Nonlinear Control of an Active Magnetic Bearing with Output Constraint

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Article Info	**ABSTRACT**

In this paper, an appropriate control strategy is proposed to handle the nonlinear dynamics of an active magnetic bearing (AMB). The goal of the control design is to drive the AMB rotor to the origin with improved transient response. In order to achieve this task, back stepping control technique with a barrier Lyapunov function are employed to keep the tracking error trajectory inside a predefined zone to avoid possible mechanical contact between rotor and stator. Besides, a speed observer is also used since information about rotor speed is not always available. The stability of the closed-loop system is proven. The effectiveness of the proposed control strategy is verified by numerical simulations.

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1. INTRODUCTION

Active magnetic bearing (AMB) has been developed in recent few decades to replace the conventional mechanical bearings. The main concept of an AMB is to use an electromagnetic force to support a body without any mechanical contact. In comparison with the traditional mechanical bearings with many drawbacks [1] and [2], AMB exhibits many advantages such as: frictionless, lubricant-free operation, active vibration control and unbalance compensation ability. Hence, AMB is a promising solution for high-speed applications such as high-speed motor [3], flywheel energy storage systems (FESS), .etc. For the proper operation of an AMB, rotor positioning control is a challenging task due to the inherent instability and nonlinearity of the system as mentioned in [4], [5], and [6]. In order to deal with system nonlinearity, authors of [7] propose a linearized AMB model and employed conventional PID control, similar approach can be found in [8]. In addition to linear control trend, an off-line tunning technique for centralized and decentralized linear AMB controllers are demonstrated in [9]. When the variation of the rotor position is small, linearizing the model in a small region around an equilibrium point in order to use the linear control technique is an appropriate approach to stabilize the AMB system. However, the performance of the positioning system may degrade significantly when the operating point is far from the desired equilibrium point. Hence, nonlinear control technique has been studied to further improve the systems performance.

In order to stabilize rotor when facing with a vibrating base, [10] design a sliding mode scheme for an AMB system. The closed-loop system exhibits robustness to uncertainties and external disturbances. Based on Lyapunov's direct method and the singular perturbation order-reduction technique, the state feedback control is derived in [11]. Through a set of numerical simulations, the state feedback control shows its strength in comparison with output feedback and deadbeat controls. Inspired by loop shaping properties of H_∞ optimization and disturbances rejection ability of the disturbance observer-based controller, [12] successfully develop a hybrid controller whose effectiveness are verified via simulations and real-time experiments. By investigating strong nonlinearity of 3-poles AMB, a feedback linearization law is composed in [13]. Rotor responses also are restricted to a safe distance from stator boundary, however, the requirement of initial conditions might be difficult

to archive in practice. In a circumstance of the AMB driving rotor with unknown mass imbalance, [14] proposes an adaptive mechanism to tackle this situation. Taking voltage input saturation into account, a robust fuzzy control that is able to stabilize the AMB rotor is developed in [15]. Apart from conventional AMB, [16] suggests a design and decoupling control for axial AMB by combining optimal algorithm and feed-forward control.

      The paper proposes an nonlinear control approach to stabilizing AMB rotor problem based on backstepping control. A speed observer is utilized in this research to acquire rotor speed information. Barrier Lyapunov candidate function is embedded into design process to drive the AMB rotor trajectory inside a defined range. The stability of the closed-loop system is proven. A set of numerical simulations are given to verify the control effectiveness.

2. MATHEMATICAL MODEL OF AMB

      Figure 1 shows the one degree of freedom (1DOF) AMB system considered in this paper. In this figure,
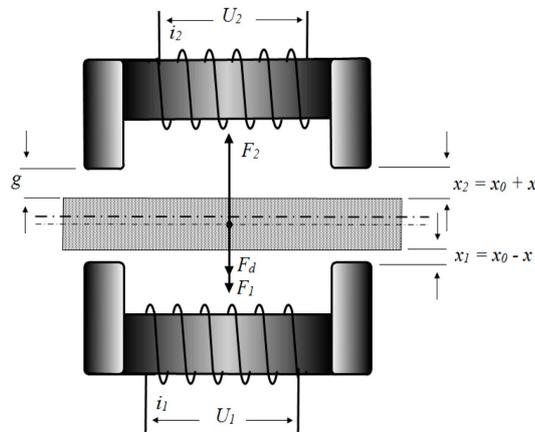

Figure 1. An AMB system.

F_k, u_k and i_k are the electromagnetic force, applied input voltage and current of the corresponding coil k, respectively, with $k = 1, 2$; F_d is the disturbance force caused by rotor mass and other uncertainties. Suppose that the displacement x of the rotor from the nominal position x_0 caused by the initial voltage and current (u_0, i_0). Denote x_1 and x_2 are the air gaps between the rotor and the left and right side stators, it yields

$$x_1 = x_0 - x, \quad x_2 = x_0 + x \tag{1}$$

$$i_1 = i_0 - i, \quad i_2 = i_0 + i \tag{2}$$

$$u_1 = u_0 - u, \quad u_2 = u_0 + u \tag{3}$$

By using (1)-(3), a fundamental operation gives

$$\begin{cases} \frac{dx}{dt} = v \\ \frac{dv}{dt} = \frac{K}{4m}\left(\frac{i_1}{x_0-x}\right)^2 - \frac{K}{4m}\left(\frac{i_2}{x_0+x}\right)^2 + \frac{F_d}{m} \\ \frac{di_1}{dt} = A\left[-Ri_1 - \frac{K}{2(x_0-x)}vi_1 + u_1\right] \\ \frac{di_2}{dt} = B\left[-Ri_2 + \frac{K}{2(x_0+x)}vi_2 + u_2\right] \end{cases} \tag{4}$$

where,

$$A = \frac{2(x_0 - x)}{2L_s(x_0 - x) + K}, \quad B = \frac{2(x_0 + x)}{2L_s(x_0 + x) + K}, \quad K = \mu_g N^2 A_g \tag{5}$$

In (5), μ_g is the permeability of air, N is the number of turns in each coil and A_g is the cross-section area of the electromagnet. Due to the fact that only the position of the rotor and the two current i_1 and i_2 of the corresponding coils are available for measurement, a speed observer is needed to estimate the rotor speed. In this research, the nonlinear observer proposed in [17] is utilized for speed estimation as follows

$$\hat{v} = \eta + \theta\lambda + kx + \frac{F_d}{m}\frac{1}{k} \tag{6}$$

where,

$$\dot{\eta} = -k\eta - k^2 x \tag{7}$$

$$\dot{\lambda} = -k\lambda + \frac{i_1^2}{(x_0 - x)^2} - \frac{i_2^2}{(x_0 + x)^2} \tag{8}$$

$$\theta = \frac{K}{4m} \tag{9}$$

where k is a positive constant. Suppose that the initial conditions of the system is

$$\eta(0) = 0, \quad \lambda(0) = 0 \tag{10}$$

Denote the speed estimation error ϵ as

$$\epsilon = \upsilon - \hat{\upsilon} \tag{11}$$

Then, by differentiating both side of (11) and based on (4) and (6)-(9), it gives

$$
\begin{aligned}
\dot{\epsilon} &= \dot{\upsilon} - \dot{\hat{\upsilon}} \\
&= \theta \left(\frac{i_1}{x_0 - x} \right)^2 - \theta \left(\frac{i_2}{x_0 + x} \right)^2 + \frac{F_d}{m} - \dot{\eta} - \theta\dot{\lambda} - k\dot{x} \\
&= k \left(\eta + \theta\lambda + kx + \frac{1}{k}\frac{F_d}{m} - \upsilon \right) \\
&= -k\epsilon
\end{aligned}
\tag{12}
$$

Relation (12) means that the speed estimation error ϵ exponentially converges to zero and the convergence speed depends on k. By using the speed observer (6), the state space model of the AMB system can be rewritten as

$$
\begin{cases}
\dot{\hat{x}} = \hat{\upsilon} + \epsilon \\
\hat{\upsilon} = \eta + \theta\lambda + kx + \frac{1}{k}\frac{F_d}{m} \\
\dot{\lambda} = -k\lambda + \theta \left(\frac{i_1}{x_0 - x} \right)^2 - \theta \left(\frac{i_2}{x_0 + x} \right)^2 \\
\dot{\eta} = -k\eta - kx^2 \\
\dot{i_1} = A \left[-Ri_1 - \frac{K}{2(x_0 - x)}(\hat{\upsilon} + \epsilon)i_1 + u_1 \right] \\
\dot{i_2} = B \left[-Ri_2 + \frac{K}{2(x_0 + x)}(\hat{\upsilon} + \epsilon)i_2 + u_2 \right]
\end{cases}
\tag{13}
$$

3. CONTROL DESIGN

It can be observed from (13) that the model of the AMB has the form of a strict-feedback system. Hence, backstepping control technique is chosen to stabilize the system. The control design is implemented step-by-step as follows.

Step 1: The goal of the control design in this step is to drive the rotor to a desired position y_r with minimized tracking error. Denote z_1 as the position error

$$z_1 = x - y_r \tag{14}$$

Differentiating both side of (14) gives

$$
\begin{aligned}
\dot{z}_1 &= \dot{x} - \dot{y}_r \\
&= \eta + \theta\lambda + k(z_1 + y_r) + \frac{g}{k} + \epsilon - \dot{y}_r
\end{aligned}
\tag{15}
$$

Consider the following barrier Lyapunov candidate function

$$V_1 = \frac{1}{2} \log \left(\frac{k_b^2}{k_b^2 - z_1^2} \right) + \frac{1}{2kd_1}\epsilon \tag{16}$$

with $d_1 > 0$. By using the proposed barrier Lyapunov function, the position error z_1 is kept in a region restricted by $(-k_b, k_b)$ where k_b is a positive number. This means the overshoot of the AMB in transient state can be

handle by k_b. This property is meaningful in practice since the mechanical contact between the rotor and stator can be avoided. Differentiating both sides of (16), it gives

$$
\begin{aligned}
\dot{V}_1 &= \frac{z_1 \dot{z}_1}{k_b^2 - z_1^2} - \frac{\epsilon^2}{d_1} \\
&= \frac{z_1(\eta + \theta\lambda + kz_1 + \epsilon - \dot{Y}_r + kY_r)}{k_b^2 - z_1^2} - \frac{\epsilon^2}{d_1}
\end{aligned}
\tag{17}
$$

in order to render $\dot{V}_1 \leq 0$, the virtual control action λ_v is selected as

$$
\lambda_v = \frac{1}{\theta}\left[-c_1 z_1(k_b^2 - z_1^2) - \frac{d_1 z_1}{k_b^2 - z_1^2} - k(z_1 + Y_r) + \dot{Y}_r \right]
\tag{18}
$$

with $c_1 > 0$.

        Step 2: As seen in the previous step, if $\lambda = \lambda_v$, then $\dot{V}_1 \leq 0$ which results in $\epsilon = 0$ and $z_1 = 0$. Hence, the goal of this step is to guarantee that λ approaches to λ_v. Denote z_2 as

$$
z_2 = \lambda - \lambda_v
\tag{19}
$$

The candidate Lyapunov function is chosen as

$$
V_2 = \frac{1}{2}\log\left(\frac{k_b^2}{k_b^2 - z_1^2}\right) + \frac{1}{2}z_2^2 + \frac{\epsilon^2}{2k}\left(\frac{1}{d_1} + \frac{1}{d_2}\right)
\tag{20}
$$

with $d_2 > 0$. Differentiating both sides of (20) gives

$$
\dot{V}_2 = \frac{z_1 \dot{z}_1}{k_b^2 - z_1^2} + z_2 \dot{z}_2 - \epsilon^2\left(\frac{1}{d_1} + \frac{1}{d_2}\right)
\tag{21}
$$

Taking λ from (17) and substitute it into (15) give

$$
\dot{z}_1 = \theta z_2 - c_1 z_1(k_b^2 - z_1^2) - \frac{d_1 z_1}{k_b^2 - z_1^2} + \epsilon
\tag{22}
$$

Differentiating both sides of (19) gives

$$
\dot{z}_2 = \dot{\lambda} - \frac{\partial \lambda_v}{\partial z_1}\hat{v} + \epsilon - \dot{Y}_r - \frac{\partial \lambda_v}{\partial \eta} - \frac{\ddot{Y}_r}{\theta} + \frac{k\dot{Y}_r}{\theta}
\tag{23}
$$

in which,

$$
\frac{\partial \lambda_v}{\partial z_1} = \frac{1}{\theta}\left[-k - c_1 k_b^2 + 3c_1 z_2^2 - \frac{d_1(k_b^2 + z_1^2)}{(k_b^2 - z_1^2)^2} \right]
\tag{24}
$$

$$
\frac{\partial \lambda_v}{\eta} = -\frac{1}{\theta}
\tag{25}
$$

Then, by substituting (22) and (23) into (21), it results in

$$
\begin{aligned}
\dot{V}_2 = &- c_1 z_1 - d_1\left(\frac{z_1}{k_b^2 - z_1^2} - \frac{\epsilon}{2d_1}\right)^2 - \epsilon^2\left(\frac{1}{d_2} + \frac{3}{4d_1}\right) \\
&+ z_2\left(\frac{\theta z_1}{k_b^2 - z_1^2} + \dot{\lambda} - \frac{\partial \lambda_v}{\partial \eta}\dot{\eta} - \frac{\partial \lambda_v}{\partial z_1}\dot{z}_1 - \frac{\ddot{Y}_r}{\theta} + \frac{k\dot{Y}_r}{\theta}\right)
\end{aligned}
\tag{26}
$$

From (26), to guarantee that $\dot{V}_2 < 0$, the virtual control signal $\dot{\lambda}_v$ is chosen as

$$
\dot{\lambda}_v = -c_2 z_2 + \frac{\partial \lambda_v}{\partial \eta}\dot{\eta} + \frac{\partial \lambda_v}{\partial z_1}(\hat{v} - \dot{Y}_r) - d_2 z_2 \frac{\partial \lambda_v}{\partial z_1}^2 - \frac{\theta z_1}{k_b^2 - z_1^2} + \frac{\ddot{Y}_r}{\theta} - \frac{k\dot{Y}_r}{\theta}
$$

with $c_2 > 0$. If

$$
\dot{\lambda} = \dot{\lambda}_v
\tag{27}
$$

then $\dot{V}_2 < 0$. However, the solution for (27) can not be obtained since it depends on two variables i_1 and i_2. In oder to solve the abovementioned problem and to save the energy, a switching control strategy is employed in which either i_1 or i_2 is used. If $x < 0$, coil 2 is turned off which means $i_2 = 0$, then

$$\dot{\lambda} = -k\lambda + \frac{i_1^2}{(x_0 - x)^2} \tag{28}$$

Substitute (28) into (27), the desired current of coil 1 is

$$i_1 = i_{1,d} = x_0 - x\sqrt{\dot{\lambda}_d + k\lambda} \tag{29}$$

Denote $\alpha = \dot{\lambda}_d + k\lambda$, it gives

$$i_1 = i_{1,d} = x_0 - x\sqrt{\alpha} \tag{30}$$

Similarly, if $x \geq 0$, then $i_1 = 0$ and hence

$$\dot{\lambda} = -k\lambda - \frac{i_2^2}{(x_0 + x)^2} \tag{31}$$

The reference current for coil 2 is

$$i_2 = i_{2,d} = x_0 + x\sqrt{-\alpha} \tag{32}$$

Remark: Equations (28), (31), and the definition of α indicate that square root operations in (29) and (32) always hold.

Step 3: The goal of this final step is to obtain u_1 and u_2 which drives i_1 and i_2 to their desired values $i_{1,d}$ and $i_{2,d}$, respectively. *Case 1:* $x < 0$. Denote z_3 as

$$z_3 = i_1 - i_{1,d} \tag{33}$$

Differentiate (33), it gives

$$
\begin{aligned}
\dot{z}_3 =& \dot{i}_1 - \dot{i}_{1,d} \\
=& -ARi_1 + \left[-\frac{AKi_1}{2(x_0 - x)^2} - \frac{\partial i_{1,d}}{\partial z_1}(\hat{v} + \epsilon) - \dot{Y}_r \right] \\
&+ Au_1 - \frac{\partial i_{1,d}}{\partial \lambda}\dot{\lambda} - \frac{\partial i_{1,d}}{\partial \dot{Y}_r}\ddot{Y}_r - \frac{\partial i_{1,d}}{\partial \ddot{Y}_r}\dddot{Y}_r - \left[\frac{AKi_1}{2(x_0 - x)^2} + \frac{\partial i_{1,d}}{\partial Y_r} \right] \dot{Y}_r
\end{aligned}
\tag{34}
$$

in which

$$\frac{\partial i_{1,d}}{\partial z_1} = -\sqrt{\alpha} + (x_0 - x)\frac{1}{2\sqrt{\alpha}}\frac{\partial \alpha}{\partial z_1} \tag{35}$$

$$\frac{\partial i_{1,d}}{\partial \lambda} = (x_0 - x)\frac{1}{2\sqrt{\alpha}}\frac{\partial \alpha}{\partial \lambda} \tag{36}$$

$$\frac{\partial i_{1,d}}{\partial \eta} = (x_0 - x)\frac{1}{2\sqrt{\alpha}}\frac{\partial \alpha}{\partial \eta} \tag{37}$$

$$\frac{\partial i_{1,d}}{\partial Y_r} = -\sqrt{\alpha} + (x_0 - x)\frac{1}{2\sqrt{\alpha}}\frac{\partial \alpha}{\partial Y_r} \tag{38}$$

$$\frac{\partial i_{1,d}}{\partial \dot{Y}_r} = (x_0 - x)\frac{1}{2\sqrt{\alpha}}\frac{\partial \alpha}{\partial \dot{Y}_r} \tag{39}$$

$$\frac{\partial i_{1,d}}{\partial \ddot{Y}_r} = (x_0 - x)\frac{1}{2\sqrt{\alpha}}\frac{\partial \alpha}{\partial \ddot{Y}_r} \tag{40}$$

with,

$$\frac{\partial \alpha}{\partial \eta} = -\left(\frac{\partial \lambda_v}{\partial z_1}\right)^2 \frac{d_2}{\theta} - \frac{c_2}{\theta} + \frac{\partial \lambda_v}{\partial z_1} + \frac{k}{\theta} \tag{41}$$

$$\frac{\partial \alpha}{\partial \lambda} = -\left(\frac{\partial \lambda_v}{\partial z_1}\right)^2 d_2 - c_2 + k + \theta \frac{\partial \lambda_v}{\partial z_1} \tag{42}$$

$$\frac{\partial \lambda}{\partial Y_r} = c_2 \left(\frac{-k}{\theta}\right) + d_2 \left(\frac{\partial \lambda_v}{\partial z_1}\right)^2 \left(\frac{-k}{\theta}\right) + \frac{\partial \lambda_v}{\partial z_1} k \tag{43}$$

$$\frac{\partial \alpha}{\partial \dot{Y}_r} = c_2 \frac{1}{\theta} + d_2 \left(\frac{\partial \lambda_v}{\partial z_1}\right)^2 \frac{1}{\theta} - \frac{\partial \lambda_v}{\partial z_1} - \frac{k}{\theta} \tag{44}$$

$$\frac{\partial \alpha}{\partial \ddot{Y}_r} = \frac{1}{\theta} \tag{45}$$

$$\frac{\partial \alpha}{\partial z_1} = \frac{\partial^2 \lambda_v}{\partial z_1^2} \hat{v} + c_2 \frac{\partial \lambda_v}{\partial z_1} + d_2 \left(\frac{\partial \lambda_v}{\partial z_3}\right)^3 + k \frac{\partial \lambda_v}{\partial z_1}$$
$$+ \frac{k^2}{\theta} + 2d_2(\lambda_v - \lambda) \frac{\partial \lambda_v}{\partial z_1} \frac{\partial^2 \lambda_v}{\partial z^2} - \frac{\theta(k_b^2 + z_1^2)}{(k_b^2 - z_1^2)^2} \tag{46}$$

$$\frac{\partial^2 \lambda_v}{\partial z_1^2} = \frac{1}{\theta} \left[6c_1 z_1 - \frac{2z_1 d_1(3k_b^2 + z_1^2)}{(k_b^2 - z_1^2)^3} \right] \tag{47}$$

The third Lyapunov candidate function is chosen as

$$V_3 = V_2 + \frac{1}{2} z_3^2 + \frac{\epsilon^2}{2kd_3} \tag{48}$$

with $d_3 > 0$. The differentiation of (48) is

$$\dot{V}_3 = -c_1 z_1^2 - c_2 z_2^2 + z_3 \dot{z}_3 - \epsilon^2 \left(\frac{3}{4d_1} + \frac{3}{4d_2} + \frac{1}{d_3}\right) - d_1 \left(\frac{z_1}{k_b^2 - z_1^2} - \frac{\epsilon}{2d_1}\right)^2 - d_2 \left(z_2 \frac{\partial \lambda_v}{\partial z_1} - \frac{\epsilon}{2d_2}\right)^2 \tag{49}$$

To make $\dot{V}_3 \leq 0$, $\dot{z}_3$ is chose as

$$\dot{z}_3 = -c_3 z_3 - d_3 z_3 F^2 + \epsilon F \tag{50}$$

where

$$F = -\frac{AKi_1}{2(x_0 - x)^2} - \frac{\partial i_{1,d}}{\partial z_1} \tag{51}$$

and $c_3 > 0$. The control signal u_1 which stabilizes the AMB system around the equilibrium point x_0 in *Case 1* can be obtained from (34) and (50) as follows

$$u_1 = \frac{1}{A} \left[ARi_1 - F\hat{v} - \dot{Y}_r - c_3 z_3 - d_3 z_3 F^2 + \frac{\partial i_{1,d}}{\partial \lambda} \dot{\lambda} + \frac{\partial i_{1,d}}{\partial \eta} \dot{\eta} \right]$$
$$+ \frac{1}{A} \left[\left(\frac{AKi_1}{2(x_0 - x)^2} + \frac{\partial i_{1,d}}{\partial Y_r}\right) \dot{Y}_r + \frac{\partial i_{1,d}}{\dot{Y}_r} \ddot{Y}_r + \frac{\partial i_{1,d}}{\partial \ddot{Y}_r} \dddot{Y}_r \right] \tag{52}$$

Case 2: In this case,

$$x > 0, \quad i_1^2 = 0, u_1 = 0, \quad _{2,d} = x_0 + x\sqrt{-\alpha} \tag{53}$$

Similar to *Case 1*, the control action u_2 in this case is

$$u_2 = \frac{1}{B} \left[BRi_2 - G\hat{v} - \dot{Y}_r - c_4 z_4 - d_4 z_4 F^2 + \frac{\partial i_{2,d}}{\partial \lambda} \dot{\lambda} \right]$$
$$+ \frac{1}{B} \left[\frac{\partial i_{2,d}}{\partial \eta} \dot{\eta} - \left(\frac{BKi_2}{2(x_0 + x)^2} - \frac{\partial i_{2,d}}{\partial Y_r}\right) \dot{Y}_r + \frac{\partial i_{2,d}}{\partial \dot{Y}_r} \ddot{Y}_r \right] + \frac{1}{B} \left[\frac{\partial i_{2,d}}{\partial \ddot{Y}_r} \dddot{Y}_r \right] \tag{54}$$

with $c_4 > 0$ and

$$G = \frac{BKi_2}{2(x_0 + x)^2} - \frac{\partial i_{2,d}}{\partial z_1} \tag{55}$$

4. STABILITY ANALYSIS

Similar to the control design section, the stability analysis of the AMB control system is also divided into two cases.

Case 1: $x < 0$ and $i_2 = 0$. The closed-loop system can be described by

$$\begin{cases} \dot{z}_1 = \theta z_2 - c_1 z_1 (k_b^2 - z_1^2) - \frac{d_1 z_1}{k_b^2 - z_1^2} + \epsilon \\ \dot{z}_2 = -c_2 z_2 - d_2 z_2 \left(\frac{\partial \lambda_v}{\partial z_1}^2 \right) - \frac{\theta z_1}{k_b^2 - z_1^2} - \frac{\partial \lambda_v}{\partial z_1} \epsilon \\ \dot{z}_3 = \dot{i}_1 - \dot{i}_{1,d} \end{cases} \quad (56)$$

where,

$$\dot{i}_1 = A \left[-R i_1 - \frac{K(\hat{v} + \epsilon) i_1}{2(x_0 - x)^2} + u_1 \right] \quad (57)$$

$$\dot{i}_{1,d} = \frac{\partial i_{1,d}}{\partial z_1} \dot{z}_1 + \frac{\partial i_{1,d}}{\partial \lambda} \dot{\lambda} + \frac{\partial i_{1,d}}{\partial \eta} \dot{\eta} + \frac{\partial i_{1,d}}{\partial Y_r} \dot{Y}_r + \frac{\partial i_{1,d}}{\partial \dot{Y}_r} \ddot{Y}_r + \frac{\partial i_{1,d}}{\partial \ddot{Y}_r} \dddot{Y}_r \quad (58)$$

$$x = z_1 + Y_r \quad (59)$$

The velocity observer in this case is

$$\hat{v} = \eta + \theta \lambda + kx + \frac{g}{k} \quad (60)$$

with,

$$\begin{cases} \dot{\eta} = -k\eta - k^2 x \\ \dot{\lambda} = -k\lambda - \frac{i_2^2}{(x_0 + x)^2} \end{cases} \quad (61)$$

and

$$\frac{\partial \alpha}{\partial \eta} = - \left(\frac{\partial \lambda_v}{\partial z_1} \right)^2 \frac{d_2}{\theta} - \frac{c_2}{\theta} + \frac{\partial \lambda_v}{\partial z_1} + \frac{k}{\theta} \quad (62)$$

$$\frac{\partial \alpha}{\partial \lambda} = - \left(\frac{\partial \lambda_v}{\partial z_1} \right)^2 d_2 - c_2 + k + \theta \frac{\partial \lambda_v}{\partial z_1} \quad (63)$$

$$\frac{\partial \alpha}{\partial Y_r} = -c_2 \frac{k}{\theta} - d_2 \left(\frac{\partial \lambda_v}{\partial z_1}^2 \right) \frac{k}{\theta} + \frac{\partial \lambda_v}{\partial z_1} k \quad (64)$$

$$\frac{\partial \alpha}{\partial \dot{Y}_r} = \frac{c_2}{\theta} + d_2 \left(\frac{\partial \lambda_v}{\partial z_1} \right)^2 \frac{1}{\theta} - \frac{\partial \lambda_v}{\partial z_1} - \frac{k}{\theta} \quad (65)$$

$$\frac{\partial \alpha}{\ddot{Y}_r} = \frac{1}{\theta} \quad (66)$$

$$\frac{\partial \alpha}{\partial z_1} = \frac{\partial^2 \lambda_v}{\partial z_1^2} \hat{v} + c_2 \frac{\partial \lambda_v}{\partial z_1} + d_2 \left(\frac{\partial \lambda_v}{\partial z_1} \right)^3 + k \frac{\partial \lambda_v}{\partial z_1}$$
$$+ \frac{k^2}{\theta} + 2 d_2 (\lambda_v - \lambda) \frac{\partial \lambda_v}{\partial z_1} \frac{\partial^2 \lambda_v}{\partial z^2} - \frac{\theta(k_b^2 + z_1^2)}{(k_b^2 - z_1^2)^2} \quad (67)$$

$$\frac{\partial^2 \lambda_v}{\partial z_1^2} = \frac{1}{\theta} \left[6 c_1 z_1 - \frac{2 z_1 d_1 3 k_b^2 + z_1^2}{(k_b^2 - z_1^2)^3} \right] \quad (68)$$

$$\frac{\partial i_{1,d}}{\partial z_1} = -\sqrt{\alpha} + (x_0 - x) \frac{1}{2\sqrt{\alpha}} \frac{\partial \alpha}{\partial z_1} \quad (69)$$

$$\frac{\partial i_{1,d}}{\partial \lambda} = (x_0 - x) \frac{1}{2\sqrt{\alpha}} \frac{\partial \alpha}{\partial \lambda} \quad (70)$$

$$\frac{\partial i_{1,d}}{\partial \eta} = (x_0 - x) \frac{1}{2\sqrt{\alpha}} \frac{\partial \alpha}{\partial \eta} \quad (71)$$

$$\frac{\partial i_{1,d}}{\partial Y_r} = -\sqrt{\alpha} + (x_0 - x) \frac{1}{2\sqrt{\alpha}} \frac{\partial \alpha}{\partial Y_r} \quad (72)$$

$$\frac{\partial i_{1,d}}{\partial \dot{Y}_r} = (x_0 - x)\frac{1}{2\sqrt{\alpha}}\frac{\partial \alpha}{\partial \dot{Y}_r} \tag{73}$$

$$\frac{\partial i_{1,d}}{\partial \ddot{Y}_r} = (x_0 - x)\frac{1}{2\sqrt{\alpha}}\frac{\partial \alpha}{\partial \ddot{Y}_r} \tag{74}$$

To show the stability of the AMB system in this case, consider the following candidate Lyapunov function

$$V_{C1} = \frac{1}{2}\log\left(\frac{k_b^2}{k_b^2 - z_1^2}\right) + \frac{1}{2}z_2^2 + \frac{1}{2}z_3^2 + \frac{\epsilon^2}{2k}\left(\frac{1}{d_1} + \frac{1}{d_2} + \frac{1}{d_3}\right) \tag{75}$$

Differentiate both sides of (75) yields

$$\dot{V}_{C1} = -c_1 z_1^2 - c_2 z_2^2 - c_3 z_3^2 - d_1\left(\frac{z_1}{k_b^2 - z_1^2} - \frac{\epsilon}{2d_1}\right)^2$$
$$- d_2\left(\frac{\partial \lambda_v}{\partial z_1} - \frac{\epsilon}{2d_2}\right)^2 - d_3\left(z_3 F - \frac{\epsilon}{2d_3}\right)^2 - \left(\frac{3}{4d_1} + \frac{3}{4d_2}\frac{3}{4d_3}\right)\epsilon^2 \tag{76}$$

Since c_1, c_2, c_3, d_1, d_2, d_3 are positive constants, then $\dot{V}_{C1} < 0$ which means system (56) is stable. Proof of bounded output is given in [18].

Case 2: $x > 0$ and $i_1 = 0$. In this case, the stability analysis can be treated in the same manner as in *Case 1*.

5. SIMULATION RESULTS

The closed-loop system is numerically tested to verify the ability of the proposed control design. The AMB used in the numerical simulation has the following parameters:

Parameters	Value
Nominal air-gap	0.001m
Number of turns	400
Coil resistance	1Ω
Cross-section area	0.001m^2
Rotor mass	2.6Kg
Rotor initial position	0.0004Kg
Air-gap permeability	$1.256.10^{-6}$

Assume, initially the rotor is at a distance of 0.0004m away from the x_0 ie. attached to coil 1. The simulation is carried out in two scenarios. First, a conventional direct Lyapunov is applied and then Lyapunov function with appended term to limit system output response is embedded into the system.
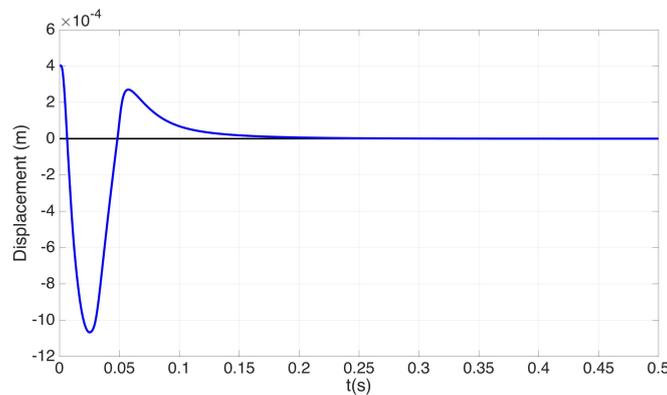

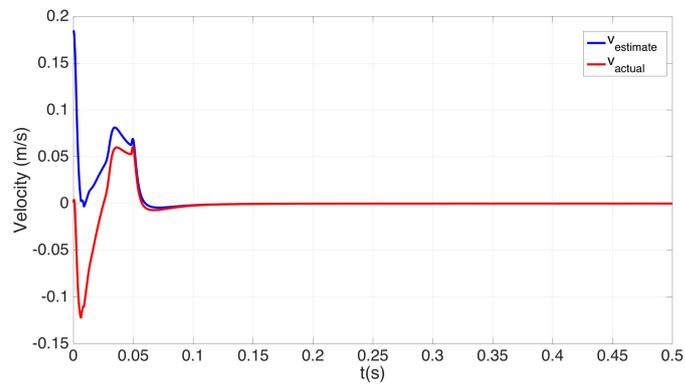Figure 2. Displacement.

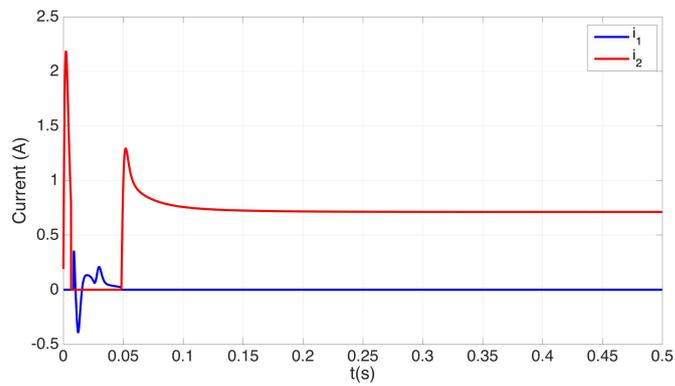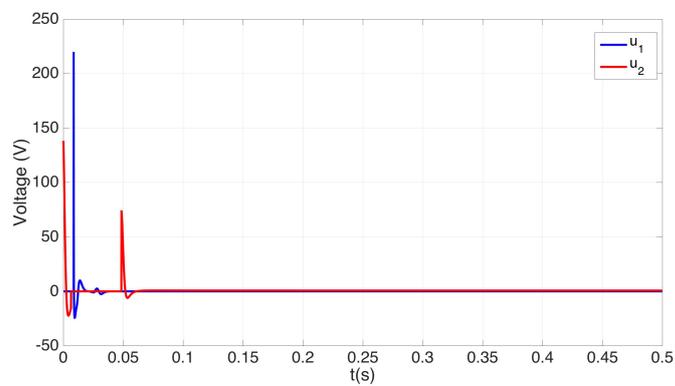Figure 3. Actual and estimated velocity.

Figure 4. Current.

Figure 5. Control voltage.

It can be seen from the system responses that the rotor displacement has a maximum value of 0.001m. The phenomenon implies the rotor hits coil 2 during transient period , this is undesirable in practice.
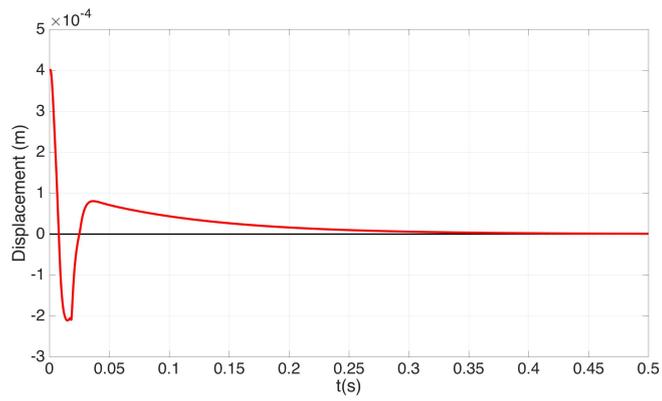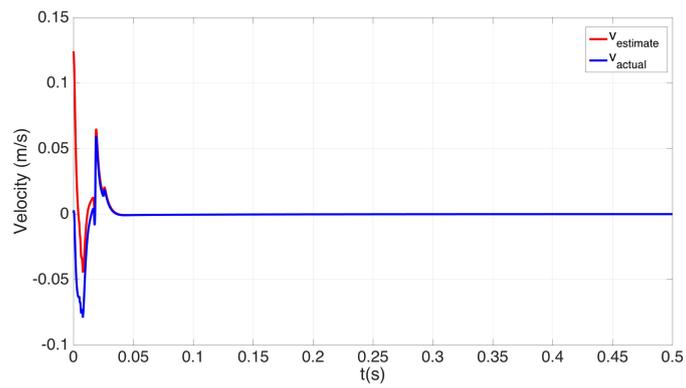
Figure 6. Displacement.
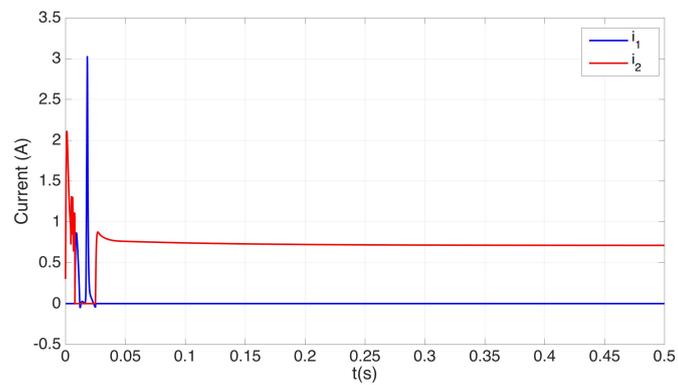

Figure 7. Actual and estimated velocity.
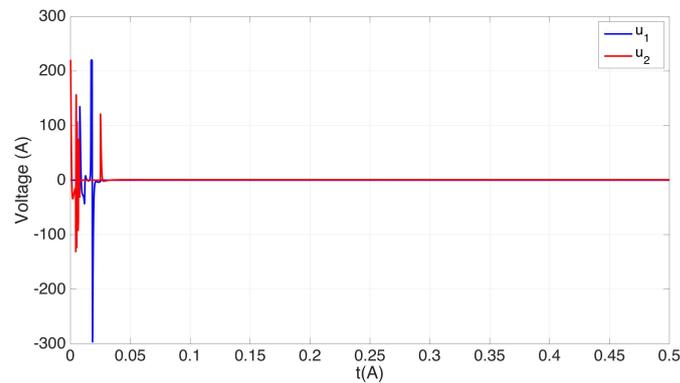

Figure 8. Current.

Figure 9. Control voltage.

In order to maintain the rotor position in safe range, the barrier Lyapunov candidate function is employed instead of the conventional one. System responses indicate better tracking performance. The rotor is kept in the predefined zone and possible mechanical contact with stator is avoided. In addition, the closed-loop system dynamics also prove the ability of the speed observer. It is noted that the control current and voltage are in a practical area.

6.   CONCLUSION

This paper presents the positioning control design for an AMB system. To handle the inherent instability and nonlinearity of the AMB, back stepping control technique is employed. Besides, by introducing a barrier Lyapunov function in control design, the mechanical contact which may happen in transient response can be avoided. An nonlinear velocity is also used to reduce the cost of the system. The feasibility and effectiveness of the proposed control strategy are verified by simulations results. The simulations show good tracking performances of the system and all control inputs are in applicable values.

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