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Identification of Nonlinear Systems Structured by Wiener-Hammerstein Model

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ABSTRACT

Wiener-Hammerstein systems consist of a seriesconnection including a nonlinear static element sandwichedwith two linear subsystems. The problem of identifying Wiener-Hammerstein models is addressed in the presence of hard nonlinearity and two linear subsystems of structure entirely unknown (asymptotically stable). Furthermore, the static nonlinearity is not required to be invertible. Given the system nonparametric nature, the identification problem is presently dealt with by developing a two-stage frequency identification method, involving simple inputs.

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1. INTRODUCTION

Wiener-Hammerstein systems consist of a series connection including a nonlinear static element sandwiched with two linear subsystems (Fig. 1). Accordingly, this structure of models can be viewed as a generalization of Hammerstein and Wiener models and so it is expected to feature a superior modeling capability. This has been confirmed by several practical applications e.g. paralyzed skeletal muscle dynamics [1]. Note that, the internal signals: $v(t)$, $w(t)$, $x(t)$ and $\xi(t)$ are not accessible to measurements. The only measurable signals are the system input $u(t)$ and output $y(t)$.The available methods have been developed following three main approaches i.e. iterative nonlinear optimization procedures e.g. [2]; stochastic methods e.g. [3]; frequency methods [4].

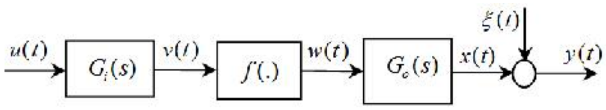

Figure 1.Wiener-Hammerstein Model structure.

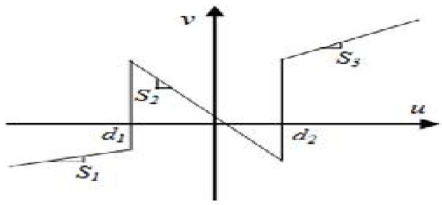

Figure 2. Hard nonlinearity with preload.

In this paper, a frequency-domain identification scheme is designed for Wiener-Hammerstein systems involving two linear subsystems (asymptotically stable) of entirely unknown structure, unlike many previous works. Quite a few previous studies have dealt with block-oriented systems (of any type) that involve piecewise affine nonlinearities (Fig. 2) that are, possibly discontinuous and of a priori unknown structure. The system nonlinearity can have several effects [4]. In particular, it may contain a saturation effect or dead zone. One key contribution of the present work is to show that the system identification is possible without passing by an orthogonal series expansion of the (possibly) discontinuous input nonlinearity. Given the system nonparametric nature, the identification problem is presently dealt with by developing a two-stage frequency identification method. First, the identification of system nonlinearity can be achieved by using a set of constant points. Then, the linear subsystems can be dealt by developing a frequency identification method.

The outline of the remaining part of this paper consists of 4 sections. The identification problem is formally described in Section 2. Section 3 is devoted to the identification of the system nonlinear element. The linear subsystems identification is discussed in Section 4. Simulations are presented in Section 5.

2. IDENTIFICATION PROBLEM STATEMENT

We are interested in systems that can be described by the Wiener-Hammerstein structure (Fig. 1) with hard nonlinearity (Fig. 2) with known segments number q. This model is analytically described by the following equations:

$$v(t)=g_i(t)*u(t). \tag{1a}$$

$$w(t) = f\left(v(t)\right) = f\left(g_i(t)*u(t)\right). \tag{1b}$$

$$y(t) = g_o(t) * w(t) + \xi(t) = g_o(t) * f\left(v(t)\right) + \xi(t). \tag{1c}$$

where $g_i(t)=L^{-1}(G_i(s))$ and $g_o(t)=L^{-1}(G_o(s))$ are the inverse Laplace transform of $G_i(s)$ and $G_o(s)$ (respectively); * refers to the convolution operation. The linear subsystems are of entirely unknown structure, but are BIBO stable (because system identification is carried out in open loop) with non-null static gain. For a problem of identifiability, at least one of nonlinearity segment has nonzero slope. The external noise $\xi(t)$ is supposed to be a zero-mean stationary sequence of independent random variables and ergodic.

Let $\left[u_m \quad u_M\right]$ be the working interval. The problem complexity also lies in the fact that the internal signals are not uniquely defined from an input-output view point. In effect, if $\left(G_i(s), f(v), G_o(s)\right)$ is representative of the system then, any model of the form $\left(G_i(s)/k_1\ ,\ k_2 f(k_1 v)\ ,\ G_o(s)/k_2\right)$ is also representative whatever the real numbers $k_1 \neq 0$ and $k_2 \neq 0$. To get benefit of model plurality, these constants can be chosen as follows: $k_2 = G_o(0)$ and $k_1 = G_i(0)$. Then, the focus model is characterized by the following properties:

$$G_i(0) = G_o(0) = 1. \tag{2}$$

Equation (2) implies that, if $u(t)$ is constant then the steady-state undisturbed output $x(t)$ depends only on the input value and the nonlinearity $f(.)$. Specifically, let:

$$u(t) = U \text{ for } t \in \left[0 \quad kT_r\right], \text{ with } k>1. \tag{3}$$

where k is any integer and T_r is comparable to the system rise time. Then, the internal signal $x(t)$, in the steady-state, is of the form of:

$$x(t) = f(U). \tag{4}$$

3. IDENTIFICATION OF SYSTEM NONLINEARITY

The Wiener-Hammerstein system is excited by a set of constant inputs $u(t) \in \{U_1; \ldots ; U_N\}$, where the number N is arbitrarily chosen by the user and $U_1 < U_2 < \ldots < U_N$. Afterward, using the fact that $y(t) = x(t) + \xi(t)$, it is readily obtained from (4) that, the steady state of the system output $y(t)$ can be expressed as follows:

$$y(t) = f(U_j) + \xi(t) \quad \text{where } j \in \{1; \ldots ; N\}. \tag{5}$$

Hence, the estimate of $f(U_j)$, for any input U_j, can be recovered by averaging $y(t)$ on a sufficiently large interval (the noise $\xi(t)$ is zero-mean). The above results suggest the following estimator for $f(.)$:

$$\hat{f}(U_j) = \hat{X}_j(k) = \frac{1}{kT_r} \int_0^{kT_r} y(t)\, dt. \tag{6}$$

with $j = 1 \ldots N$. Accordingly, a number of points of the nonlinear function $f(.)$ can thus be accurately estimated (i.e. the resulting system, in the steady state, boils down to the linearity part $f(.)$). This yields the following statement:

Proposition 1

The couple of points $\left(U_j, \hat{X}_j(k)\right)$, for $j = 1 \ldots N$, determined in the Nonlinearity Estimator, converge (in probability) to the trajectory of $f(.)$.

Accordingly, from (6), one gets estimates of N points $\left(U_j, f(U_j)\right)$ belonging to $f(.)$. Furthermore, the larger the parameter N is, the better the estimation accuracy. Then, by successively connecting allavailable points $\left\{(U_j, f(U_j)); k = 1 \cdots N\right\}$, a piecewise affine approximation of $f(.)$ is obtained. If the number of obtained segment is less than q, the nonlinear system is excited by other constant inputs. Finally, let choose any segment l of the identified nonlinearity with nonzero slope.

4. LINEAR SUBSYSTEMS IDENTIFICATION

In this section, an identification method is proposed to obtain estimates of the complex gain corresponding to the two linear subsystems $G_i(j\omega)$ and $G_o(j\omega)$ for a set offrequencies $\{\omega_1; \ldots ; \omega_m\}$. Let $\varphi_i(\omega) = \arg\left(G_i(j\omega)\right)$ and $\varphi_o(\omega) = \arg\left(G_o(j\omega)\right)$. For simplicity, we presently suppose that the nonlinearity identification have been exactly determined. Then, let define the variables (for any ω):

$$\varphi(\omega) = \varphi_i(\omega) + \varphi_o(\omega). \tag{7a}$$

$$\left|G(j\omega)\right| = \left|G_i(j\omega)\right| \left|G_o(j\omega)\right|. \tag{7b}$$

The subsystem identification can be implemented in two stages:

Firstly, an accurate estimates of $\left|G(j\omega_k)\right|$ and $\varphi(\omega_k)$, for any frequency $\omega_k \in \{\omega_1; \ldots ; \omega_m\}$, can be determined.

In the 2nd stage, the estimates of complex gains (modulus gains and phases) the two linear subsystems ($G_i(j\omega)$ and $G_o(j\omega)$) can be separated.

The identification problem under study is dealt using a method based on the frequency approach. The Wiener-Hammerstein systemis excited with a given sine input:

$$u(t) = u_o + U\sin(\omega_k t)\,. \tag{8}$$

where the amplitude $U > 0$ is a priori small value. The choice of u_o can be performed using the experimental data of nonlinearity estimation. It can take any value in the vicinity from the center of segment l. Then, as the linear subsystem $G_i(s)$ is asymptotically stable, it follows from (2)-(8), one has in the steady state:

$$v(t) = u_o + U\left|G_i(j\omega_k)\right|\sin(\omega_k t + \varphi_i(\omega_k))\,. \tag{9}$$

If $v(t)$ spans only the chosen segment, one gets:

$$w(t) = S^* v(t) + P^*. \tag{10}$$

where S^* is the slope of segment l and P^* is the value of $w(t)$ when $v(t) = 0$. In practice, this case can be easily detected by a simple inspection of the output signal. For a small value of the amplitude U, the steady state of system output $y(t)$ is a sine signal (up to noise). As soon as, $v(t)$ spans at least two segments, $y(t)$ is not a sine signal. Accordingly, a judicious choice for U can be given practically. Then, from (9)-(10), the internal signal $w(t)$ is written in the following form:

$$w(t) = US^*\left|G_i(j\omega_k)\right|\sin(\omega_k t + \varphi_i(\omega_k)) + S^* u_o + P^*\,. \tag{11}$$

As the linear subsystem $G_o(s)$ is asymptotically stable, it follows from (11) and (7a-b) that, the steady state undisturbed output $x(t)$ can be expressed as follows:

$$x(t) = S^* u_o + P^* + US^*\left|G(j\omega_k)\right|\sin\left(\omega_k t + \varphi(\omega_k)\right). \tag{12}$$

Finally, as $y(t) = x(t) + \xi(t)$, one immediately gets:

$$y(t) = US^*\left|G(j\omega_k)\right|\sin\left(\omega_k t + \varphi(\omega_k)\right) + y_o + \xi(t)\,. \tag{13}$$

where:

$$y_o = S^* u_o + P^*\,. \tag{14}$$

On the other hand, recall that sine signals that oscillate at the same frequency as $\sin\left(\omega_k t + \varphi(\omega_k)\right)$ and having the amplitude U are of the form:

$$z_\delta(t) = U\sin\left(\omega_k t + \delta\right). \tag{15}$$

where $\delta \in IR$ is arbitrary and IR denotes the set of real numbers. It is readily seen that:

$$U\sin\left(\omega_k t + \varphi(\omega_k)\right) = z_{\varphi(\omega_k)}(t)\,. \tag{16}$$

Let $C_\delta^{\omega_k,U,u_o}$ is the parameterized locus constituted of all points of coordinates $\left(z_\delta(t), x(t)\right)$. These curves are viewed as a generalization of the Lissajous curves used in linear system frequency analysis [5].

These ideas are formalized in the following proposition:

Proposition 2

Consider the Wiener-Hammerstein system described by equations (1a-c) and excited by the input (8), with u_0 and U are judiciously chosen so that the system output $y(t)$ is sine signal (up to noise). Then, one has:

The locus $C_\delta^{\omega_k,U,u_o}$ is a linear curve if and only if $\delta = \varphi(\omega_k)$ (modulo π).

Another key idea (getting benefit from Proposition 2) is to determine the gain modulus $\left|G(j\omega_k)\right| = \left|G_i(j\omega_k)\right|\left|G_o(j\omega_k)\right|$ and the phase $\varphi(\omega_k) = \varphi_i(\omega_k) + \varphi_o(\omega_k)$ by tuning the parameter δ until the locus $C_\delta^{\omega_k,U,u_o}$ shows linear curve.The point is that the locus $C_\delta^{\omega_k,U,u_o}$ depends on the signal $x(t)$ which is not accessible to measurement. This is presently coped with making full use of the information at hand, namely the periodicity (with period $2\pi/\omega_k$) of both $z_\delta(t)$ and $x(t)$ and the ergodicity of the noise $\xi(t)$. Bearing these in mind, the relation $y(t)=x(t)+\xi(t)$ suggests the following estimator:

$$\hat{x}(t,M)=\frac{1}{M}\sum_{j=1}^{M} y(t+jT_k)\,.\,t\in[0,T_k)\,. \tag{17a}$$

$$\hat{x}(t+jT_k,M)=\hat{x}(t,M) \text{ for any integer } j>0\,. \tag{17b}$$

where $T_k=2\pi/\omega_k$ and M is a sufficiently large integer. Specifically, for a fixed time instant t, the quantity $\hat{x}(t,M)$ turns out to be the mean value of the (measured) sequence $\{y(t+jT_k);\, j=0\ 1\,...\}$. Then, an estimate $\hat{C}_{\delta,M}^{\omega_k,U,u_o}$ of $C_{\delta}^{\omega_k,U,u_o}$ is simply obtained substituting $\hat{x}(t,M)$ to $x(t)$ when constructing $C_{\delta}^{\omega_k,U,u_o}$. These remarks lead to the following proposition:

Proposition 3

Consider the problem statement of Proposition 2. Then, one has:

1) $\hat{x}(t,M)$ Converges in probability to $x(t)$ (as $M\to\infty$).

2) $\hat{C}_{\delta,M}^{\omega_k,U,u_o}$ Converges in probability to $C_{\delta}^{\omega_k,U,u_o}$ (as $M\to\infty$).

On the other hand, let δ^* the corresponding value of δ and $s(\omega_k)$ the slop of the obtained curve $C_{\delta^*}^{\omega_k,U,u_o}$. Knowing the sign of S^*, the phase $\varphi(\omega_k)$ can be recovered modulo 2π. Let us consider the parameter γ defined as follows:

$$\gamma=0 \text{ if } \operatorname{sign}(S^*)=\operatorname{sign}\left(s(\omega_k)\right) \text{ else } \gamma=1\,. \tag{18}$$

Let $\hat{s}_M(\omega_k)$ denotes the estimate of $s(\omega_k)$. Then, an estimate $\left(\hat{\varphi}_M(\omega_k),\left|\hat{G}_M(j\omega_k)\right|\right)$ of $\left(\varphi(\omega_k),\left|G(j\omega_k)\right|\right)$ can be determined, one has thus, for any frequency ω_k :

$$\hat{\varphi}_M(\omega_k)=\hat{\varphi}_i(\omega_k,M)+\hat{\varphi}_o(\omega_k,M)=\delta^*+\gamma\pi \text{ (modulo } \pi). \tag{19a}$$

$$\left|\hat{G}_M(j\omega_k)\right|=\left|\hat{G}_i(j\omega_k,M)\right|\left|\hat{G}_o(j\omega_k,M)\right|=\left|\frac{\hat{s}_M(\omega_k)}{S^*}\right|. \tag{19b}$$

5. SIMULATION

The identification method described in this paper will now be illustrated by simulation using Matlab/Simulink. Presently, the system is characterized by:

$$G_i(s)=\frac{0.2}{(s+1)(s+0.2)} \text{ and } G_o(s)=\frac{0.05}{(s+0.1)(s+0.5)}\,. \tag{20}$$

The considered nonlinear element is illustrated by Fig. 4. The noise $\xi(t)$ is a sequence of normally distributed (pseudo) random numbers, with zero-mean and standard deviation $\sigma_\xi=0.02$. Firstly, the aim is to estimate the system nonlinearity. The identification method described in Section 3 will now be applied and, accordingly, the system is successively excited by $N=11$ constant inputs, $\{U_j;\, j=1\ldots N\}$, where the values U_j and the obtained estimates of $f(U_j)$ are shown in Fig. 5. The true nonlinearity and the set ofpoints $\left(U_j\,,\hat{f}_L(U_j)\right)$ $(j=1\ldots 11)$, are represented in Fig. 6. By connecting the set of collinear points (Fig.6). The $q=3$ segments are then obtained.

The nonlinear system is excited by (8). Fig.7 shows example of obtained results when $v(t)$ spans at least two segments. Then, $y(t)$ is not a sine signal. This confirms the result already obtained using the plot $\hat{C}_{\delta,M}^{\omega_k,U,u_o}$. This latter, turns out to be a non static curve, whatever $\delta\in[0\ 2\pi)$ (Fig. 8a-b). For a small value of U, e.g. $U=0.25$, $\omega_1=0.01\,(rd/s)$ and $u_o=-0.75$, Fig. 9a shows the measured output $y(t)$. This signal turns out to be a sine signal (up to noise). Then, using the estimator (17a-b), the filtered output $\hat{x}(t,M)$ is generated. $\hat{x}(t,M)$ is represented in (Fig. 9b). The locus $\hat{C}_{\delta,M}^{\omega_1,U,u_o}$ is plotted for different $\delta \in [0 \quad 2\pi)$ and a sample of the obtained curves is shown by Figs. 10a-b. It is seen that $\hat{C}_{\delta,M}^{\omega_1,U,u_o}$ associated to $\delta = 2$ is not linear ($\delta \neq \varphi(\omega_k)$). Further, the curve $\hat{C}_{\delta,M}^{\omega_1,U,u_o}$ associated to $\delta^* = 3.01$ is affine portion. Additionally, it is seen that the sign of S^* is different from that of $\hat{C}_{\delta,M}^{\omega_1,U,u_o}$. We have thus shown that $\hat{\varphi}_M(\omega_1) = \delta^* + \pi = 6.15\,(rd)$ (modulo 2π). From Fig. 10b, one has $\hat{s}_M(\omega_k) = 0.97$.

The estimator (19b) is used to get estimates of the modulus $|G(j\omega_k)|$. Accordingly, $|\hat{G}_M(j\omega_k)| \approx 0.97$.

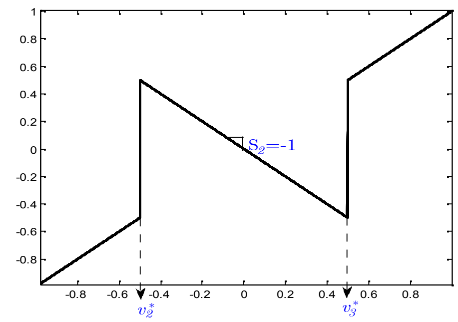

Figure 4. Nonlinear hard element *f*(.) considered in simulation.

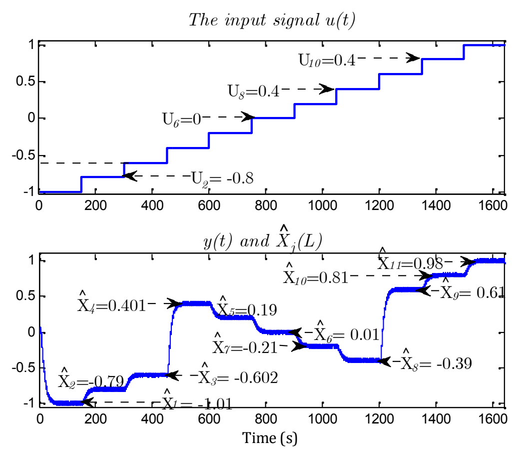

Figure 5. $u(t)$, $y(t)$ and the undisturbed output estimate.

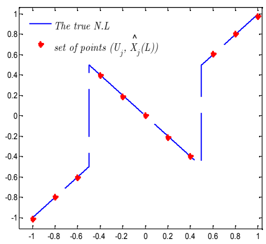

Figure 6. The true N.L and set of estimated points.

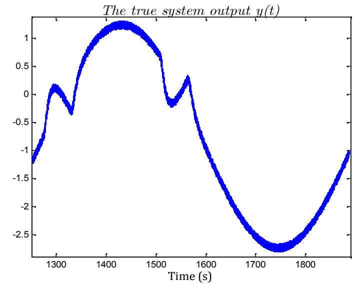

Figure 7. The steady-state output $y(t)$ obtained over one period.

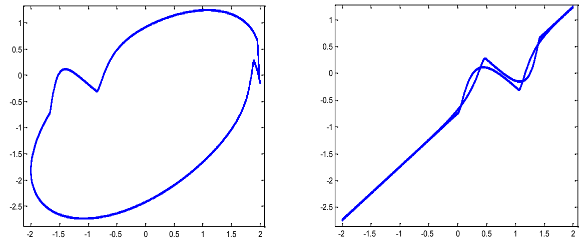

Figure 8.a. The locus $\hat{C}_{\delta,M}^{\omega_1,U,u_o}$ for $\delta = 5.1 \neq \varphi(\omega_k)$; b. $\hat{C}_{\delta,M}^{\omega_1,U,u_o}$ for $\delta = 6.1 = \varphi(\omega_k)$.

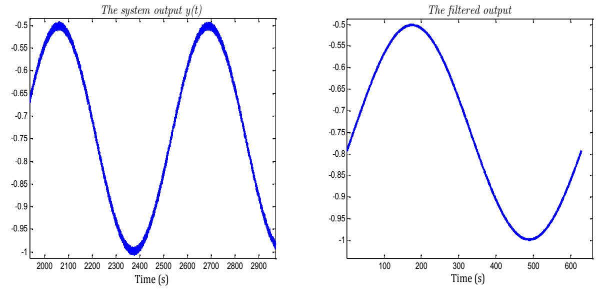

Figure 9. a. The steady-state of $y(t)$; b. One period of $\hat{x}(t, M)$.

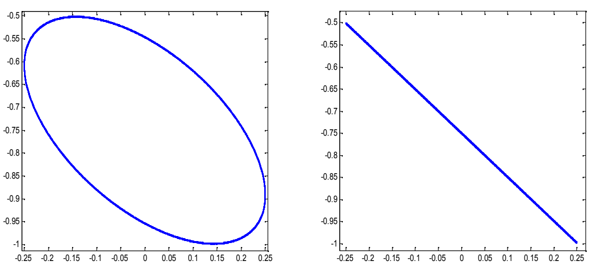

Figure 10. a. $\hat{C}_{\delta,M}^{\omega_1,U,u_o}$ for $\delta = 2\,(rd)$; b. $\hat{C}_{\delta,M}^{\omega_1,U,u_o}$ for $\delta = 3.01\,(rd)$.

TABLE 1.

Frequency gain estimates $\hat{G}_M(j\omega_k)$			
ω_k (rd/s)	0.01	0.05	0.1
$\varphi(\omega_k)$ (rd)	6.1	5.42	4.74
$\hat{\varphi}_M(\omega_k)$ (rd)	6.15	5.38	4.78
$\left\|G(j\omega_k)\right\|$	0.99	0.86	0.62
$\left\|\hat{G}_M(j\omega_k)\right\|$	0.97	0.88	0.65

6. CONCLUSION

We have developed a new frequency identification method to deal with Wiener-Hammerstein systems; the identification problem is addressed in presence of hard nonlinearity and two linear subsystems of structure entirely unknown. The present study constitutes a significant progressin frequency-domain identification of block-oriented nonlinear system identification.The originality of the present study lies in the fact that the system is not necessarily parametric and of structure totally unknown. Another feature of the method is the fact that the exciting signals are easily generated and the estimation algorithms can be simply implemented.Then, the complex gains (modulus gains and phases) of linear subsystems can be obtained.

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