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An efficient application of Particle Swarm Optimization in control of constrained two-tank system

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ABSTRACT

Despite all the Model Predictive Control (MPC) based solution advantages such as a guarantee of stability, the main disadvantage such as an exponential growth of the number of the polyhedral region by increasing the prediction horizon exists. This causes the increment in computation complexity of control law. In this paper, we present the efficiency of particle swarm optimization in optimal control of a two-tank system modeled as piecewise affine. The solution of constrained final time-optimal control problem (CFTOC) is derived, and then particle swarm optimization algorithm is used to reduce the computational complexity of control law and set the physical parameters of the system to improve performance simultaneously. On other hand, a new combined algorithm based on PSO is going to be used to reduce the complexity of explicit MPC-based solution CFTOC of the two-tank system; consequently, that the number of polyhedral is minimized and system performance is more desirable simultaneously. The proposed algorithm is applied in simulation and our desired subjects are reached.

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1. INTRODUCTION

According to the system modeling, it is possible to introduce Piecewise Affine (PWA) systems, a particular hybrid system class. It is defined by partitioning the vast input-state space into Polyhedron regions and assigning a PWA equation to each of these regions. Discrete-time PWA systems are a very effective tool for modeling most hybrid systems [1]. These systems have established themselves as a powerful class for identifying and approximating generic nonlinear systems by multi-linearization at Equilibrium [2]. A practical method in designing controllers of nonlinear systems is optimal control concepts in constrained and non-constrained processes by linear discrete-time models in the form of state space. a constrained discrete model predictive control strategy for a greenhouse inside temperature is presented[3]. Given what has been said about the modeling advantages of most systems based on the PWA class, in recent years, there has been a great deal of interest in computing the optimal form-package controller for PWA systems. These problems became known as the Constrained Final Time-Optimal Control (CFTOC)[4,5]. The most important methods of analysis of this problem are multi-parameter programming, RHC, or MPC. MPC is an effective way to deal with constrained control problems and has found many applications and advances in industry and academic research. This method requires that the next position of the variables be predicted based on the current position, the controller input, and the process model. In other words, the sequence of control inputs that optimize the objective function is computed and applied to the process. This control concept is called the MPC [6]. the control of boiler turbine process with three manipulated variables and three controlled variables has been attempted using MPC technique[7]. Optimal control sequence allows recalculation and feedback performance in MPC whenever a new measurement arrives; this is known as MPC. The stability of the control system and the fulfillment of conditions, constraints, and requirements during operation are ensured by the proper formulation of the objective function. RHC has the complexity and high volume of online computing related to optimizing and reducing system robustness due to the difference between the actual and MPC processes. Optimal methods based on the use of MP-LP and MP-QP were provided [8,9]. The offline calculation of the optimal control rule for constrained discrete-time linear systems was performed using these methods. The resulting rules were made available as a PWA function on the polyhedrons. At present, explicit MPC techniques enable a standard method in controller design for nonlinear processes that are modeled in the PWA form, creating a substitute for intelligent controller design methods such as fuzzy logic and neural networks in high-performance applications [10]. Unfortunately, one of the main problems is the increasing complexity of the control rule obtained by increasing the prediction horizon and its effect on system performance. On the other hand, it is necessary to increase the prediction horizon for the system's optimal performance. For linear systems with parametric uncertainty by the Lyapunov function, the PWA controller is designed with low complexity [11]. Effective representation and approximation are provided by in-depth learning to MPC of LTI systems. Theoretically, at least neurons and hidden layers are considered [12]. A nonlinear robust MPC with input-dependent perturbations and states and uncertainty is presented [13]. The MPC algorithm with PWA control rules is presented for discrete-time linear systems in the presence of finite *perturbation* [14]. The online computational burden of the linear MPC can be transferred offline using multi-parameter programming, which is called explicit MPC [15]. A flexibility algorithm is proposed to reduce the calculation volume in [15] that the designer can balance between time and storage Complexities. This is done by hash tables and the associated hash functions. Two modified controllers instead of the standard MP-QP are used [16] to reduce the complexity of the multi-parameter programming of MPC. The problem of reducing the complexity of explicit MPC for linear systems is considered by PWA employing separating functions [17]. A Semi-Continuous PWA model based on the optimal control method for the nonlinear system is proposed [18]. First, the nonlinear system is approximated by multi-linear subsystems, then these subsystems are combined into a PWA system and formulated as an optimal control problem. A computational method for optimizing and controlling a two-tank system with three control valves is presented [19]. The main advantages of PSO are easy implementation and the ability to optimize complex objective functions with many local minimums. Furthermore, PSO can search the much-extended space of candidate solutions. The dynamics of the tank system are nonlinear. The linear model is considered, and the parameters are adjusted so that the difference between the actual system and the model is minimized by solving the optimal control problem. PSO has been used to solve the problem of constrained optimization [20]. SAPSO is recommended to increase PSO performance. Theoretically, the convergence of the method has been investigated. Considerable interest has recently been generated to use PSO in optimization and engineering problems [21]. a new algorithm which is a combination of model predictive control with particle swarm optimization is presented to optimal control of constrained DC-DC power system modeled as piecewise affine[22].The present paper organized as follows, First, the CFTOC of PWA systems is expressed briefly. Having introduced the two-tank optimal control in section III, in section IV the application of PSO to solve the expressed problem is discussed, and eventually, the simulation results and conclusion are presented.

2. CFTOC problem and solution

We will focus on the constrained PWA systems as follows [4]:

$$x(t+1) = f_{PWA}\big(x(t), u(t)\big) := A_i x(t) + B_i u(t) + f_i \quad \begin{pmatrix} x(t) \\ u(t) \end{pmatrix} \in D_i \tag{1}$$

Where $t \geq 0$, the domain $D := \bigcup_{i=1}^{N_D} D_i$ of $f_{pwa}(.,.)$ is a non-empty compact set in $R^{n_x+n_u}$, $N_D < \infty$ is the number of system dynamics and $D := \bigcup_{i=1}^{N_D} D_i$ denotes a polyhedral partition of the domain D. i.e. the closure of D_i is

$\bar{D_i} := \left\{ \begin{pmatrix} x \\ u \end{pmatrix} \in R^{n_x+n_u} \middle| D_i{}^x x + D_i{}^u u \leq D_i{}^0 \right\}$ and $\mathrm{int}(D_i) \cap \mathrm{int}(D_j) = \emptyset \ \ \forall i \neq j$. We define CFTOC problem for piecewise affine system (1) in the form below [4]:

$$J_T^*(x(0)) := \min_{U_T} J_T(x(0), U_T) \quad (2.a)$$

$$\text{subject to} \begin{cases} x(t+1) = f_{PWA}(x(0), u(t) \\ \quad x(t) \in \chi^f \end{cases} \quad (2.b)$$

$$J_T(x(0), U_T) := l_T(x(T)) + \sum_{t=0}^{T-1} l(x(t), u(t)) \quad (2.c)$$

Where $J_T(.,.)$ is the cost function, l(.,.) the stage cost, $l_T(.)$ the final penalty, U_T optimization variable described as the input sequence $U_T := \{u(t)\}_{t=0}^{T-1}$, T<∞ receding horizon and χf is a compact terminal target set inR^{n_x}. If the solution of CFTOC problem is not unique, $u_T^*(x(0)) := \{u^*(t)\}_{t=0}^{T-1}$ determines one realization from the set of possible optimizer.

CFTOC problem determine a set of initial state and feasible inputs as $\chi_T \subset R^{n_x}(x(0) \in \chi_T)$, $U_{T-t} \subset R^{n_u}(u(t) \in U_{T-t}, t = 0, \dots, T-1)$ respectively.

The explicit closed form solution can be expressed as $u^*(t){:}\ \chi_T \rightarrow U_{T-t}$,t=0,…,T-1. The considered system is PWA (1) and the cost is based on 1, ∞ norm. i.e.

$$l(x(t), u(t)) := \|Qx(t)\|_p + \|Ru(t)\|_p \quad (3.a)$$

$$l(x(T)) := \|Px(T)\|_p \quad (3.b)$$

Where $\|.\|_p$ with p={1,∞} represent the standard vector norm 1,∞. The solution of (2) with aforesaid restrictions is time-varying PWA function of the initial state x(0)∈ $\mathcal{P}_i$

$$u^*(t) = \mu_{PWA}(x(0), t) = K_{T-t,i}x(0) + L_{T-t,i} \quad (4)$$

Where $t = 0, \dots, T-$, $\{\mathcal{P}_i\}_{i=1}^{N_p}$ is the polyhedral partition of a set of feasible state x(0), $\chi_T = \cup_{i=1}^{N_p} \mathcal{P}_i$,with the closure of $\mathcal{P}_i$ stated as $\bar{\mathcal{P}}_i = \{x \in R^{n_x} | \mathcal{P}_i^x x \le \mathcal{P}_i^o\}$ [2].

If a receding horizon control strategy is used for closed loop, the control law is stated as time-varying PWA state feedback of the form [4]:

$$\mu_{RH}(x(t)) := K_{T,i}x(t) + L_{T,i} \quad if\ x(t) \in \mathcal{P}_i \quad (5)$$

Where i=1,…,Np and for t≥0, $u^*(t) = \mu_{RH}(x(t))$. CFTOC problem can be presented and solved for any selection of P,Q,R, albeit here it is assumed that the parameters T,Q,R,P and χ^f are selected by the following assumptions[3]. To avoid additional control actions in steering states to the origin (equilibrium point), matrices R, Q are required to have a full column rank.

3. Optimal Control of Two-Tank System

The two-tank [23] shown in Figure 1 is a basic benchmark model to investigate and analyze the control issues for PWA system.

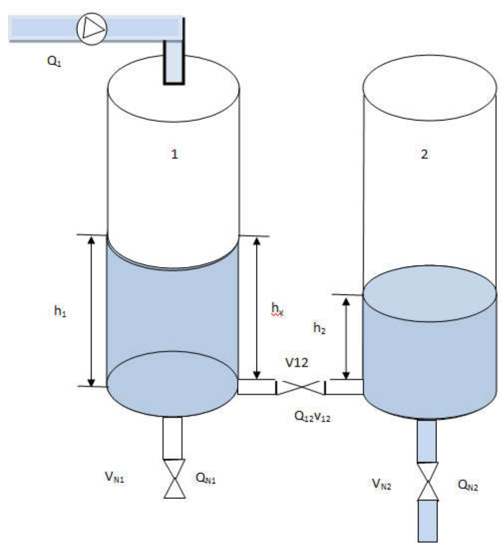

Figure 1. Two-tank system schematic

It consist of two tanks that connected to each other. We assumed that:

- The valves behavior is linear.
- The initial volume of liquid in tanks is zero.
- The inflow of liquid to the first tank is constant and has the maximum value.

The liquid volume of tank 1 is defined by a time varying equation as:

$V_1(t)=V_{0,1}+(Q_1-Q_{1,2})\times t$ (6.a)

$V_1(t)=A_1\times h_1(t)$ (6.b)

Where $V_{0,1}$ is initial volume of liquid in tank 1and Q_1,$Q_{1,2}$ are inflow and outflow liquid of tank1 ($Q_{1,2}$can be defined as inflow of liquid to tank 2), A_1 and h_1 are base area and the time varying height of liquid in tank1 respectively. The Eq.(6) can be repeated for the tank 2 with similar definition as:

$V_2(t)=V_{0,2}+(Q_{1,2}-Q_2)\times t$ (7.a)

$V_2(t)=A_2\times h_2(t)$ (7.b)

By combining equations (6),(7),

$$h_1(t)=\frac{1}{A_1}\left(V_{0,1}+\left(Q_1-Q_{1,2}\right)\times t\right) \quad (8.a)$$

$$h_2(t)=\frac{1}{A_2}\left(V_{0,2}+\left(Q_{1,2}-Q_2\right)\times t\right) \quad (8.b)$$

The operation of instruction is following :

"The tanks are filled by a pump acting on tank 1, continuously manipulated from 0 up to a maximum flow Q1. A switching valve V_{12} controls the flow between the tanks. This valve is assumed to be either completely opened or closed (V_{12}=0 or 1 respectively). The V_{N2} manual valve controls the nominal outflow of the second tank. It is assumed in the simulations that the manual valves, V_{N1} is always closed and that V_{N2} is open. The liquid levels to be controlled are denoted by h_1,h_2 for each tank respectively"[18].

The system is expressed as a discrete time model with a sampling time (T_s=10s) by Eq.(9):

$$\begin{cases} h_1(k+1)=h_1(k)+\frac{T_s}{A_1}\left(Q_1(k)-k_{12}V_{12}^*(h_1(k)-h_2(k)\right) \\ h_2(k+1)=h_2(k)+\frac{T_s}{A_2}\left(k_{12}V_{12}^*(h_1(k)-h_2(k))-k_{N2}V_{N2}^*h_2(k)\right) \end{cases} \quad (9)$$

This model can be formulated as a piecewise affine system of form (1), with four subsystems (four modes), described as follows:

- Mode one V_{12}^* open, $h_1\geq h_v$
- Mode two V_{12}^* open, $h_1\leq h_v$
- Mode three V_{12}^* closed, $h_1\geq h_v$
- Mode four V_{12}^* closed, $h_1\leq h_v$

For instance, for mode one the system matrices are:

$$A_1=\begin{bmatrix}0.9542 & -0.0393\\ 0.0941 & 0.9670\end{bmatrix}\ B_1=\begin{bmatrix}0.0699 & 0\\ 0 & 0\end{bmatrix}\ C_1=[1\ 0]\ D_1=[0]\ and\ f_1=\begin{bmatrix}0.0164\\ -0.0164\end{bmatrix}$$

The CFTOC problem of the presented PWA system is solved by MPT [25] based on MPC for the prediction horizon=3,norm =1, Q = eye(2),R = (1e-5)*eye(2) and the explicit PWA control law has 78 polyhedral region as shown in Figure 2 and the close loop system performance from a given initial condition is presented in Figure 3.

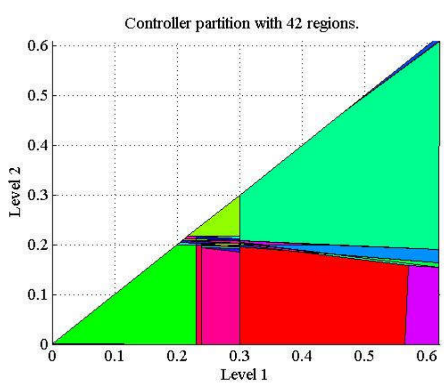

Figure 2. controller partitions

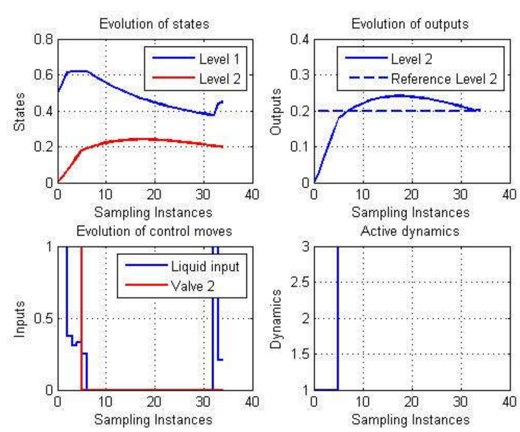

Figure 3. closed loop system performance

Using PSO algorithm, the considered purposes are going to be fulfilled simultaneously:

- The number of polyhedral of explicit MPC-based control law is minimized to get the complexity reduction.
- The liquid reaches a certain height in tanks in a short time and desirable manner.

4. PSO algorithm Application for solving defined problem

Soon afterwards, a brief review of PSO algorithm is presented and then PSO Application is investigated to solve the defined problem.

Particle swarm optimization is a heuristic global optimization method put forward originally by Kennedy and Eberhart in 1995[24]. It is developed from swarm intelligence and is based on the research of bird and fish flock movement behavior. Each particle's movement is influenced by its best known local position and is also guided toward the best known positions in the search-space, which are updated as better positions are found by other particles. According to Figure 4, the basis of methods is as follows:

Each particle can be shown by its current speed and position, the most optimal position of each individual and the most optimal position of the surrounding [24].

Having chosen the initial population X_i, V_i, the speed and position of each particle change around search space according to the equality(10) [24]:

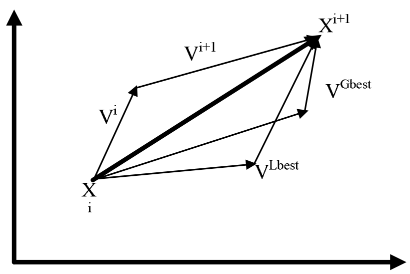

Figure 4. The basis of evolutionary PSO algorithm

$$X_i = [x_{1,i}\ x_{i,2} \cdots x_{n,i}]$$
$$V_i = [v_{1,i}\ v_{i,2} \cdots v_{n,i}]$$
$$V_{id}^{k+1} = V_{id}^{k} + c_1 \times r_1^k \times \left(V_{id}^{Lbest}\right) + c_2 \times r_2^k \times \left(V_{id}^{Gbest}\right) \tag{10.a}$$
$$X_{id}^{k+1} = X_{id}^{k} + V_{id}^{k} \tag{10.b}$$
$$V_{id}^{Lbest} = pbest_{id}^{k} - X_{id}^{k} \tag{10.c}$$
$$V_{id}^{Gbest} = gbest_{id}^{k} - X_{id}^{k} \tag{10.d}$$

Where In this equality, $V_{id}^k$and X_{id}^k separately stand for the speed of the particle “i” at its “k” times and the d-dimension quantity of its position; $pbest_{id}^k$ represents the d-dimension quantity of the individual “i” at its most optimal position at its “k” times. $gbest_{id}^k$ is the d-dimension quantity of the swarm at its most optimal position. In order to prevent a particle being far away from the searching space, the speed of the particle created at its each direction is confined between -v_{dmax}, and v_{dmax}. If the number of v_{dmax} is too big, the solution is far from the best,, otherwise the solution will be the local optimum; c1 and c2 represent the speeding figure, regulating the length when flying to the most particle of the whole swarm and to the most optimal individual particle. If the figure is too small, the particle is probably far away from the target field, if the figure is too big, the particle mayfly to the target field suddenly or fly beyond the target field. The proper figures for c1 and c2 can control the speed of the particles flying and the solution will not be the partial optimum. c1 usually is equal to c2 and they are equal to 2; r_1 and r_2 represent random fiction, and 0-1 is a random number.

As mentioned before, our new aim is using PSO for complexity reduction of explicit MPC-based control law by reduction the number of its polyhedral and setting the physical parameter of system to improve the system performance simultaneously. Therefore, the following objective function has been defined:

Fitness-Function ≜Number of polyhedral + Output specifications

Where output specifications are determined as a summation of operational specifications such as settling time, over shoot, under Shoot, steady state deviation, time constant, and so forth.

Now, the explicit controller obtained in previous section as a part of PSO should be consider and the following new performance index is being defined:

$$\mathrm{J_{new}}_{\mathrm{T}}^{*} \coloneqq Min(Fitness - Function) \coloneqq$$
$$Min[number\ of\ polyhedrals\ of\ \mu_{RH}(x(0) +\ Output\ speciication] \tag{11.a}$$

$$\mathrm{S.T}\begin{cases} \mathrm{u}^* = \mu_{\mathrm{RH}}(\mathrm{x(t)}) \coloneqq \mathrm{K_{T,i}x(t)} + \mathrm{L_{T,i}} \quad \text{if } \mathrm{x(t)} \in \mathcal{P}_\mathrm{i} \\ \mathrm{Output - Spec.} \triangleq \sum \text{desired output characteristic} \end{cases} \tag{11.b}$$

It is used according the flowchart shown in Figure 5.

The problem is solved as following steps:

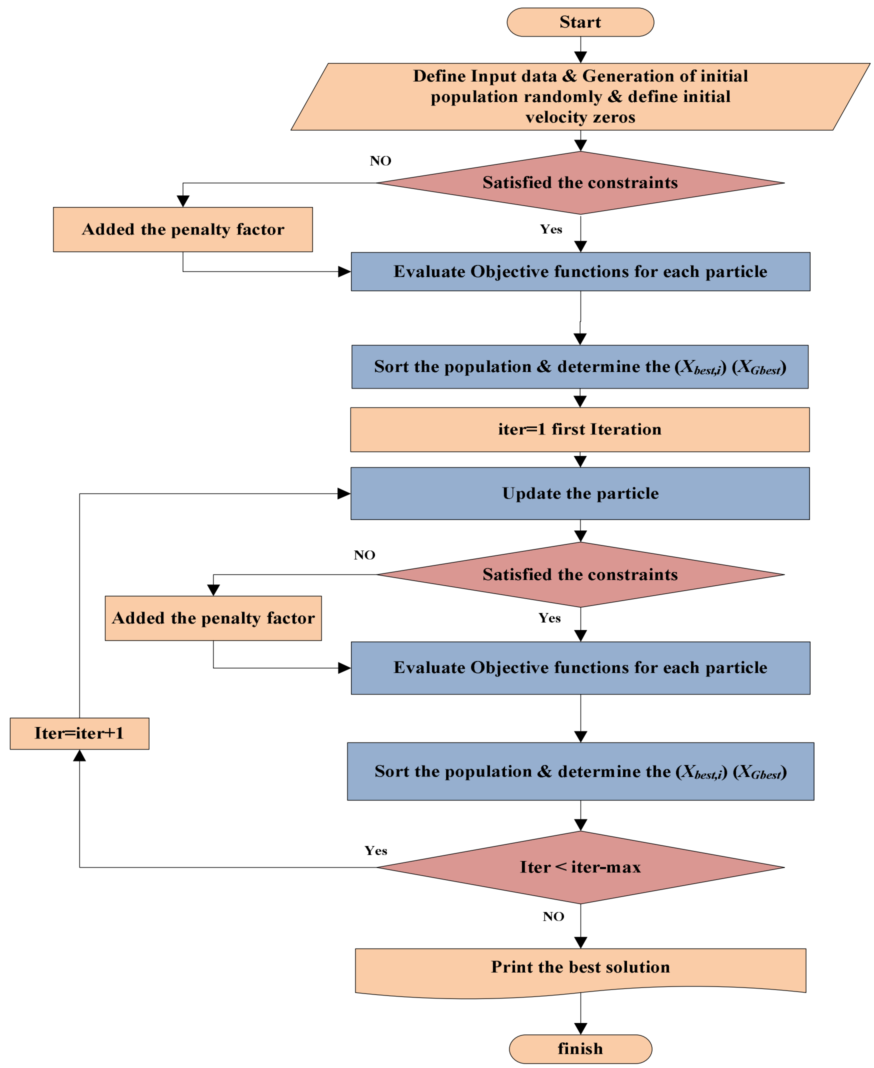

Figure 5. Flowchart of PSO algorithm Application

1- Required data such as time step, liquid height, algorithm parameters like number of population and number of iteration are considered.
2- Initial population is created as following:

$$X_i = [\text{height of valve1, cross section of valves, maximum height, base area of each tank}]$$

$$\text{initial population} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_{N\ pop} \end{bmatrix}$$

3- The system output specifications are measured based on initial parameters then the control law(obtained by MPT) is applied and the number of polyhedral regions is calculated. Eventually objective function is defined as
Fitness-Function=Number of polyhedral+ output specifications. Where we consider output specifications can be assumed as:

$$\text{Output specifications} \triangleq \text{settling time} + \text{certain height of liquid}$$

The best solution among the total population is determined and population is updated based on (10.a, 10.b).
4- For predefined iteration, steps 3,4 are done iteratively.
5- The convergence condition is checked and the best solution is shown in output finally.

The optimal parameters are compiled in the table (1).

Table.1. The optimal parameters

sampling time	height of valve 1	cross-section of valves	maximum height	area of each tank	Num P	Time(S)
10 s	0.1 m	1.00E-05	0.5	0.001 m^2	10	189

T is the required time to reach the certain height of liquid. As presented, the number of control law polyhedral reduces from 42 to 10. In Figure 6, the output flow variation of tank 1 and In Figure 7, the liquid height in tank 1 is shown. These figures are repeated for tank 2 in Figure 8 and Figure 9 respectively.

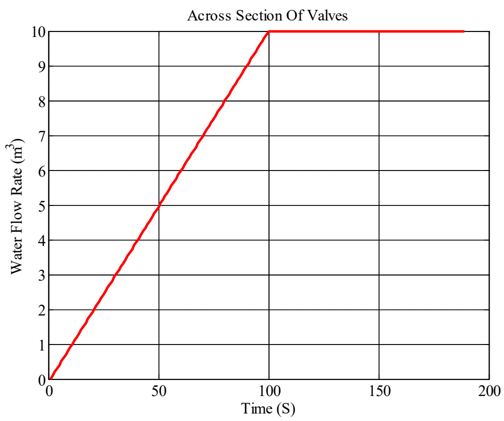

Figure 6. The output flow variation of tank 1

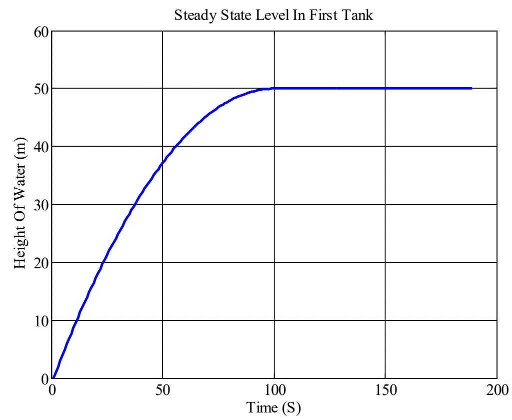

Figure 7. the liquid height in tank 1

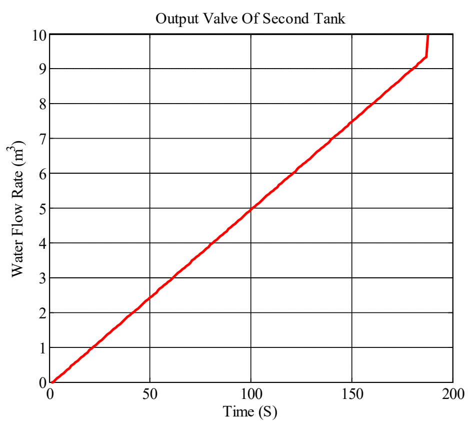

Figure 8. the output flow variation of tank 2

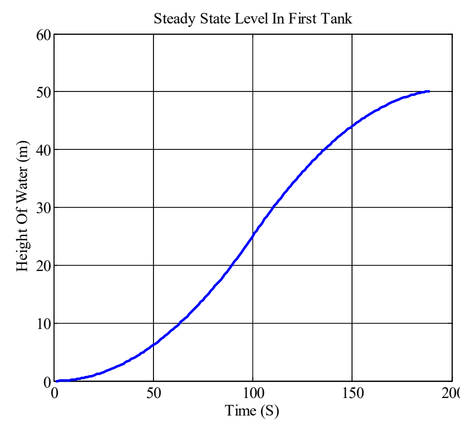

Figure 9. the liquid height in tank 2

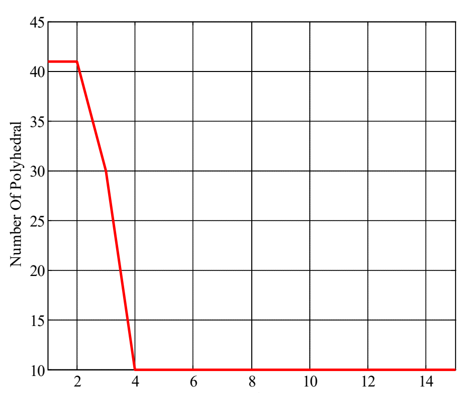

Figure 10. convergence diagram of objective function

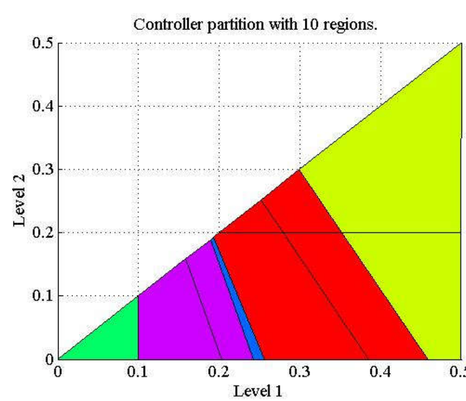

Figure 11. controller with minimum partitions

According to the results, it can be concluded that physical parameters setting through using PSO, the number of polyhedral of MPC-based control law is minimized; thus, the related complexity of solution to CFTOC will be reduced along with the improvement of output specification simultaneously so that the liquid height in both tanks reach desired certain value in 189 seconds.

5. CONCLUSION

Several analytical methods have been used for the CFTOC solution. Their main disadvantage is the computational complexity of the solution; therefore, the problem can be considered as NP-hard. The hyper-heuristic algorithm is used to solve the NP-hard optimization problems that have strategies to escape from the local optimal solution and are applicable in a wide range of issues. In general, the development of hyper-heuristic methods is taken by investigating and inspiration optimization type in nature like Particle Swarm Optimization. By using PSO and an appropriate definition of the objective function, the complexity of the MPC-based solution of CFTOC was reduced and the system performance was improved simultaneously. The most massive advantage of the recommended method is that if the mentioned purposes were not in one direction, we can define a multi-objective function to fulfill aims. According to the simulation results, it is demonstrated that the number of polyhedral and the dependent complexity of CFTOC solution are reduced, the system performance such as reaching the liquid height at a certain time is desirable and the obtained steady-state error reaches zero. the number of control law polyhedral reduces from 42 to 10. the liquid height in both tanks reaches desired certain value in 189 seconds.

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