

## Design of Observer-Based Robust Power System Stabilizers

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### ABSTRACT

Power systems are subject to undesirable small oscillations that might grow to cause system shutdown and consequently great loss of national economy. The present manuscript proposes two designs for observer-based robust power system stabilizer (PSS) using Linear Matrix Inequality (LMI) approach to damp such oscillations. A model to describe power system dynamics for different loads is derived in the norm-bounded form. The first controller design is based on the derived model to achieve robust stability against load variation. The design is based on a new Bilinear matrix inequality (BMI) condition. The BMI optimization is solved iteratively in terms of Linear Matrix Inequality (LMI) framework. The condition contains a symmetric positive definite full matrix to be obtained, rather than the commonly used block diagonal form. The difficulty in finding a feasible solution is thus alleviated. The resulting LMI is of small size, easy to solve. The second PSS design shifts the closed loop poles in a desired region so as to achieve a favorite settling time and damping ratio via a non-iterative solution to a set of LMIs. The approach provides a systematic way to design a robust output feedback PSS which guarantees good dynamic performance for different loads. Simulation results based on single-machine and multi-machine power system models verify the ability of the proposed PSS to satisfy control objectives for a wide range of load conditions.

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## 1. INTRODUCTION

Power systems exhibit undesirable oscillations at low frequencies that may decay gradually, or grow continuously resulting in system separation. The source of such oscillation is continuous load changes, series of lightning strokes and the associated auto-reclosure of circuit breakers, grid topology changes..etc. Power system generators are equipped with Automatic Voltage Regulators (AVR) to control its terminal voltage. However, the high-gain AVR may add negative damping to the system and worsen its dynamic stability. A supplementary stabilizing signal is added to the excitation using Power System Stabilizer (PSS) [1] to overcome this problem.

The main problem encountered in the Classical PSS (CPSS) design is that when the controller is designed at one operating point, there is no guarantee that it performs well on another. Many approaches, based on robust control theory, have been suggested to cope with model uncertainties. In [1], an interval plant model is considered to capture power system uncertainties and Kharitonov theorem is adopted to design a first order PSS. Robust PSS design using robust control techniques such as  $H_\infty$  optimization and  $\mu$  synthesis is given in [2],[3]. An output feedback PSS is designed using LMIs to guarantee robust stability is given in [4]. A regional pole constraint is added and a quadratic performance index is minimized, for only the nominal

plant. The problems of nominal system design are: (1) it does not guarantee adequate damping for other operating conditions, (2) the performance is guaranteed around the nominal operating point only. For this, a mixed  $H_2/H_\infty$  controller to achieve robust pole placement for Flexible AC Transmission Systems (FACTS) is given in [5]. Robust pole placement for polytopic uncertainty using an LMI-based state feedback PSS is given in [6]. FACTS controllers are used to enhance power system oscillation damping is given in [7]. Stabilization of power systems subject to drastic changes such as controllers' failure, is given in [8],[9]. [10] models power systems subject to a series of lightning strokes, and the consequent circuit breakers auto-reclosure, as a Markov chain. The PSS is then designed to tackle this situation. Recently Fractional-order PID-PSS appears in [11].

In this manuscript, power systems small oscillations around an operating point are represented by a linear model. The uncertainty due to load variation is modeled in the form of a norm-bounded structure. Two PSS designs, based on this model, are presented. The first design guarantees robust stability under all expected loads. The design of the robust output feedback controller is carried out as a BMI optimization problem. An algorithm is proposed to solve this BMI optimization problem in the numerically efficient LMI framework [12]. In addition to the constraint of robust stability, the second PSS design controls both the desired settling time  $t_s$  and damping ratio  $\zeta$  [13] under different loads by forcing the closed loop poles to lie in a desired domain.

The manuscript is organized as follows. The problem is formulated in Sec. 2 and power system dynamics, around an operating point, are modelled in the norm-bounded form. Two PSS designs are described in Sec. 3 in the form of LMIs; the first design achieves robust stability, while the other satisfies regional pole placement constraint. Simulation results for single-machine infinite-bus and multi-machine test power systems are given in Sec. 4. Sec. 5 concludes the paper.

**Notation and facts** [14]: The notation used throughout this paper is standard. Capital letters denote matrices, small letters denote vectors and small Greek letters denote scalars.  $W', W^{-1}$  denotes the transpose, and the inverse of any square matrix  $W$ , respectively.  $W > 0$  ( $W < 0$ ) denotes a symmetric positive (negative) -definite matrix  $W$ , and  $I$  denotes the identity matrix of an appropriate dimension.

The symbol  $\bullet$  is used as an ellipsis for terms in matrix expressions that are induced by symmetry.

**Fact 1:** The congruence transformation  $H'WH$  does not change the definiteness of  $W$ .

**Fact 2:** For any real matrices  $W_1, W_2$ , and  $\Delta(t)$  with appropriate dimensions, where  $\Delta'\Delta \leq I$ ,  $\leftrightarrow \|\Delta\| \leq 1$ , it follows that  $W_1 \Delta W_2 + W_2' \Delta' W_1' \leq \varepsilon^{-1} W_1 W_1' + \varepsilon W_2' W_2$ ,  $\varepsilon > 0$ , where  $\Delta(t)$  represents system bounded norm uncertainty.

**Fact 3:** (Schur complement): This fact is used to transform a non-linear matrix inequality to a linear one. Given constant matrices  $W_1, W_2$ , and  $W_3$ , where  $W_1' = W_1$ , and  $0 < W_2 = W_2'$ , it follows that

$$W_1 + W_3' W_2^{-1} W_3 < 0 \leftrightarrow \begin{bmatrix} W_1 & W_3' \\ W_3 & -W_2 \end{bmatrix} < 0$$

## 2. PROBLEM FORMULATION

The case study system is a single machine connected to an infinite-bus through a tie line. The generator is equipped with an Automatic Voltage Regulator (AVR) and a fast static exciter. The system dynamics is represented by the fourth order linearized model [15]. The following state space model represents the:

$$\dot{x} = A^* x + Bu, y = Cx \quad (1)$$

where:

$$x = [\Delta\delta \quad \Delta\omega \quad \Delta E_q' \quad \Delta E_{fd}]$$

$$A^* = \begin{bmatrix} 0 & \omega_o & 0 & 0 \\ -\frac{k_1}{M} & 0 & -\frac{k_2}{M} & 0 \\ -\frac{k_4}{T_{do}'} & 0 & -\frac{1}{T_{do}'} & -\frac{1}{T_{do}'} \\ -\frac{K_E k_5}{T_E} & 0 & -\frac{K_E k_6}{T_E} & -\frac{1}{T_E} \end{bmatrix}, B = [0 \ 0 \ 0 \ K_E/T_E], C = [0 \ 1 \ 0 \ 0],$$

$$A^* \in R^{n \times n}, B \in R^{n \times m}, C \in R^{l \times n}.$$

The symbols above have their usual meaning [15],[16]. Different PSS inputs can be used, like machine shaft speed, bus frequency or accelerating power. The matrix C is so selected, because the most commonly used is the speed variation  $\Delta\omega$ .

Typical data for the case study system is given in per unit, unless otherwise stated, as follows:  
Synchronous machine parameters:

$$x_d = 1.6, x_q = 1.55, x_d' = 0.32, T_{do}' = 6, M = 10s, V = 1, \omega_o = 314 \text{ rad/s}, \text{ Rating} = 100\text{MVA}$$

Exciter parameters:  $K_E = 50, T_E = 0.05s$  and transmission line reactance:  $x_e = 0.4$ .

The k-parameters of the model depend on the real power loading P, and the reactive power loading Q. Direct analytical expressions that relate parameters ( $k_1, k_2, \dots, k_6$ ) to (P,Q) are derived in [1]. The load conditions (P,Q) at heavy, nominal, and light load are: (1,0.5), (0.7, 0.3), and (0.4, 0.1) respectively. The corresponding model matrices are given by:

$$A^{nom} = \begin{bmatrix} 0 & 314 & 0 & 0 \\ -0.1186 & 0 & -0.0906 & 0 \\ -0.1934 & 0 & -0.4633 & 0.1667 \\ -11.864 & 0 & -511.6 & -20 \end{bmatrix}, A^{light} = \begin{bmatrix} 0 & 314 & 0 & 0 \\ -0.0875 & 0 & -0.0759 & 0 \\ -0.1619 & 0 & -0.4633 & 0.1667 \\ -54.214 & 0 & -511.94 & -20 \end{bmatrix},$$

$$A^{heavy} = \begin{bmatrix} 0 & 314 & 0 & 0 \\ -0.1445 & 0 & -0.0976 & 0 \\ -0.2082 & 0 & -0.4633 & 0.1667 \\ 32.097 & 0 & -525.44 & -20 \end{bmatrix}, B^{nom} = B^{light} = B^{heavy} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1000 \end{bmatrix}, C^{nom} = C^{light} = C^{heavy} = [0 \ 1 \ 0 \ 0]$$

At different loads, system (1) can be cast in the following norm-bounded form:

$$\begin{aligned} \dot{x} &= (A + \Delta A)x + Bu, \Delta A = M\Delta(t)N, \|\Delta(t)\| < 1 \\ y &= Cx \end{aligned} \quad (2)$$

The matrices  $M$  and  $N$  are known constant real matrices, and  $\Delta(t)$  is the uncertain parameter matrix. The matrix  $\Delta A$  has bounded norm given by  $\|\Delta(t)\| \leq 1$ . It is worth mentioning that  $\Delta(t)$  can represent power system uncertainties, unmodelled dynamics, and/or non-linearities. For the case study system,  $M = [0 \ 0 \ 0 \ 6.63]$ ,  $N = [6.63 \ 0 \ -2.08 \ 0]$ . Note that other types of uncertainties, e.g. line outages, can be tackled in a similar way. Our control targets consider two design cases as listed below:

**Design case #1:** To design an observer-based PSS that retains the stability for different loads, i.e. it preserves robust stability.

**Design case #2:** In some cases, robust stability might not be enough to provide satisfactory dynamic performance. The proposed controller has to damp power system oscillations, following any small disturbance, within 10 to 15 s [13]. This requires the desired settling time  $t_s \in [10-15]$  s. Since  $t_s = 4/\sigma$ , the closed-loop poles has to be placed to the left of the vertical line  $-\sigma$ ,  $\sigma = 0.3$ . In other words, the closed loop system has to achieve a prescribed degree of stability around 0.3, (Figure 1a). Another constraint has to be satisfied. The desired damping ratio ( $\zeta$ ) should be more than 10% [13].

If the closed-loop poles are forced to lie inside the circle, domain D, which touches the two lines of  $\zeta_{min}$  and  $-\sigma$ , both damping ratio and settling time can be achieved, Figure 1b. This is termed regional pole placement or robust- $D(r,q)$  stability. Where  $-q$ , and  $r$  are the circle center and radius, respectively. For  $\zeta_{min}=0.1$  and  $\sigma = 0.3$ , the two parameters of the desired circular region  $D(q,r)$  are computed as  $q = 59.8496$ , and  $r = 59.5496$ .

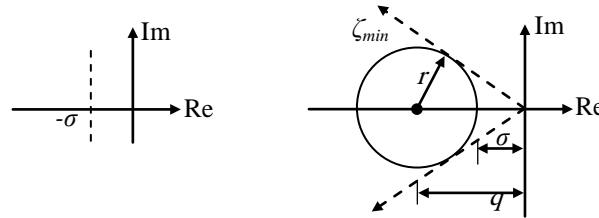


Figure 1. Stability regions: (a) Shifted region, (b) Circular region  $D(q,r)$

Note that the proposed PSS has to use the available speed measurement ( $\Delta\omega$ ) as commonly used in practice. Note that the same control objectives, is tackled using Adaptive Neurofuzzy Inference Systems (ANFIS) and improved Particle Swarm Optimization (PSO) [17], and [18] respectively. However, some training and trials are needed to properly tune the optimizer parameters. Otherwise, converge to a solution can not be attained.

**3. PROBLEM SOLUTION**

Observer-based control used to accomplish the design of the PSS because only speed measurements are available. By employing the available input and output measurement  $\Delta\omega$ , a full-order observer for system (2) is given by

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + K_0[y - \hat{y}], \\ \hat{y} &= C\hat{x}, \\ u &= K_c\hat{x} \end{aligned} \tag{3}$$

where  $\hat{x}$  is the estimate of  $x$ , and  $K_c$  and  $K_0$  are the design parameters to be calculated so as to achieve the control targets. The main results are given by the following theorems.

**3.1. Design case #1: Robust stability with desired decay rate**

Theorem 1: Given that  $(A, B)$  and  $(A, C)$  are controllable and observable pairs respectively, then the observer-based control (3) robustly stabilizes (2) if  $\alpha$  is minimized till it becomes negative and there is a feasible solution to the following BMI, i.e.

Minimize  $\alpha$   
 Subject to

$$\begin{bmatrix} (P_1A + P_1BK_c + \bullet) - \alpha P_1 + \varepsilon N'N & -P_1BK_c + P_3(A - K_oC) + (A' + K_c'B')P_3 - \alpha P_3 & (P_1 + P_3)M \\ \bullet & (-P_3'BK_c + P_2A - P_2K_oC + \bullet) - \alpha P_2 & (P_3' + P_2)M \\ \bullet & \bullet & -\varepsilon I \end{bmatrix} < 0,$$

$P = P' > 0, \varepsilon > 0$

(4)

where  $P$  is a full matrix given by:

$$P = \begin{bmatrix} P_1 & P_3 \\ \bullet & P_2 \end{bmatrix}$$

Unfortunately, the product  $\alpha P$  is not an LMI. It is a BMI optimization problem.

**Proof:**

Attaching the observer-based controller (3) to system (1), the closed-loop system is obtained. Consider that the state estimation error vector is defined as  $e = x - \hat{x}$ , and let the augmented vector to be  $g = [x \ e]$ , the closed loop system is given by

$$\dot{g} = [A_c + \Delta A_c]g \quad (5)$$

where:

$$A_c = \begin{bmatrix} A + BK_c & -BK_c \\ 0 & A - K_o C \end{bmatrix}, \quad \Delta A_c = \begin{bmatrix} \Delta A & 0 \\ \Delta A & 0 \end{bmatrix} = \begin{bmatrix} M \\ M \end{bmatrix} \Delta(t) \begin{bmatrix} N & 0 \end{bmatrix} \quad (6)$$

The closed loop (5) is robustly stable if and only if  $\alpha$  is minimized till it becomes negative and the following LMI has a feasible solution [19]

$$\begin{aligned} P = P' > 0 \\ P(A_c + \Delta A_c) + \bullet - \alpha P < 0 \end{aligned} \quad (7)$$

Note that, the closed loop poles of  $A_c$  are shifted progressively towards the left half-plane through the reduction of  $\alpha$ . As mentioned in [19], if (7) holds, the closed-loop poles lie to the left-hand side of the line  $\sigma = \alpha/2$  in the complex plane. If a negative  $\alpha$ , which satisfies (7), can be found,  $K_c$  and  $K_o$  are obtained and the observer-based stabilization problem is solved. The matrix  $P$  in (7), is partitioned as  $\begin{bmatrix} P_1 & P_3 \\ P_3 & P_2 \end{bmatrix}$ , where robust stability is guaranteed if the following inequality is satisfied.

$$\left( \begin{bmatrix} P_1 & P_3 \\ \bullet & P_2 \end{bmatrix} \begin{bmatrix} A + BK_c & -BK_c \\ 0 & A - K_o C \end{bmatrix} + \bullet \right) - \begin{bmatrix} \alpha P_1 & \alpha P_3 \\ \bullet & \alpha P_2 \end{bmatrix} + \left( \begin{bmatrix} P_1 & P_3 \\ \bullet & P_2 \end{bmatrix} \begin{bmatrix} \Delta A & 0 \\ \Delta A & 0 \end{bmatrix} + \bullet \right) < 0$$

Or equivalently

$$\begin{bmatrix} (P_1 A + P_1 B K_c + \bullet) - \alpha P_1 & -P_1 B K_c + P_3 (A - K_o C) + (A' + K_c' B') P_3 - \alpha P_3 \\ \bullet & (-P_3 B K_c + P_2 A - P_2 K_o C + \bullet) - \alpha P_2 \end{bmatrix} + \begin{bmatrix} P_1 & P_3 \\ \bullet & P_2 \end{bmatrix} \begin{bmatrix} \Delta A & 0 \\ \Delta A & 0 \end{bmatrix} + \bullet < 0 \quad (8)$$

For  $\Delta A = M \Delta(t) N$ , (8) is satisfied if the following equation is fulfilled.

$$\begin{bmatrix} (P_1 A + P_1 B K_c + \bullet) - \alpha P_1 & -P_1 B K_c + P_3 (A - K_o C) + (A' + K_c' B') P_3 - \alpha P_3 \\ \bullet & (-P_3 B K_c + P_2 A - P_2 K_o C + \bullet) - \alpha P_2 \end{bmatrix} + \varepsilon^{-1} \begin{bmatrix} P_1 & P_3 \\ \bullet & P_2 \end{bmatrix} \begin{bmatrix} M & M \\ M & M \end{bmatrix} \begin{bmatrix} P_1 & P_3 \\ \bullet & P_2 \end{bmatrix} + \varepsilon \begin{bmatrix} N' & N' \\ 0 & 0 \end{bmatrix} < 0 \quad (9)$$

Applying Schur complement to (9), Theorem 1 is obtained. The following iterative algorithm is proposed to solve the BMI given by Theorem 1.

Note that the BMI (4) is a nonlinear matrix inequality, which is difficult to solve as being a non-convex. Note that if  $P$  is given, inequality (4) becomes linear in  $K_c$ ,  $K_o$ ,  $\varepsilon$ , and  $\alpha$ . This can be easily solved by LMI control toolbox [12]. The idea, proposed in this manuscript, is to minimize  $\alpha$  by matrix,  $P$ , which is generated by a non-linear minimizer, e.g. `fminsearch` of MATLAB [20], in an outer loop of the iteration as explained below.

**Algorithm 3.1:**

Step 0: Initialize a vector  $z \in R^{2n}$ .

Step 1: Cast  $z$  into its matrix equivalent  $Z$ . Form  $P = ZZ'$ . In this way  $P = P' > 0$  is generated. Set  $i=1$  and  $P_i = P$ .

Step 2: Solve the following optimization problem for  $K_c$ ,  $K_o$  and  $\alpha_i$ .

Minimize  $\alpha_i$  subject to the following LMI constraints

$$\begin{bmatrix} (P_{1i}A + P_{1i}BK_c + \bullet) - \alpha P_{1i} + \varepsilon N'N & -P_{1i}BK_c + P_{3i}(A - K_oC) + (A' + K_c'B')P_{3i} - \alpha P_{3i} & (P_{1i} + P_{3i})M \\ \bullet & (-P_{3i}'BK_c + P_{2i}A - P_{2i}'K_oC + \bullet) - \alpha P_{2i} & (P_{3i}' + P_{2i}')M \\ \bullet & \bullet & -\varepsilon I \end{bmatrix} < 0, \\ P_i = P_i' > 0, \varepsilon > 0 \tag{10}$$

Denote  $\alpha_i^*$  as the minimized value of  $\alpha_i$ .

Step 3: If  $\alpha_i^* < 0$ ,  $K_c, K_o$  are the stabilizing controller-observer gains. Stop.

If no  $\alpha_i^* < 0$  is found, the system cannot be stabilized.

Step 4: Use `fminsearch` to solve the following optimization problem

$$\underset{z \in R^*}{\text{Min}} J = \alpha_i^*(z) \tag{11}$$

Set  $i \rightarrow i+1$ , then go to Step 1.

### 3.2. Design case #2: Robust D-stability

Our control objective is to develop a condition which guarantees regional pole placement, thus achieving the chosen  $t_s$ , and  $\zeta_{min}$ . The main target is established by the following theorem.

**Theorem 2:** If  $(A, B)$  and  $(A, C)$  are controllable and observable pairs respectively, then the observer-based controller (3), robustly stabilizes the system (2), with closed loop poles lie in a disk  $D(q,r)$  if there is a feasible solution to the following LMI optimization

$$Y_1 = Y_1' > 0, Y_2 = Y_2' > 0, S_1, S_2, S_3, \varepsilon > 0, \rho > 0 \\ \begin{bmatrix} -r^2Y_1 & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & -r^2Y_2 & \bullet & \bullet & \bullet & \bullet \\ AY_1 + BS_1 + qY_1 & -BS_3 & -Y_1 + \varepsilon MM' & \bullet & \bullet & \bullet \\ 0 & AY_2 - S_2 + qY_2 & 0 & -Y_2 + \rho MM' & \bullet & \bullet \\ NY_1 & 0 & 0 & 0 & -\varepsilon I & \bullet \\ NY_1 & 0 & 0 & 0 & 0 & -\rho I \end{bmatrix} < 0 \tag{12}$$

and the gains are given by  $K_c = S_1Y_1^{-1}, K_o = S_2Y_2^{-1}C^+$ , where  $C^+ = C'(CC')^{-1}$  is the pseudo-inverse of  $C$ .

**Proof:**

According to [21], the poles of a matrix  $A$  lie in the disk  $D(q,r)$  if and only if there exists a matrix  $P=P' > 0$ , such that

$$\begin{bmatrix} -r^2P & \bullet \\ A + qI & -P^{-1} \end{bmatrix} < 0 \tag{13}$$

Replacing  $A$  with the closed loop matrix  $A_c + \Delta A_c$  (6),  $P$  with  $\text{blockdiag}[P_1, P_2]$ , and substituting  $\Delta A_c = M\Delta(t)N$ ; the closed loop eigenvalues of the uncertain system lie inside the disc  $D(q,r)$  if and only if the following matrix inequalities are satisfied.

$$P_1 = P_1' > 0, P_2 = P_2' > 0, \\ \begin{bmatrix} -r^2P_1 & \bullet & \bullet & \bullet \\ 0 & -r^2P_2 & \bullet & \bullet \\ A + BK_c + qI & -BK_c & -P_1^{-1} & \bullet \\ 0 & A - K_oC + qI & 0 & -P_2^{-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ M \\ 0 \end{bmatrix} \Delta(t) \begin{bmatrix} N' \\ 0 \\ 0 \\ 0 \end{bmatrix} + \bullet + \begin{bmatrix} 0 \\ 0 \\ 0 \\ M \end{bmatrix} \Delta(t) \begin{bmatrix} N' \\ 0 \\ 0 \\ 0 \end{bmatrix} + \bullet < 0$$

Applying Fact 2, the last inequality is satisfied if the following inequality is satisfied.

$$P_1 = P_1' > 0, P_2 = P_2' > 0, \varepsilon > 0, \rho > 0,$$

$$\begin{bmatrix} -r^2 P_1 & \bullet & \bullet & \bullet \\ 0 & -r^2 P_2 & \bullet & \bullet \\ A + BK_c + qI & -BK_c & -P_1^{-1} & \bullet \\ 0 & A - K_o C + qI & 0 & -P_2^{-1} \end{bmatrix} + (\varepsilon \begin{bmatrix} 0 & 0 \\ M & M \end{bmatrix} + \varepsilon^{-1} \begin{bmatrix} N' & N' \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}) + (\rho \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ M & M \end{bmatrix} + \rho^{-1} \begin{bmatrix} N' & N' \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}) < 0$$

The above inequality can be linearized by pre and post-multiplying by  $\text{Blockdiag}[P_1^{-1}, P_2^{-1}, I, I]$ , and letting  $P_1^{-1} = Y_1, P_2^{-1} = Y_2, K_c P_1^{-1} = S_1, K_c P_2^{-1} = S_3, K_o C P_2^{-1} = S_2$ , (12) is obtained.

**4. SIMULATION RESULTS**

In this section, the proposed design is evaluated by applying it to both single- machine infinite-bus and multi-machine models.

**4.1. Application to single-machine infinite-bus test power system**

In design case #1, although  $\alpha < 0$  is sufficient for robust stability, the minimizer is kept running till  $\alpha$  is minimized to -0.1416; thus achieving a satisfactory settling time. The linear matrix inequalities are solved using the MATLAB LMI control toolbox [22] to get the feedback matrices for the design cases mentioned above. The resulting controller and observer gains are summarized in Table 1.

Table 1. Proposed controllers

Controller	Observer-regulator gains	Comments
Design Case #1	$K_c = [0.419 \quad 63.915 \quad -2.777 \quad -0.051]$ , $K_o = [-220 \quad 5 \quad -578 \quad 18778]^T$	$\alpha = -0.1416$
Design Case #2	$K_c = [-0.9936 \quad 12.3551 \quad -0.5636 \quad -0.0494]$ , $K_o = [156.376 \quad 59.688 \quad 0.956 \quad 0.7227]^T$	$r = 59.5496,$ $q = 59.8496$

System's dominant poles at different loads, without and with PSS, for design cases #1 and #2, are shown in Figure 2a, 2b, 2c respectively.

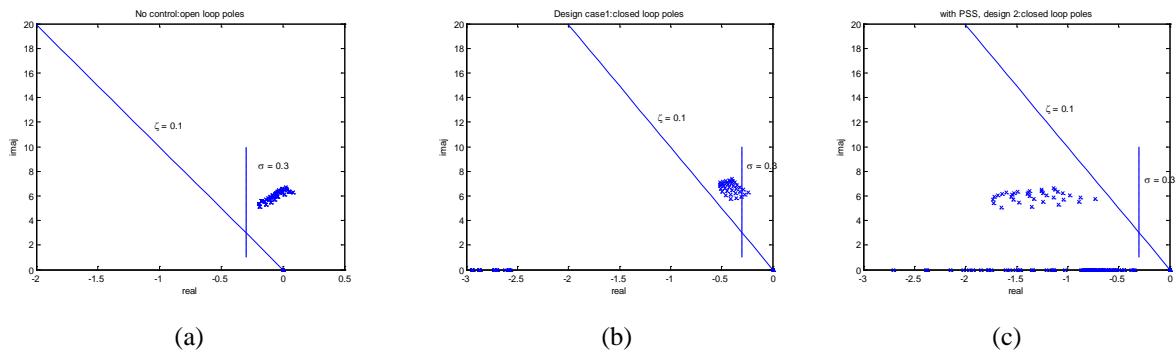


Figure 2. (a) Dominant poles: no control, (b) Dominant poles with PSS1, (c) Dominant poles with PSS2

It is clear from Figure 2a that the system without control does not achieve the desired  $t_s, \zeta_{min}$ , and even becomes unstable for some loads. Figure 2b shows that the control objective of achieving a desired  $\zeta$  is violated. As shown in Figure 2c, controller design #2 satisfies the control objectives ( $t_s, \zeta_{min}$ ).

**4.2. Assessment of the proposed PSSs at extreme loads**

Consider two extremities, namely heavy and light machine loading. A cleared three phase fault at the end of the transmission line at  $t=0$  is used in all simulation cases. For the two proposed designs, PSS1

and PSS2, the time response of the system, under the two extreme operations, shows that the settling time of the system is indeed within the desired range, as shown in Figure 3, 4.

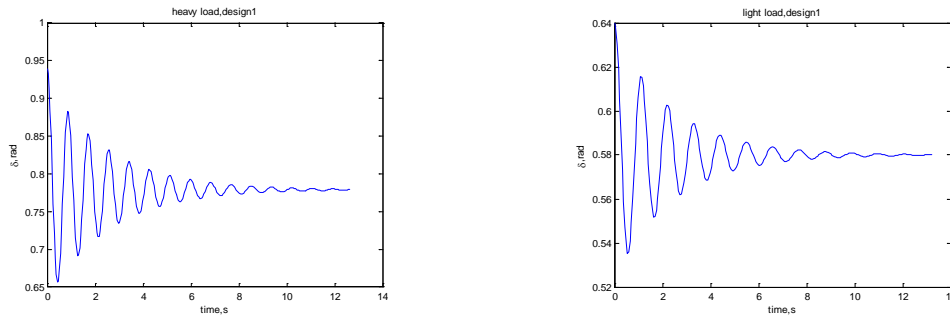


Figure 3. Power system responses at heavy and light loads with PSS1

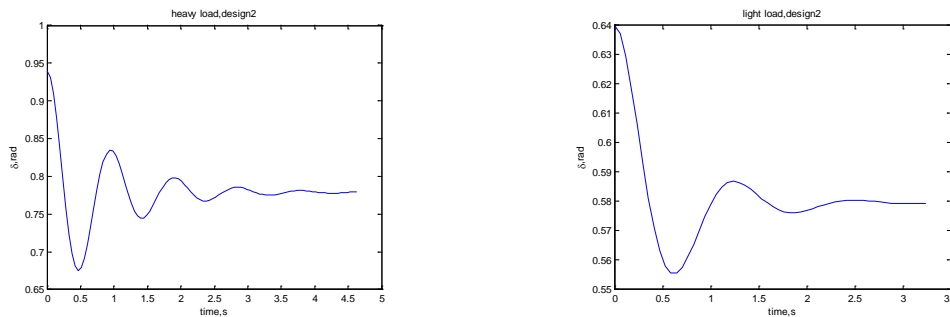


Figure 4. Power system responses at heavy and light loads with PSS2

However, PSS2 provides better response than PSS1, since it achieves the desired damping ratio as well. In addition, PSS2 results in oscillations with lower frequencies; and consequently, less generator's shaft fatigue. For loads between the extremities, similar results are obtained. Note that, in the proposed design, the estimated states are fast enough to follow the true ones, not shown due to space limitations.

#### 4.3. Proposed versus conventional PSS (CPSS)

Some simulations are carried out to demonstrate the effectiveness of the proposed designs as compared with the conventional one. The performance of our closed-loop system is compared to a conventional PSS. Simulations are done in the presence of the same previous disturbances and load variations. For this purpose, a quite popular structure for the CPSS, with the following double lead transfer function, is considered [15]. Many existing generators are commissioned with CPSSs having the following form:

$$u = K_{CPSS} \frac{(1 + sT_1)(1 + sT_3)}{(1 + sT_2)(1 + sT_4)(1 + sT_w)} sT_w \Delta\omega$$

where  $T_w$  is the time constant of a washout circuit which eliminates the controller action in steady state. The CPSS parameters are typically [0.001–50] for  $K_{CPSS}$  and [0.06–1.0s] for  $T_1$  and  $T_3$ . The time constants  $T_w$ ,  $T_2$  and  $T_4$  are set as 5s, 0.05s, and 0.05s, respectively [15]. For the problem at hand, the gain and the time constants of conventional PSS are properly selected as  $K_{CPSS}=15$ ,  $T_1=T_3=0.66s$ . The parameters of CPSS are selected to provide the same desired settling time as the proposed one. The performance of the CPSS is evaluated at heavy and light loads as the first test, Figure 5.



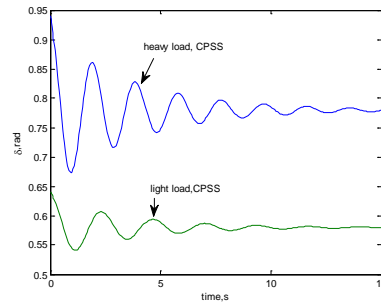


Figure 5. Power system responses at heavy and light loads with CPSS

The simulation results of both CPSS and the proposed PSS2 show that the latter attains robustness against plant uncertainties, quite better performance, higher damping, and lower settling time.

#### 4.4. Decentralized control of multi-machine test power system

Although considerable research is being done in designing PSSs for a multi-machine system [15], no definitive results have been applied in the field. The design can still be done on the basis of a single-machine infinite-bus system. The parameters are then tuned on-line to suppress both the local and inter-area modes. The proposed design can be applied to multi-machine power systems by designing the PSS for one machine at a time and considering the rest of the system as an infinite bus. The resulting controller is decentralized or local as it uses speed deviation  $\Delta\omega$  only from the generator on which it is installed. Local PSSs have three basic advantages. First, they are effective in damping local modes. Second, no communication network is needed to transfer data to a centralized controller; thus they are cost-effective. Third, communication time delays are avoided.

The four-machine two-area test power system of [15]. The CPSS parameters are:

$$K_{CPSS} = 30, T_w = 10s, T_1 = 0.05s, T_2 = 0.02s, T_3 = 3.0s, T_4 = 5.4s, U_{\min} = -0.15, U_{\max} = 0.15$$

Our proposed design is compared to CPSSs to clarify the effectiveness of the former in damping both local and inter-area modes of oscillations. Bus 3 is considered as a slack-bus to provide an angular reference for the system. The equivalent single-machine subsystems are roughly identical, due to system symmetry. Since the inter-area mode is strongly affected by the amount of the tie line power, three operating conditions are considered for the test system. These conditions represent the base case (415 MW), 20% increment and 20% decrement in the tie line power.

The state-space matrices for the base case point are given by:

$$A = \begin{bmatrix} 0 & 377 & 0 & 0 \\ -0.0993 & 0 & -0.104 & 0 \\ -0.2472 & 0 & -0.418 & 0.125 \\ -2631 & 0 & -78013 & -1000 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 200000 \end{bmatrix}, C = [0 \ 1 \ 0 \ 0]$$

The corresponding  $M$  and  $N$  matrices for these test points are computed and are given by  $M = [0 \ 0 \ 0 \ -116.1]$  and  $N = [-109.62 \ 0 \ -58.9 \ 0]$ . The gain matrices of the observer-based regulator, i.e., PSS2, are computed based on Theorem 2, and are given by:

$$K_o = [167.4 \ 54.15 \ -1.558 \ -1143.5], K_c = [-0.02067 \ 12.693 \ 0.333 \ 0.0044]$$

Generators # 1, 2, and 4 are equipped with the same observer-based regulator (PSS2). The effectiveness of such stabilizer is tested by simulating the nonlinear model of the test power system and by comparing it with the CPSS as well. The proposed design is firstly tested for a disturbance initiated by a 5% step change in the reference voltage of Generator #1. The system has recovered within 100ms, at the nominal tie line power, as shown in Figure 6.

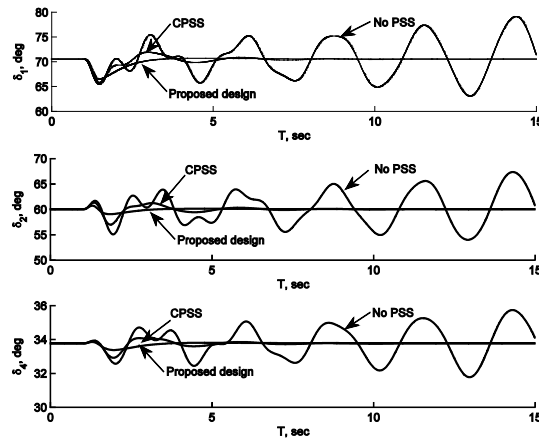


Figure 6. Rotor angles of Gen #1, 2 & 4 due to 5% step change in  $V_{ref1}$  with full recovery after 0.1sec ( $P_{Tic}=415MW$ )

It is worth mentioning that although the proposed PSS is based on a linearized model when the system is subject to small-disturbances (dynamic stability), it is tested under severe large disturbances and it shows the very effective oscillation damping (transient stability).

Note that in the proposed PSS design the effect of damper bars is neglected, thus low-order state equation is obtained, easy to handle. Had the dampers been included, more damping is provided.

## 5. CONCLUSION

Power system dynamics under different loads is represented by a model with uncertainty in the form of norm-bounded structure. Two new designs of dynamic output feedback power system stabilizers (PSSs) are described in this paper. The first PSS design guarantees robust stability to control only the settling time. The synthesis of PSS leads to a bilinear matrix inequality (BMI) optimization problem. The derived BMI sufficient condition is transformed into a Linear Matrix Inequality LMI, easy to solve, using a parameter matrix which is iteratively updated to minimize a certain objective function. In addition to robust stability, the second PSS design attains robust pole placement to control both settling time and damping ratio. The design is based on a non-iterative solution to a sufficient LMI condition.

Simulation results in a single machine infinite-bus and multi-machine test power systems illustrate the validity of the proposed design procedure.

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