

A new chaotic system with line of equilibria: dynamics, passive control and circuit design

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ABSTRACT

A new chaotic system with line equilibrium is introduced in this paper. This system consists of five terms with two transcendental nonlinearities and two quadratic nonlinearities. Various tools of dynamical system such as phase portraits, Lyapunov exponents, Kaplan-Yorke dimension, bifurcation diagram and Poincarè map are used. It is interesting that this system has a line of fixed points and can display chaotic attractors. Next, this paper discusses control using passive control method. One example is given to insure the theoretical analysis. Finally, for the new chaotic system, an electronic circuit for realizing the chaotic system has been implemented. The numerical simulation by using MATLAB 2010 and implementation of circuit simulations by using MultiSIM 10.0 have been performed in this study.

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1. INTRODUCTION

Discovering chaotic attractor is an important issue in chaotic systems. We can classify two kinds of chaotic attractors: self-excited attractors and hidden attractors [1-2]. The chaotic system such as Lorenz system [3], Rössler [4], Lü [5], Chen [6], Rucklidge [7] Sprott [8] etc, belongs to the self-excited attractors. The chaotic systems with hidden attractors are divided into three parts: (a) system with no equilibria [9] (b) system with stable equilibria [10] and (c) system with infinite number of equilibria [11]. Hidden attractors have been used in applied models such as a model of the phase-locked loop (PLL) [12], aircraft flight control systems [13], drilling system actuated by induction motor [14], Lorenz-like system describing convective fluid motion in rotating cavity [15] and a multilevel DC/DC converter [16].

Motivated by the major work of Jafari and Sprott, researchers focused on chaotic systems with line of equilibria. The nine simple chaotic flows with line of equilibria were proposed by Jafari and Sprott [17]. Five novel chaotic system with a line of equilibria and two parallel lines were proposed by Li and Sprott [18]. Li and Sprott have presented chaotic systems with a line of equilibria and two perpendicular lines of equilibria by using signum functions and absolute-value functions [19]. In addition, Li et al reported a hyperchaotic system with an infinite number of equilibria and circuit design [20]. Hyperchaos and horseshoe in a 4D memristive system with a line of equilibria were proposed in [21]. The simplest 4-D chaotic system

with line of equilibria, by applying a state feed-back control to Rössler equation was represented by Singh and Roy [22].

Various control methods have been used for controlling chaotic systems such as adaptive control [23], sliding mode control [24], active control [25], passive control [26], Pecora–Carroll control [27], impulsive control [28], fuzzy control [29], optimal control [30], digital redesign control [31] and many others [32-33]. The concept of passivity theory has been found to be a nice tool in various domains of science and engineering such as robotic [34], signal processing [35], permanent-magnet synchronous motors chaotic system [36], hopfield neural network [37], nuclear spin generator system [38], synchronization [39], compass-like biped robot [40] and complexity [41].

This paper introduces a new chaotic system with line of equilibria. Our new chaotic system has five terms with two transcendental nonlinearities and two quadratic nonlinearities. Basic properties of the new chaotic system are analyzed, including such as equilibrium points, Lyapunov exponents, Kaplan-Yorke dimension, bifurcation diagram and Poincaré map. Finally, passive control and circuit design.

The rest of our paper is organized as follows. In Section 2, description of the new chaotic system with line of equilibria is presented. In addition, the dynamics properties of the new chaotic system are investigated. Stability analysis of the system is presented in Section 3. Some definitions and theorems of passive control are presented in Section 4 and control of this chaotic system using passive control is described in section 5. In Section 6, a new electronic implementation of the new chaotic system is described and examined in MultiSIM. Finally, conclusions are drawn in Section 7.

2. DYNAMICAL ANALYSIS OF A NEW CHAOTIC SYSTEM WITH LINE OF EQUILIBRIA

In this part, inspired by the method and structure proposed in [19], we present a new chaotic system as:

$$\begin{cases} \dot{x} = yz, \\ \dot{y} = x|x| - y|y|, \\ \dot{z} = a|x| - by^2, \end{cases} \quad (1)$$

where x, y, z are state variables a and b are positive system parameters. Here the parameter a is a control parameter to control the amplitude and frequency of all variables.

The new chaotic system (1) exhibits chaotic behavior as shown in Figure 1

$$a = 1.6, b = 0.8, \quad (2)$$

and with the initial conditions

$$x(0) = 0.2, y(0) = 0.2, z(0) = 0.2. \quad (3)$$

The fourth order Runge–Kutta method is used for employing the numerical simulations. Moreover the Lyapunov exponents of the new system (1) are calculated using Wolf algorithm [42].

$$LE_1 = 0.11026, LE_2 = 0, LE_3 = -1.66103 \quad (4)$$

As seen in Figure 2 (a). A positive Lyapunov exponent reveals the presence of chaotic system. Simulation is run for 50.000 seconds.

The Kaplan-Yorke dimension of the new chaotic system (1) is calculated as

$$D_{KY} = 2 + \frac{LE_1 + LE_2}{|LE_3|} = 2.066 \quad (5)$$

Therefore, system (1) is really chaotic with fractional dimension.

a. Fix $a=1.6$ and vary b :

For this case $b=0.46$, the Lyapunov exponents for system (1) are: $LE_1=0.0230$, $LE_2=-0.0169$, $LE_3=-2.4458$. It is clear that (1) has chaotic attractors for $b \geq 0.46$. For $b = 0.45$, the Lyapunov exponents for system (1) are: $LE_1=0.0223$, $LE_2=-0.0284$, $LE_3=-2.4847$. Thus, periodic behavior can be seen in the system for $b < 0.46$.

b. Fix $b=0.8$ and vary a

For $a=0.1$, the Lyapunov exponents for system (1) are: $LE_1=0.0063$, $LE_2=-0.0053$, $LE_3=-0.1072$. It is clear that (1) has chaotic attractors for $a \geq 0.1$ and for $a=0.09$, the Lyapunov exponents for system (1) are: $LE_1=0.0048$, $LE_2=-0.0078$, $LE_3=-0.1048$. Thus, periodic behavior can be seen in the system for $a < 0.09$.

The bifurcation diagram and Lyapunov exponent spectrum of new chaotic system (1) for $a=1.6$, $b=0.8$ and initial conditions $x(0) = 0.2$, $y(0) = 0.2$, $z(0) = 0.2$ are plotted in Figures 2(b) and 2(c), respectively. As shown in Figures 2(b) and 2(c), the system (1) has periodic behavior or chaotic behavior by varying the value of the parameter b . When the value of $b < 0.46$, system (1) exhibits periodic state and when $b \geq 0.46$, the system (1) shows complex behavior. In addition, the Poincaré map of the system (1) in Figure 2(d) also reflects properties of chaos.

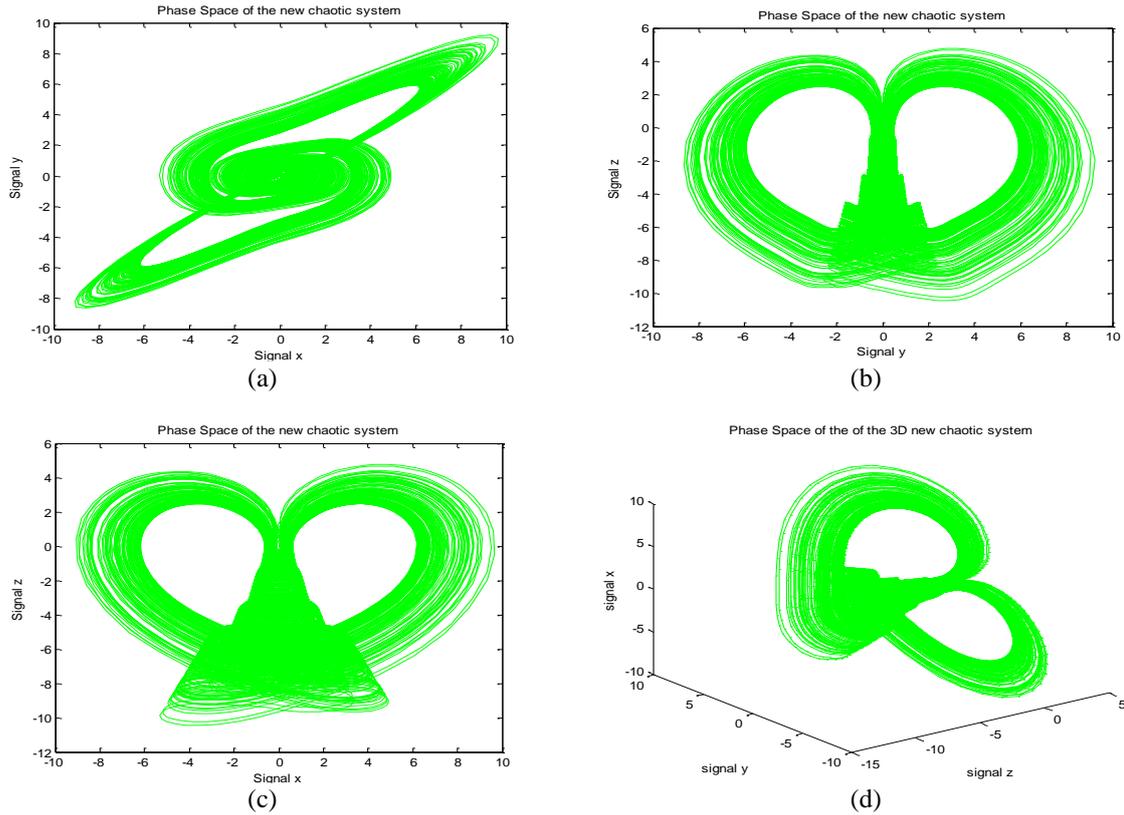


Figure 1. Numerical simulation results using MATLAB, for $a=1.6$ and $b=0.8$, in (a) x - y plane, (b) y - z plane, (c) x - z plane and (d) x - y - z plane

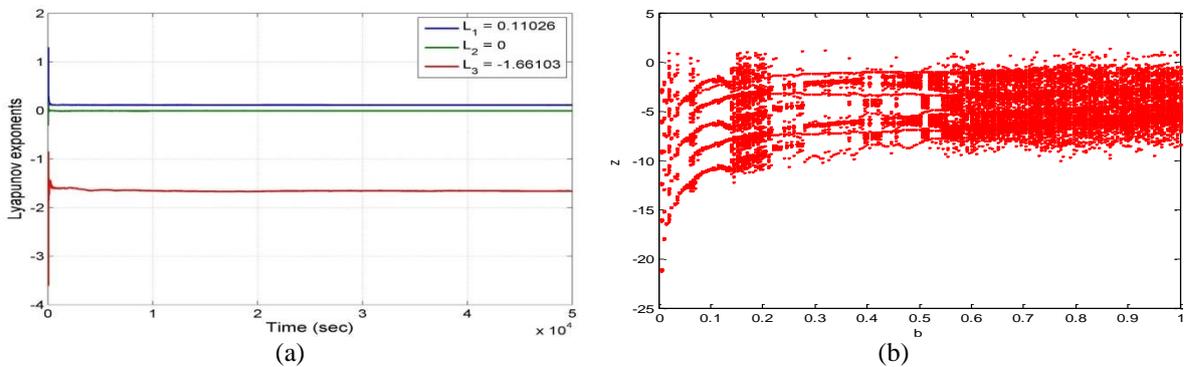


Figure 2 Complex analysis of new chaotic system (1) using MATLAB for $a = 1.6$
 (a) The Lyapunov exponents of the new system (1) (b) Bifurcation diagram of the new chaotic system (1) with b as varying parameter

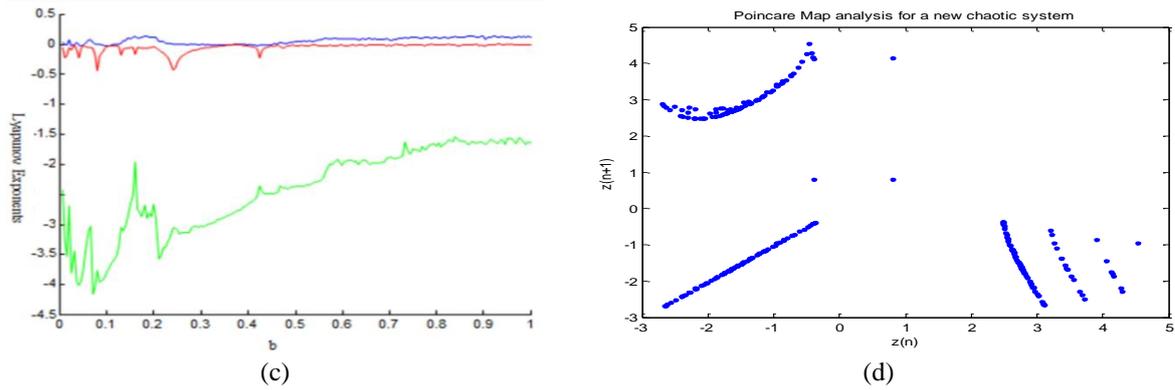


Figure 2 Complex analysis of new chaotic system (1) using MATLAB for $a = 1.6$
 (c) Lyapunov spectrum of system (1) when varying the parameter b (d) Poincaré map of system (1)

3. EQUILIBRIUM AND STABILITY

The equilibrium points of the new chaotic system (1) are obtained by solving the following system.

$$\begin{cases} 0 = yz \\ 0 = x|x| - y|y| \\ 0 = a|x| - by^2 \end{cases} \tag{6}$$

System (1) has a line equilibrium $E_z = (0, 0, z)$ and four nontrivial fixed points which are $(\pm \frac{a}{b}, \pm \frac{a}{b}, 0)$ and $(\pm \frac{a}{b}, \mp \frac{a}{b}, 0)$. The Jacobian matrix of the system (1) is given by

$$J = \begin{bmatrix} 0 & z & y \\ |x| + x * \text{sign}(x) & -(|y| + y * \text{sign}(y)) & 0 \\ a * \text{sign}(x) & -2 * b * y & 0 \end{bmatrix} \tag{7}$$

The characteristic equation can be written as

$$\lambda^3 + (|x| + x * \text{sign}(x)) \lambda^2 + (-z|x| - (ay + xz) * \text{sign}(x)) \lambda + 2by^2|x| - axy + (2bxy^2 - ay|x|) \text{sign}(x) = 0 \tag{8}$$

It is clear that the eigenvalues for system (1) at the line equilibrium E_z are $\lambda_1 = \lambda_2 = \lambda_3 = 0$.

System (1) has four equilibrium points at $E_{1,2}(\pm 2, \pm 2, 0)$, $E_{3,4}(\mp 2, \mp 2, 0)$ with $a=1.6$ and $b=0.8$. The eigenvalues at E_1 are

$$\lambda_1 = -5.11488, \lambda_{2,3} = 0.55744 \pm 1.48046i \tag{9}$$

Here λ_1 is a negative real number, while λ_2 and λ_3 are a pair of complex conjugate eigenvalues with positive real parts. Thus, the equilibrium E_1 is a *saddle-focus* point, which is unstable. For the second equilibrium E_2 $(-2, -2, 0)$, the eigenvalues are identical to those of E_1 . Similarly, for the equilibrium $E_{3,4}$, The eigenvalues are

$$\lambda_1 = -4.93056, \lambda_{2,3} = 0.465279 \pm 2.75167i \tag{10}$$

which also are unstable.

4. THE THEORY OF PASSIVE CONTROL

Consider the following differential:

$$\begin{aligned}\dot{u} &= \Lambda(u) + Y(u)\theta, \\ v &= \Delta(u),\end{aligned}\tag{11}$$

where $u \in \mathbb{R}^n$ is state variable, $\Lambda(u)$ and $Y(u)$ are the smooth vector fields, $\theta \in \mathbb{R}^m$ is the control function, $n > m$ and $\Delta(u)$ is a smooth mapping.

Definition 1 [26, 43] If the matrix $L_Y \Delta(0) = \frac{\partial \Delta}{\partial u} Y(u)$ is nonsingular, system (11) is said to have relative degree $[1, 1, \dots, 1]$ at $u = 0$.

Definition 2 [26, 43] System (11) is said to be C^r -passive if there exists a C^r -positive real valued function $V(u)$, $V(0) = 0$, called storage function, such that $\forall t \geq 0$, the following dissipation inequality holds:

$$V(u) - V(0) \leq \int_0^t \theta^T(s) v(s) ds.\tag{12}$$

The parametric version of the normal form of system (11) can be written as follows:

$$\begin{aligned}\dot{\zeta} &= \phi_0(\zeta) + \chi(\zeta, v)v, \\ \dot{v} &= \varphi(\zeta, v) + \mu(\zeta, v)\theta,\end{aligned}\tag{13}$$

where a new coordinate of the system (13) is (ζ, v) , locally defined in the neighborhood of the origin, $\zeta \in \mathbb{R}^{n-m}$ and $\mu(\zeta, v)$ is nonsingular for all (ζ, v) in the neighborhood of the origin [26, 43].

Remark 1 Setting $v = 0$ in system (13), yields the zero dynamic system:

$$\dot{\zeta} = \phi_0(\zeta),\tag{14}$$

where the stability of zero dynamics is a necessary condition for passivity control design.

Definition 3 [26, 43] Suppose $L_Y \Delta(0)$ is nonsingular, then system (11) is said to be minimum phase if its zero dynamics is asymptotically stable. In other words, there exists the function $W(\zeta)$ (called Lyapunov function of $\phi_0(\zeta)$) which is positive-definite and differentiable in ζ such that:

$$\frac{\partial W(\zeta)}{\partial \zeta} \phi_0(\zeta) \leq 0,\tag{15}$$

$\forall \zeta$ in a neighborhood of $\zeta = 0$.

Theorem 1 [26, 44] If the system (11) is a minimum phase system, the system (12) will be equivalent to a passive system and asymptotically stabilized at an equilibrium point if we let the local feedback control as follows:

$$\theta = \mu^{-1}(\zeta, v) \left(-\varphi(\zeta, v) - \left(\frac{\partial W(\zeta)}{\partial \zeta} \chi(\zeta, v) \right)^T - \gamma v + \beta \right),\tag{16}$$

where γ is a positive real value and β is an external signal vector that is connected with the reference input. System (17) after control as shown in Figure 3.

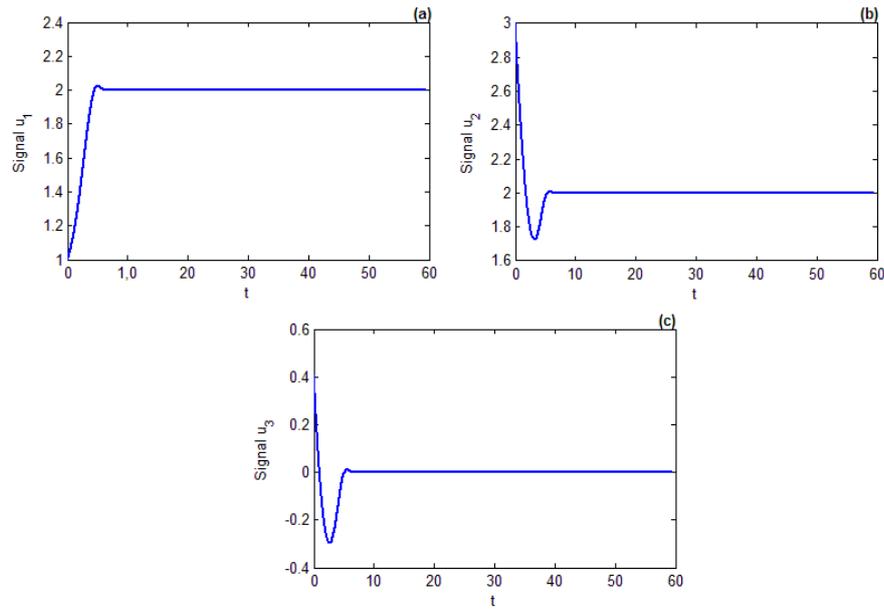


Figure 3. System (17) after control

5. THE CONTROL OF SYSTEM (1) USING PASSIVE CONTROL

To control system (1), we add the control function to the first equation. So, the controller system can be written as:

$$\begin{aligned}
 \dot{u}_1 &= u_2 u_3 + \theta, \\
 \dot{u}_2 &= u_1 |u_1| - u_2 |u_2|, \\
 \dot{u}_3 &= a |u_1| - b u_2^2.
 \end{aligned}
 \tag{17}$$

The main goal is to design the appreciate controller function θ to stabilize system (17).

Theorem 2 If we choose the controller as follows

$$\theta = -u_2 u_3 - u_2 |u_1| - \gamma u_1 + \beta,
 \tag{18}$$

where γ is a positive real constant, then the chaotic system (17) will be asymptotically stabilized at the fixed point.

Proof. Clearly, $L_Y \Delta(0) = 1$, where

$$\Delta = u_1, \quad Y = [1, 0, 0]^T,
 \tag{19}$$

so according to definition 1, system (17) have relative degree $[1, 1, \dots, 1]$. Let $\zeta_1 = u_2, \zeta_2 = u_3, v = u_1$, the (17) can be rewritten as:

$$\begin{aligned}
 \dot{\zeta}_1 &= v |v| - \zeta_1 |\zeta_1|, \\
 \dot{\zeta}_2 &= a |v| - b \zeta_1^2, \\
 \dot{v} &= \zeta_1 \zeta_2 + \theta.
 \end{aligned}
 \tag{20}$$

Comparing (20) by (13) one has

$$\begin{aligned}
 \phi_0(\zeta) &= [-\zeta_1 |\zeta_1| \quad -b \zeta_1^2]^T, \\
 \chi(\zeta, v) &= [|v| \quad a \text{sign}(v)]^T, \\
 \varphi(\zeta, v) &= \zeta_1 \zeta_2, \\
 \mu(\zeta, v) &= 1.
 \end{aligned}
 \tag{21}$$

Let

$$W(\zeta) = \frac{1}{2} \zeta_1^2, \quad (22)$$

with $W(0) = 0$, then we have

$$\begin{aligned} \dot{W}(\zeta) &= \frac{\partial W(\zeta)}{\partial \zeta} \dot{\zeta} = \frac{\partial W(\zeta)}{\partial \zeta} \phi_0(\zeta) \\ &= [\zeta_1 \quad 0] \begin{bmatrix} -\zeta_1 |\zeta_1| & -b\zeta_1^2 \end{bmatrix}^T \\ &= -\zeta_1^2 |\zeta_1| \leq 0. \end{aligned} \quad (23)$$

Regarding to definition 3, system (17) is minimum phase system. Consequently, based on theorem 1, one can design the controller as

$$\theta = -\zeta_1 \zeta_2 - \zeta_1 |v| - \gamma v + \beta, \quad (24)$$

Namely

$$\theta = -u_2 u_3 - u_2 |u_1| - \gamma u_1 + \beta. \quad (25)$$

Remark [26] the attractors of the new chaotic system (17) after control are converted to non-trivial equilibrium E_1 point if $\beta = 2\gamma + 2|2|$. For the numerical simulation, the fourth-order Runge-Kutta method is used to solve the system of differential (17), with step size equal 0.001 in numerical simulations. By taking γ equals to 0.2 and the initial conditions of (17) are $u_1(0) = 1, u_2(0) = 3, u_3 = 0.4$. As expected, one can observe that the trajectories of the new chaotic system (17) asymptotically stabilized at equilibrium point E_1 as illustrated in Figure 3.

6. CIRCUIT DESIGN OF THE NEW CHAOTIC SYSTEM

Chaos phenomenon is widely applied in the field of engineering. Specifically, electronic circuits [44-50], secure communication [51], robotic [52], random bits generator [53], and voice encryption [54]. In this section, we describe a possible circuit to implement new chaotic system with line of equilibria (1) as presented in Figure 4. The circuit consists of twenty-one resistors, three capacitors, three integrators (U1A-U3A), three inverting amplifiers (U4A-U6A), four operational amplifiers (U7A-U10A) for absolute nonlinearity, which are implemented with the operational amplifier TL082CD. The circuit has two diodes (D1 (1N4148), D2(1N4148), which provide the signal $|Y|$, two diodes (D3 (1N4148), D4(1N4148) which produce the signal $|X|$ and four multipliers (AD633JN).

In this study, a linear scaling is considered as follows:

$$\begin{cases} \dot{x} = 2yz \\ \dot{y} = 2x|x| - 2y|y| \\ \dot{z} = a|x| - 2by^2 \end{cases} \quad (26)$$

By applying Kirchhoff's circuit laws, the corresponding circuital equations of the designed circuit can be written as

$$\begin{cases} \dot{x} = \frac{1}{C_1 R_1} yz \\ \dot{y} = \frac{1}{C_2 R_2} x|x| - \frac{1}{C_2 R_3} y|y| \\ \dot{z} = \frac{1}{C_3 R_4} |x| - \frac{1}{C_3 R_5} y^2 \end{cases} \quad (27)$$

We choose the values of the circuital elements as: $R_1 = R_2 = R_3 = 20 \text{ k}\Omega$, $R_4 = R_5 = 25 \text{ k}\Omega$, $R_6 = R_7 = R_8 = R_9 = R_{10} = R_{11} = R_{12} = R_{13} = R_{14} = R_{15} = R_{16} = R_{17} = R_{18} = R_{19} = R_{20} = R_{21} = 10 \text{ k}\Omega$, $C_1 = C_2 = C_3 = 10 \text{ nF}$

In system (27), the variables x , y and z correspond to the voltages in the outputs of the integrators U1A-U3A. The supplies of all active devices are ± 15 volt. The MultiSIM projections of chaotic attractors with line equilibria are described in Figures 5 (a-c). The numerical simulations with MATLAB see Figure 1 are similar with the circuital ones see Figure 5.

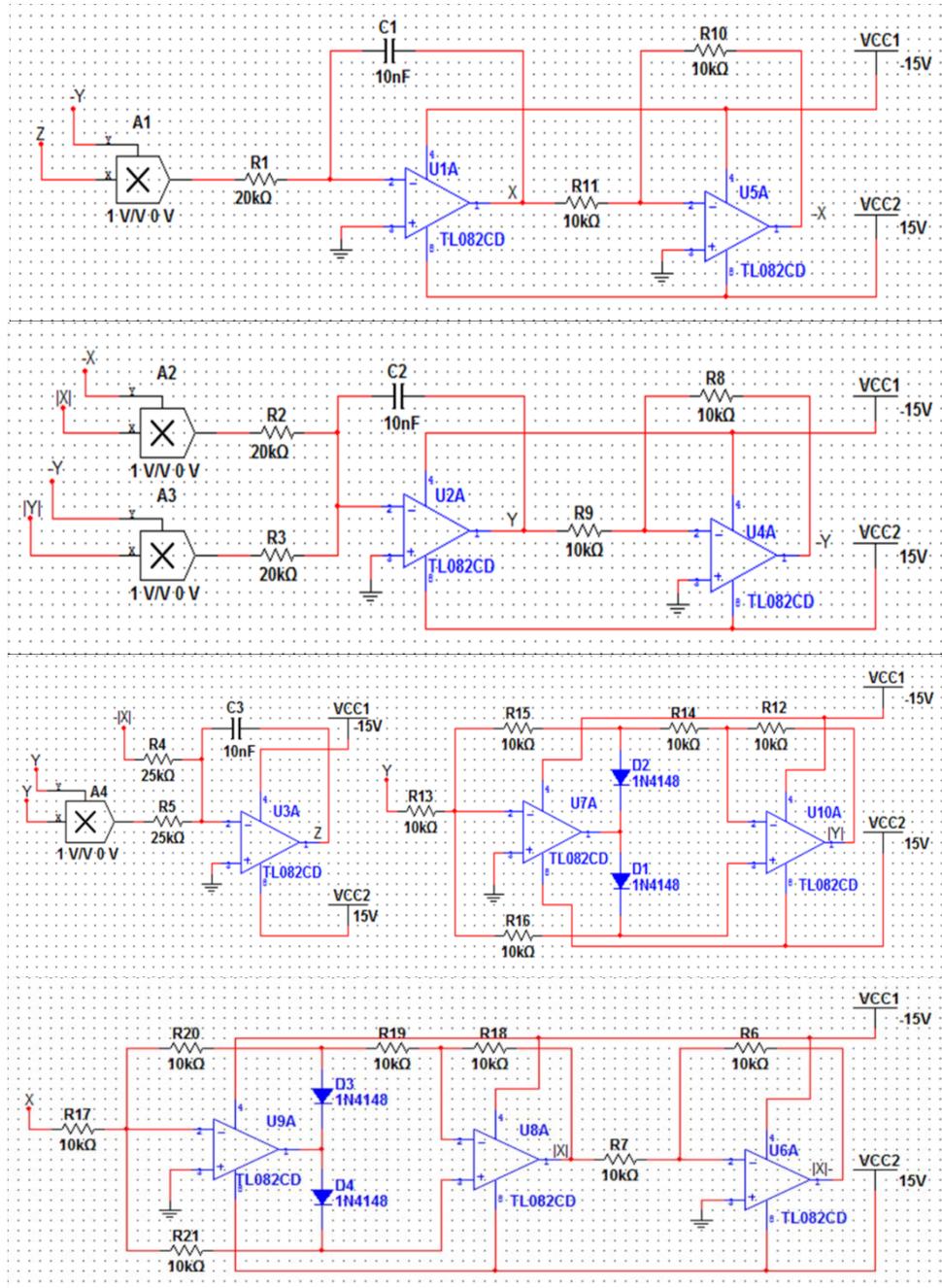
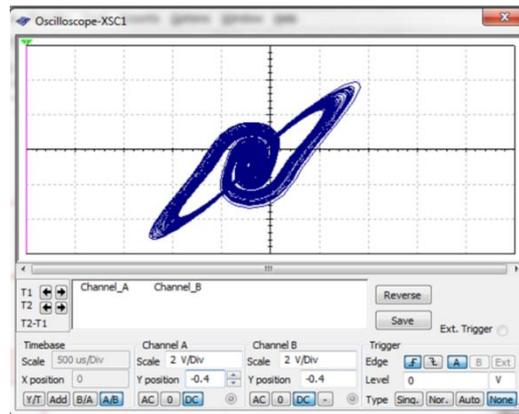
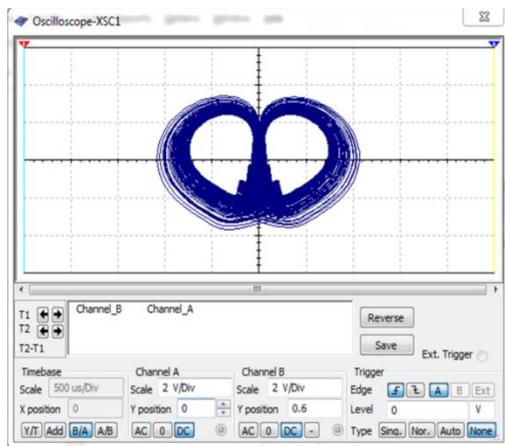


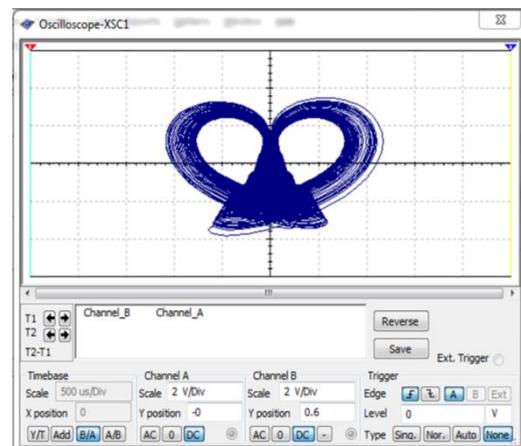
Figure 4. Schematic of the proposed new chaotic system by using MultiSIM



(a)



(b)



(c)

Figures 5. The phase portraits of new chaotic system (1) observed on the oscilloscope in different planes (a) x-y plane, (b) y-z plane and (c) x-z plane by MultiSIM

7. CONCLUSION

A new chaotic system with line of equilibria has been investigated. The proposed new chaotic system has rich dynamics as confirmed by eigenvalue structure, chaotic attractors, Lyapunov exponents, bifurcation diagram and Poincaré map. In addition, the possibility of passive control of a new chaotic system with line of equilibria has been analyzed and confirmed. Moreover, electronic circuit has been implemented and tested using the MultiSIM software. Comparison of the oscilloscope output and numerical simulations using MATLAB, showed good qualitative agreement between the chaotic system and circuit design. Further analyses like engineering application on robotic, random bits generator and secure communication system are interesting issues for future work.

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