Modeling the Vibrational Dynamics of Piezoelectric Actuator by System Identification Technique

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ABSTRACT

Actuators based on smart materials such as piezoelectric actuators (PEAs) are widely used in many applications to transform electrical signal to mechanical signal and vice versa. However, the major drawbacks for these smart actuators are hysteresis nonlinear, creep and residual vibration. In this paper, PEAs are used for active vibration application. Therefore, a model of PEA must be established to control the vibration that occurs in the system. The frequencies of 1 Hz, 20 Hz and 50 Hz were tested on the PEAs. The results obtained from the experiment were used to develop transfer function model by employing system identification technique. Meanwhile, the model validation was based on level of models fitness to estimation data, mean squared error (MSE), final prediction error (FPE) and correlation test. The experimental result showed that the displacement of the actuator is inversely proportional to the frequency. The following consequences caused the time response criteria at 50 Hz achieved smallest overshoot and fastest response of rise time and settling time.

Keyword: Actuator Piezoelectric System identification Vibration

1. INTRODUCTION

Piezoelectricity from the Greek word, piezo or piezin means pressure electricity was first discovered by J. and P. Curie in 1880 in quartz crystal. Piezoelectricity is a fundamental process in electromechanical energy conversion in certain crystalline materials [1]. These crystalline materials turn out to be electrically polarized when subjected to mechanical stress and conversely change shape in the presence of electric field. Soon after, it was discovered that materials showed deformation as electrical signal is applied presented for actuators material. Piezoelectric actuators are smart material devices become competitive choices for micro applications [2]. Piezoelectric actuators have been increasingly used because of high stiffness, high resolution, fast response, no magnetic field and high precision [3]-[4]. As an example, piezoelectric actuator is used in atomic force microscopy (AFM) and scanning probe microscopy (SPM) for micro and nanoscales applications [3]. They are also widely used in mechatronic applications such as car fuel injector, medical surgery equipment and active vibration control [5]. Nevertheless, piezoelectric actuators reveal undesirable displacement behavior that limits the accuracy and stability of the actuators. The major issues are:

i. Hysteresis: The system has different output with the same input applied to piezoelectric actuator. The error generates 10-15 % error under open-loop voltage control. Though, the error reaches 35 % when the frequency of the input signal is increased [6].

ii. Creep: The amplitude of displacement output is change over time at constant input voltage signal [7].

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iii. Residual vibration: The actuator vibrates before reach steady state or final position. This is due to mechanical properties such as mass, stiffness and damping [8].

The hysteresis behavior in piezoelectric actuator causes inaccuracy and instability to its applied system. Hysteresis is characterized as nonlinear because the voltage - displacement curve is not one-one mapping [3],[9]. The same input voltage produces different output displacement for increasing and decreasing input voltage. In addition, hysteresis behavior also characterized as rate-dependency because the hysteresis curve orientation and shape is changed as the frequency of input voltage is changed.

Figure 1 showed the hysteresis nonlinear phenomenon in the piezoelectric actuators for open loop and close loop system. The close loop system shows the hysteresis effect for the actuator is minimized by using controller. Figure 2 showed effect of creep phenomena in piezoelectric actuator. This phenomenon will give significant error when the PEA extended periods of time. It can be observed that, at the same input voltage signal, the displacement output is changed overtime. According to [10], the hysteresis and creep strongly affect the actuator at frequency lower than 10 Hz. Moreover, vibration is repetitive motions that occur in time interval with respect to a point of reference. The vibration is caused by potential energy storage in spring, kinetic energy storage for mass and gradual energy damping for damper. These effects reduced the performance accuracy and increased the uncertainty of measurements [11]. Moreover, vibration is a repetitive motion that occur in time interval with respect to a point of reference. The vibration occurred because of potential energy storage in spring, kinetic energy storage for mass and gradual energy damping for damper. This effect reduced the performance accuracy and increased the uncertainty of measurements [10]. Furthermore, the vibration frequency restrained the development of high speed-SPM to be used in semiconductor industries, to study the mechanism of fast surface processes, and to developed SPM-based nanofabrication equipments.

![Figure 1. Hysteresis nonlinear behaviour in open loop and close loop system](image1)

![Figure 2. Creep properties over time](image2)
The mathematical model of PEA can be developed by using physics law and parameter estimation approach. The physics law need full understanding of the system and physical interactions related to it. Meanwhile, parameter estimation approach only required input-output data [12]. The parameters of mathematical model that obtained based on physics law then need to be optimized. According to [13], the parameters of model was obtained by using artificial neural network (ANN). The neural networks had to be trained until the simple model was achieved. Besides, the faster training of simple model resulting less computation in actual control. Though, the neural network need to be created, trained, and evaluated for various feedforward backpropagation networks until the network gave accurate model. Furthermore, the parametric identification such as recursive least square method, genetic algorithm and particle swarm optimization also can be used for parameters optimization. The genetic algorithm (GA) is a heuristic method to estimate the parameters based on evolutionary ideas of natural selection and genetics [14]. The parameters that need to be recognized known as chromosomes that form a string called gene. The process of GA was started with initialized the values of chromosomes. The values were randomly chosen to form a population. Then, the evaluation fitness and algorithm were calculated until the best estimation value was accomplished. The advantage for this identification optimization technique is, the wrong initialization value for estimation parameters can be identified since the optimal parameters lies outside interval. Hence, results tend to be clustered near the side of the interval that should be tuned. Nevertheless, GA method cannot keep the memory when the population was changed [15]. Therefore, researcher [16] was implementing particle swarm optimization (PSO) technique. PSO is known as a new heuristic method which is easy to understand and implement to optimize the parameters. In addition, PSO provided advantages with high optimization performance and good memory where each particle in PSO was prepared with a small memory comprising its previous best position. This technique was based on the idea of collaborative behaviour and swarming in biological population [17]. The particles hold a position ($x_i$) and velocity ($v_i$) with trial solution of the optimum problem. However, the main drawback for this technique was the fitness function must be selected based on suitability of models and PSO has slow convergence speed [18]. Next, parameter estimation approach such as system identification (SI) is a technique to obtained the mathematical description from measurements of the input and output. This technique only required data collection of input-output data to find the exact or approximate models of dynamic system [19]. The objective of SI is to find the exact or approximate models of dynamic system based on knowledge of the observed input and output data.

2. **Methodology**

The main component of the experiment system used in this study was piezoelectric actuator (AE0505D16DF, NEC Tokin). This actuator provided output displacement corresponding of 17.4 ± 2.0 µm with 150 VDC maximum voltage. Figure 1 showed the experimental setup that has been used as a platform to model the PEA. The frame of tip PEA movement was captured by a software that was connected to inverted microscope. Meanwhile, the microcontroller and switching circuit was connected to the laptop to control the frequency of PEA.

![Platform for conducting the experiments](image_url)

The vibration of PEA was controlled by the frequency that has been generated by Arduino Uno microcontroller and switching circuit. The square wave input signal was supplied to the PEA with 50 % duty cycle. The switching circuit used the transistor C546B as a switch. When the square wave at 0 V, the
transistor was in cut-off region where at this point no current at collector caused the voltage of PEA dropped nearly to zero. However, the high voltage let the transistor to saturate and resulting in maximum collector current and minimum voltage dropped. Therefore, the input voltage of PEA at this moment reached maximum peak. The vibration phenomenon video was recorded by using DigiAcquis software. Then, this video was adapted into still images to measure the displacement of PEA.

The system identification technique was employed to develop mathematical models of a dynamic system based on a set of measured data samples. The developed model was a prediction model that represented the vibration phenomenon of PEA. The system identification approach is shown in Figure 4 where the PEA model was estimated based on input-output data collection, followed by model structure and model validation.

The model structure was selected by trying different combination of zeros and poles until the model gives optimum fit for test data and lower mean squared error (MSE) and fitness prediction error (FPE). The experimental data collection were imported into workspace variable, where input data were identified as voltage and displacement data were identified as output. The toolbox identification in Matlab R2013a as shown in Figure 2.3(a) was used for modelling purpose. In the identification window, press the popup menu-import data and select time domain data to import the data. In this research, there are 300 data of voltage and displacement. Therefore, first 150 data were used for estimation and the remaining 150 for validation. Next, transfer function model was obtained as shown in Figure 2.3(b). In this paper, model validation was evaluated using correlation test which were autocorrelation and cross correlation, MSE and FPE. The confidence interval level used for residual test were 99.9% for all models.

Figure 4. System Identification approach

Figure 5. (a) System identification toolbox window (b) Transfer function estimation in system identification toolbox
3. RESULTS AND ANALYSIS

At this point, the mathematical model of PEA are presented. Three transfer functions have been developed based on data collection for vibration phenomenon. The frequencies used for vibration experiment were 1 Hz, 20 Hz and 50 Hz. The purpose of vibration experimental was to investigate the displacement behavior of PEA at various frequencies.

i) Frequency: 1 Hz

The model structure obtained at frequency of 1 Hz was third order system with three poles and two zeros. The transfer function is shown in equation (1). System identification results presented that the established model fit validation data by 96.58 % fitness level with mean squared error (MSE) of 0.04948 and final prediction error (FPE) of 0.0421. High percentage of model fitness and lower error level recognized the validity of model. Then, residual analysis was continued to validate the model. The majority of sampled data were within the 99.9 % confidence interval limit for autocorrelation function, similarly, all sampled data were within 99.9 % confidence interval limit for cross correlation function. Therefore, the model has passed the validation test. The step response at 1 Hz is shown in Figure 6 and time response criteria can be observed in Table 1.

\[
G(S) = \frac{-8.633s^2-5427s-3523}{s^3+80.36s^2+202.5s+7651}
\]  

(1)

![Figure 6. Step Response for vibration at 1 Hz](image)

Table 1. Time response criteria for vibration at 1 Hz

<table>
<thead>
<tr>
<th>Rise Time (s)</th>
<th>Settling Time (s)</th>
<th>Overshoot(%OS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0103</td>
<td>6.9571</td>
<td>135</td>
</tr>
</tbody>
</table>

ii) Frequency: 20 Hz

The transfer function for PEA model at 20 Hz was given in equation (2). The best prediction model was third order with three poles and two zeros. The system identification stated that the model fits time domain validation data with 73.7 %, MSE of 0.5271 and FPE of 0.5722. Meanwhile, the residual analysis showed that most of sampled data were within the 99.9 % confidence boundary for autocorrelation and cross correlation. Therefore, the model has passed the validation test from residual analysis perspective. The step response for this model is shown in Figure 7 and the time response performance is presented in Table 2.

\[
G(S) = \frac{-0.732s^2-2639s+738.3}{s^3+128.2s^2+462.3s+6390}
\]  

(2)

![Figure 7. Step Response for vibration at 20 Hz](image)

Table 2. Time response criteria for vibration at 20Hz

<table>
<thead>
<tr>
<th>Rise Time (s)</th>
<th>Settling Time (s)</th>
<th>Overshoot(%OS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0089</td>
<td>3.5408</td>
<td>901.5389</td>
</tr>
</tbody>
</table>
iii) Frequency: 50 Hz

The data collection from experimental were used to predict the PEA model using system identification approach. Hence, third order transfer function model with three poles and two zeros has been established for vibration effect at 50 Hz as shown in (3). The step response for this model is shown in Figure 8 and the time response criteria is presented in Table 3. The validation of transfer function as given by equation (3) for vibration was optimized by 66.33 % of fit to validation data, MSE of 0.4047 and FPE of 0.3774. This result recognized the validity of the model with high percentage model fitness and lower error level. In addition, majority of sampled data were within the confidence interval limit for autocorrelation and cross correlation. Therefore, the model has passed the validation test and can be used for this project.

\[
G(S) = \frac{-1.91s^2-1.562e4s-1.19e5}{s^3+926.9s^2+4.326e4s+3.218e6}
\]  

(3)

Table 3. Time response criteria for vibration at 50Hz

<table>
<thead>
<tr>
<th>Rise Time (s)</th>
<th>Settling Time (s)</th>
<th>Overshoot (%OS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00245</td>
<td>1.2016</td>
<td>448.2456</td>
</tr>
</tbody>
</table>

The similar experimental setup has been used to established the PEA model. Therefore, three PEA’s model have been developed at various frequencies of 1 Hz, 20 Hz and 50 Hz. The displacement behaviour of PEA can be observed based on experimental data collection. At 1 Hz, the displacement of PEA was increasingly more than displacement at 50 Hz. The frequency is inversely proportional to displacement of...
PEA. Thus, the model that has been developed by SI technique showed that overshoot was high at 1 Hz and decreased as the frequency up to 50 Hz. The overshoot of the system dropped to 66.42% from initial frequency and reach fast response as the frequency at 50 Hz.

4. CONCLUSION

This paper discussed about modeling of piezoelectric actuator for vibration phenomenon by implementing system identification technique. System identification approach has been used to estimate the dynamic model of PEA by input-output experimental data. The model of PEA must be established first in order to design the controller to minimize the residual vibration in the system. The models of PEA that have been developed by employing this technique were selected based on high fit to validation data, lowest MSE and FPE, autocorrelation and cross correlation for validation test. Then, the controller can be design based on model requirement.

REFERENCES