and ε_2 are very small positive numbers.

Solutions (25), (26) and (27) depend on $\lambda_m, m=\overline{1,M}$.

When the optimal value of λ_m is obtained, the values of $P_{mn,} P_{Tmj}$ and P_{Tjm} will reach their optimum. A gradient procedure is used to calculate the optimal value of $\lambda_m, m=\overline{1,M}$, as follows:

$$\lambda_m^{l+1} = \lambda_m^l + \alpha_{\lambda m} e_{\lambda m}^l, m=\overline{1,M} \quad (29)$$

Where $\alpha_{\lambda m}, m=\overline{1,M}$ are the steps of the gradient procedure and $e_{\lambda m}^l, m=\overline{1,M}$ is given by equation (18). The gradient procedure on the second level stops when

$$e^{l}_{\lambda m} \le \varepsilon_3,\ \varepsilon_3 \ge 0, m = \overline{1,M} \tag{30}$$

Or when l reaches the maximum number of iterations $L_{\lambda m}$.

4. ALGORITHM OF THE LAGRANGE'S DECOMPOSITION-COORDINATING METHOD FOR CALCULATION OF THE MAEED PROBLEM

The steps of the Lagrange's decomposition-coordinating method are:

1. Values of all coefficients are given and the number of iterations of the second $L_{\lambda m}\left(k_m, m = \overline{1,M}\right)$ are given.
2. Initial values of the coordinating variables $\lambda^{l}_{m}, m = \overline{1,M}$ are guessed, l=1
3. Initial values of the tie-lines active powers are guessed $P^{s_m}_{Tmj}, P^{g_m}_{Tjm}, m = \overline{1,M}, j = \overline{1,M}, j \ne m$
4. Calculation on the first level are done for every subsystem $m = \overline{1,M}$ using Equation (25).
5. The solutions of the first level $P^{l}_{mn}, n = \overline{1,N_m}, m = \overline{1,M}$ are checked according to the constraints (12), and are sent to the second level.
6. On the second level the values of the tie-lines are calculated by gradient procedures:
   i. Equations (26) and (13) are solved
   ii. Equations (27) and (14) are solved

   The gradient procedures (26) and (27) stop when the conditions (28) are fulfilled respectively or the maximum number of iterations $k_m, m = \overline{1,M}$ is reached.
7. The solutions of the second level for P^{l}_{Tmj} and P^{l}_{Tjm}, $m = \overline{1,M}, j = \overline{1,M}, j = m$ are checked according to the constraints (13) and (14).
8. On the second level the improved values of the Lagrange's variables are determined. Equations (18) and (29) are calculated. The gradient procedure (29) stops when the condition (30) is fulfilled or the maximum number of iterations $L_{\lambda m}$ is reached. In this case the optimal solution for $\lambda_m, m = \overline{1,M}$ is obtained and the corresponding to it optimal solutions of the problems on the first and second levels are obtained. The iterations on the second level for calculation of $\lambda_m, m = \overline{1,M}$ can stop before reaching the optimum solution if a maximum number of iteration $L_{\lambda m}$ is determined.

The hierarchical calculating structure is shown in Figure 4. In the above algorithm for one-step of the gradient procedure for optimisation of λ_m on the 2nd level, k_m maximum number of iterations on the second level are performed to obtain the optimal solutions for the tie-line real powers and one calculation of the generators real power is done on the first level.

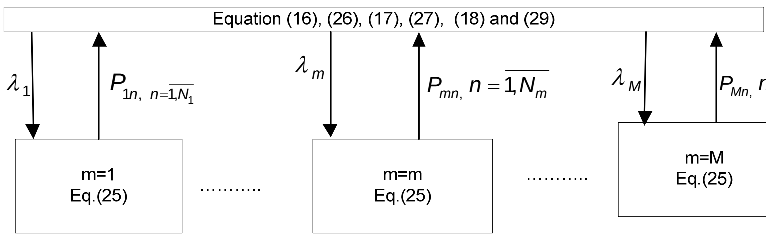


Figure 4. Hierarchical structure for the MAEED problem solution using the developed Lagrange's decomposition-coordinating algorithm

5. STUDIES OF THE SINGLE AREA AND MULTI-AREA CEED PROBLEM SOLUTIONS

The proposed algorithm is tested using two benchmark models of single and multi-area power systems. They are:

(i) Four areas with ten generators in each area without considering the transmission line losses
(ii) Four area with three generators in each area with considering the transmission line losses

The two bench mark models are applied for two considered scenarios, they are:

a) The whole power system is considered as a single area one prior to decompose the power system into multi-areas. The tie-line constraints are not considered.
b) The whole power system is decomposed into multi-areas with tie-line constraints.

5.1. Single Area Combined Economic Emission Dispatch (SACEED) problem solutions

5.1.1. Case study 1: Four areas power system with ten generators in each area without considering the transmission line losses

The fuel cost and emission data of the system are given in [32]. The generators in each area have different fuel and emission characteristics and the tie-lines have different transfer limits. The system has a total power demand of 10500 [MW]. The transmission cost is neglected in the process of problem solution since it is normally small as compared with the total fuel cost. The initial value of λ_m for *m=1* is assumed as 4 and the maximum number of iterations is set to 10000. The single area economic dispatch problem is solved for power demand of 10500 [MW] in a sequential way and the results are given in Table 1.

The flowchart of the Lagrange's decomposition-coordinating algorithm for calculation of parallel solution of a MACEED problem is given in Figure 5.

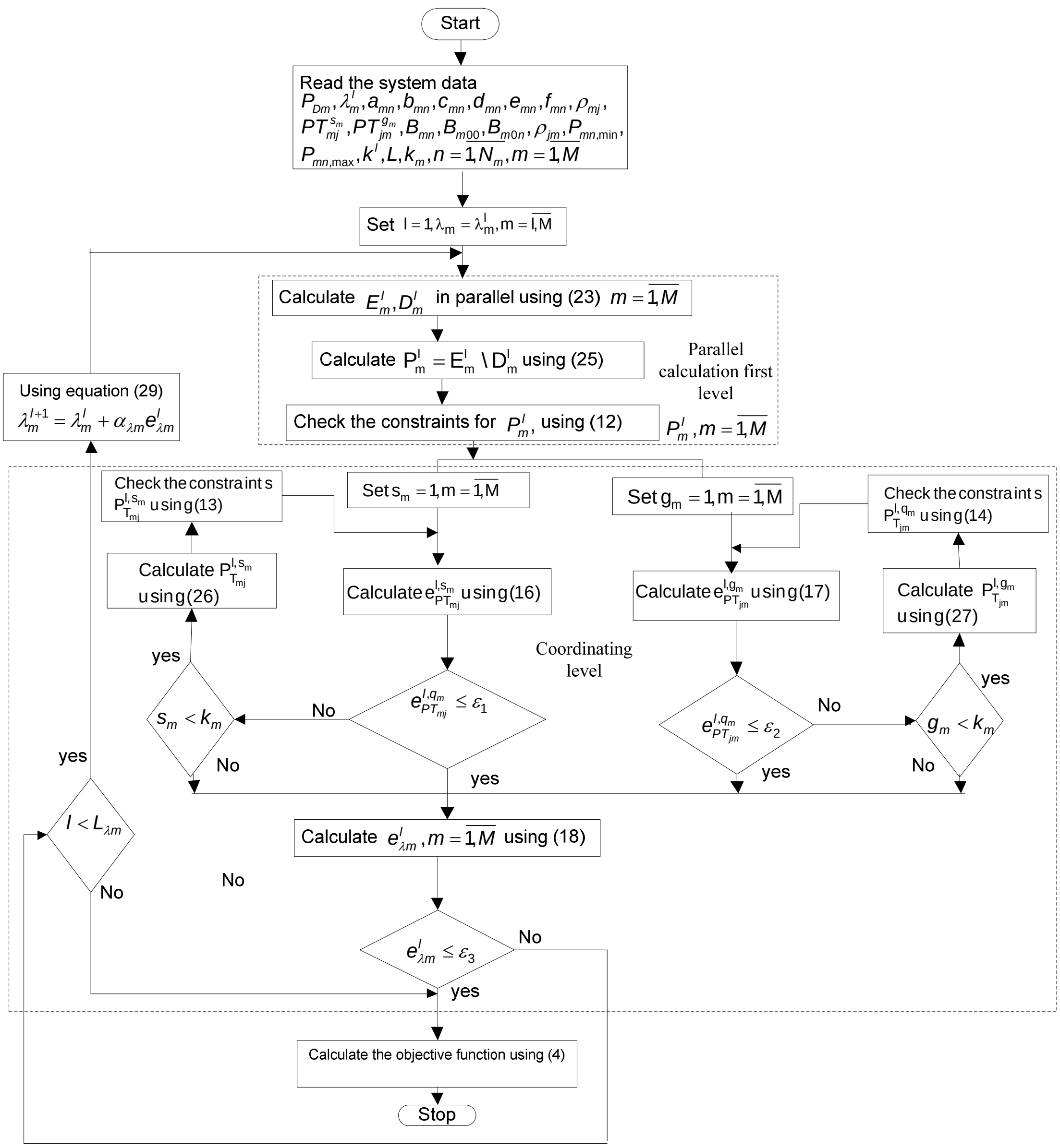


Figure 5. Decomposition-coordinating method flow diagram for parallel solution of the MACEED problem

Table 1. Results from the single area CEED problem solution for the forty generator system with the power demand of 10500 MW

P_D[MW]	λ	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
10500	34.09	114.00	114.00	108.98	166.32	97.00	131.45	286.81	300.00	300.00	168.41
P11	P12	P13	P14	P15	P16	P17	P18	P19	P20	P21	P22
215.73	213.77	291.36	320.92	320.35	320.35	465.51	467.27	509.65	509.65	550.00	550.00
P23	P24	P25	P26	P27	P28	P29	P30	P31	P32	P33	P34
550.00	550.00	550.00	550.00	14.03	14.03	14.03	97.00	176.59	176.59	176.59	90.00
P35	P36	P37	P38	P39	P40	F_C [$/h]	F_E[t/h]	F [$/h]	Number of iterations	Computation Time [s]	
90.00	90.00	110.00	110.00	110.00	509.65	121425.58	76610.17	220809.06	75	0.0191	

Table 2. Results from the four-area – forty generator MACEED problem solution

Generator real power values in [MW]	Area 1 P_D 1575 [MW]	Area 2 P_D 4200 [MW]	Area 3 P_D 3150 [MW]	Area 4 P_D 1575 [MW]
		Total P_D =10500 MW		
P1	110.44	281.09	514.74	163.93
P2	110.44	279.83	514.18	163.93
P3	83.05	386.32	515.62	163.93
P4	126.71	426.08	515.62	90.00
P5	97.00	424.35	487.85	90.00
P6	99.05	424.35	487.85	90.00
P7	222.90	500.00	11.88	109.08
P8	249.42	500.00	11.88	109.08
P9	252.69	550.00	11.88	109.08
P10	130.00	550.00	97.00	446.71
λ_m	26.77	43.12	29.31	30.27
F_C [$/h]	16032.35	59123.69	34066.52	14914.37
F_E [ton/h]	13852.85	42533.18	18591.71	4609.26
F [$/h]	27217.69	109912.01	62334.97	26608.50
Tie-line power in MW				
56.97	-45.83	-104.44	32.10	32.96 32.22

Table 3. Comparison of the single area ceed and the MACEED problem solutions obtained by the developed Lagrange's algorithm with the Differential Evolution (DE) solution adopted from [35]

Criteria function	Cost F_C in [$/hr] (x$10^5$)	Emission F_E in [t/hr] (x10^4)	Total cost F in [$/hr] (x$10^5$)	Number of Iterations	Computation Time [s]
SACEED problem solution for P_D 10500 [MW] using developed Lagrange's method	1.21425	7.6610	22.080906	75	0.0191
MACEED problem solution using developed Lagrange's decomposition-coordinating method	1.24136	7.9587	22.607318	3765	132.13
DE solution for P_D 10500 [MW] adopted from [35]	1.2184	3.7479	------------	---------	----------

5.2. Multi-area solution

The initial vector of the Lagrange's variable λ_m for each area is assumed as [4 4 4 4], maximum number of iterations is set to 10000. The whole power system is decomposed into four areas with considering the tie-line constraints. The loads of the four areas are subdivided into 15%, 40%, 30% and 15% of the total power demand. The multi-area economic emission dispatch problem is solved in a task-parallel way in a Cluster of Computers. 10 generators are assigned to one individual worker using Matlab Distributed Computing Engine (MDCE). The results are given in Table 2. The comparison of the single area CEED and multi-area CEED problem solutions obtained by the developed Lagrange's and by the Differential Evolution method adopted from the reference paper [35] is given in Table 3.

5.2.1. Case study 2: Four areas power system with three generators in each area and considering the transmission line losses

The fuel cost and emission data of the system are given in [20]. The generators in each area have different fuel and emission characteristics. The tie-lines have different real power transfer limits. The area power demands are 500, 410, 580 and 600 MW respectively. The single area economic dispatch problem is solved for a power demand of 2090 [MW] in a sequential way and the simulation results are given in Table 4. The optimized generator real power and tie-lines real power values of the four area three generator MACEED problem solution are given in Table 5. The fuel cost and emission values for the four-area three-generator MACEED problem solution are given in Table 6. The comparison of the SACEED and the MACEED solutions for the four-area three generator system is given in Table 7.

Table 4. Results from the solution of the SACEED problem for twelve generator system

P_D[MW]	λ	P1	P2	P3	P4	P5	P6	P7	P8
2090	178.0816	182.7879	302.6497	287.4964	130.5891	110.0000	147.6606	142.9309	181.2219
P9	P10	P11	P12	P_L [MW]	Fuel cost F_C [$/h]	Emission F_E [kg/h]	Total cost F [$/h]	Number of iterations	Computation Time [s]
168.9267	126.3394	140.9621	236.9631	68.5286	125883.5243	1684.6603	225665.1721	1360	0.3836

Table 5. Optimised generator real power and tie-line power values of the four area three generator MACEED problem for the power demand of 2090 mw

Area 1 P_D 500 [MW]		Area 2 P_D 410 [MW]		Tie-line Powers in [MW]	
P1 [MW]	131.45	P4 [MW]	150.00	P_{T12}	9.9944
P2 [MW]	209.49	P5 [MW]	110.00	P_{T13}	9.9944
P3 [MW]	202.25	P6 [MW]	191.63	P_{T14}	9.9944
Area 3 P_D 580 [MW]		Area 4 P_D 600 [MW]		P_{T23}	9.9881
P7 [MW]	175.00	P10 [MW]	164.09	P_{T24}	9.9881
P8 [MW]	215.00	P11 [MW]	180.20	P_{T34}	9.9876
P9 [MW]	236.38	P12 [MW]	306.18		

Table 6. Fuel cost and emission values for the four-area three-generator MACEED problem solution

Area	P_D in [MW]	λ	P_L [MW]	Fuel cost F_C in [$/hr]	Emission F_E in [ton/hr]	Total cost F in [$/hr]
1	500	112.04	13.22	26920.22	355.92	42755.42
2	410	240.95	11.66	28554.97	376.72	55923.87
3	580	249.14	16.41	44506.54	742.93	84539.67
4	600	248.56	20.51	44076.47	448.09	83181.94
Total	2090	--------------	61.82	144058.20	1923.70	266400.92

Table 7. Comparison of the SACEED and the maeed solutions with the PSO solution adopted from [20]

Method	Lagrange's method (Developed)		PSO adopted from [20]
Criteria	SACEED problem solution for P_D 2090 [MW]	MACEED problem solution for the area P_D of [500 410 580 600] [MW]	PSO MACEED problem solution in a sequential way for area P_D of [500 410 580 600] [MW]
P_L in [MW]	68.5286	61.82	49.54
Fuel cost F_C in [$/hr]	125883.5243	144058.20	132416.90
Emission, F_E in [kg/hr]	1684.6603	1923.70	1645.20
Total cost, F in [$/hr]	225665.1721	266400.92	269747.40
Number of iterations	1360	2392	--------------
Computation Time [s]	0.3836	101.23	--------------

5.3. Discussion on the results for the multi-area economic emission dispatch problem solution

In deregulated power system environment it is essential to formulate and solve the economic dispatch problem for multi-area cases with tie line constraints. The new structure of the power system requires new optimisation methods based on the specifics of this structure and providing fast, reliable and relevant to the requirements of the power system as a whole and to its elements solutions. These solutions can give deeper insight into the dynamics of the system. The existing situation is that most of the conventional gradient and heuristic methods are time consuming and still use a sequential method to solve the MACEED problem. Therefore this paper developed Lagrange's decomposition-coordinating method and algorithm for multi-area economic dispatch problem solution in Parallel MATLAB environment using a Cluster of Computers. The function of Lagrange is decomposed in a number of sub-functions of Lagrange according to the number of areas by using the values of the Lagrange variables as coordinating ones. Then the initial problem is decomposed in areas sub-problems and a coordinating sub-problem. The optimal solution of the coordinating sub-problem determines the optimal solutions of the areas sub-problems and the optimal solution for the initial multi-area problem. This paper provides comparison of single and multi-area dispatch problem solutions is given in Table 3 and 7 respectively for the two considered power systems. The fuel cost of a MACEED problem solution is bigger in comparison with the SACEED problem solution. The fuel cost of both single and multi-area are not the same. It is accepted that amount of tie-line power flows in multi-area system tends to increase the initial cost and operation cost in comparison with the single area one. The proposed decomposition coordinating method solution shows that the total cost is less in comparison with this of the PSO method given in [20] for the power demand of 2090 [MW]. The distribution of the demand between the areas can be done in many ways. The results show that values of the fuel cost, emission and total cost will be different for every case and the generator and tie-lines real powers are same.

6. CONCLUSION

MACEED problem is an optimisation task in power system operation for allocating amount of generation to the committed units within these areas. Its objective is to minimize the fuel cost and emission subject to the power balance, generator limit, and transmission line and tie-line constraints. Large interconnected power systems (Multi-area) are usually decomposed into areas or zones based on criteria, such as the size of the electric power system, network topology and geographical location. The Multi Area Combined Economic Emission Dispatch (MACEED) problem is solved using the Lagrange's decomposition method in this paper. The solution of the MACEED problem determines the amount of power that can be economically generated in the areas and transferred to other areas if it is needed without violating tie-line capacity constraints and the whole power network constraints. The problem formulation allows every area to determine its own cost for the power production. The impact of the cost can be seen immediately through the solution of the MACEED problem. The MACEED problem solution is obtained using a Cluster of Computers. The approach to the solution of the MACEED problem and the experience with it supports the process of deregulation of the power system. The developed new method, algorithm and software form part of the set of the energy management optimisation problems required to be solved in real-time for development and building of the Smart grid.

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