Mathematical Computing of Coniferous Tree Ignition by the Cloud-to-Ground Lightning Discharge using Joule-Lenz’s Law

Nikolay V. Baranovskiy\textsuperscript{1}, Geniy V. Kuznetsov\textsuperscript{2}, Tatiana N. Nemova\textsuperscript{3}

\textsuperscript{1,2}Departement of Theoretical and Industrial Heat Systems Engineering, Tomsk Polytechnic University, Russia
\textsuperscript{3}Departement of Heat and Gas Supply, Tomsk State University of Architecture and Building, Russia

ABSTRACT

The natural phenomenon of thunderstorm activity is one of many causes of a forest fire. Thunderstorms cause especially intensive fire danger situations within remote areas and highlands. As a rule, a cloud-to-ground lightning discharge is the fire source. The present study is based on the research results of electrical overloads in supply networks. Physical and mathematical formulation and numerical solution for the problem of a coniferous tree (pine) ignited by a cloud-to-ground lightning discharge are presented. The problem is considered in a cylindrical coordinate system in two-dimensional formulation. The features of current passage and heat transfer taking into account the reactive wood localization are investigated. The Joule-Lenz’s law is used to calculate heat production in a tree trunk. Parametric analysis has been conducted and tree trunk ignition conditions have been determined in a typical range for the influencing parameters of negative and positive discharges.

1. INTRODUCTION

The natural phenomenon of thunderstorm activity [1] is one of many causes of a forest fire [2]. Thunderstorms cause especially intensive fire danger situations within remote areas and highlands [3], [4]. As a rule, a cloud-to-ground lightning discharge is the fire source [5]. The empirical and statistical methods to assess forest fire danger (including thunderstorms) are widely used in recent years [6-8]. In addition, deterministic and probabilistic approaches have been developed [9], [10].

Moreover, the mathematical models adequate to the real physical mechanism of tree ignition caused by a cloud-to-ground lightning discharge have not been established for deterministic and probabilistic criteria at the present time. Polarity, voltage, peak current, and duration of action are the main characteristics of cloud-to-ground lightning discharges [11]. The average discharge peak current can be approximately $J=23.5$ kA for a negative discharge and approximately $J=35.3$ kA for a positive discharge [12]. At the same time, 16.5\% of the positive discharge current is less than 10 kA [13].

It is known [14] that wood ignition by an energy source is possible under certain conditions. Heat fluxes to the fuel gasification zone of gas mixture and oxygen concentrations. Wood pyrolysis gaseous products in the gas mixture reach the required values. The moisture content of wood porous structure significantly affects the wood ignition conditions. However, the ignition problem solution even for dry wood is much more complicated than, for example, for polymeric materials or rocket fuels for several reasons [15]. One of the important factors, which one should consider among many others when analyzing the conditions of real wood ignition, is a significant structural heterogeneity. The uneven distribution of branches along the...
tree trunk length affects the discharge passage conditions, heating intensity, and ignition conditions. Modeling of coniferous tree trunk heating caused by a lightning discharge in the two-dimensional formulation is appropriate for this reason.

The purpose of this study is to determine the conditions of coniferous tree ignition by a cloud-to-ground lightning discharge depending on the discharge parameters. The non-one-dimensional heat distribution in the tree trunk is taken into account.

2. FIELD RESEARCH

In the summer of 2016, we conducted field observations of thunderstorm activity in the area of Aeroport village (the Timiryazevskiy forestry in the Tomsk region). Meteorological conditions were obtained from the operational services of Tomsk airport. Thunderstorm formation conditions in the region were obtained using the data from the global network of lightning detection the World Wide Lightning Location Network (WWLL Network) [16-18]. The map of the controlled forest area is provided below (see Figure 1).

![Map of controlled forest area of Timiryazevskiy forestry in Tomsk region](image)

Figure 1. Map of controlled forest area of Timiryazevskiy forestry in Tomsk region

Figure 2 shows a video fragment when the lightning discharge ignites the pine tree trunk (a) and the thermal burn of the tree trunk after hitting it by the cloud-to-ground lightning discharge (b).

![Video fragment](image)

Figure 2. Pine tree trunk ignition (a) and thermal burn of tree trunk after hitting it by cloud-to-ground lightning discharge (b)
The visual observations and analysis of video sequences allowed formulating the physical mechanism of pine tree trunk ignition caused by a cloud-to-ground lightning discharge during the electric current passage.

3. PHYSICAL AND MATHEMATICAL STATEMENT

The present study is based on the research results of electrical overloads in supply networks [19]. The electric current passage is different in case of a lightning discharge in a coniferous tree trunk since the resistance of resinous wood considerably exceeds that of bark and inner bark layer. Therefore, the electric current of the lightning discharge in the pine tree trunk passes mainly on the outer layers without penetrating inside. The study of heat transfer is of particular interest when the localization of the so-called reactive wood is taken into account. This wood is formed at the bottom of branches in coniferous trees and is also called the wood of compression [20]. The reactive wood differs from the usual one in physical and chemical properties [20].

The following physical model has been adopted to describe the simulated process. A detached pine tree has been under consideration. The cloud-to-ground lightning discharge is characterized by certain polarity and the duration of strikes in the tree trunk at the fixed time. It is believed that the discharge current-voltage characteristics are the same for different sections of the tree trunk. The tree trunk wood heating up is due to Joule heating [21] released in the inner bark area of the tree trunk. Electric current passes and heats the wood. As a result, the critical heat occurs, fluxes from the inner bark to surface area, leads to high temperatures, and ignites the tree. The wood moisture content influence [22] on the ignition is neglected.

The decision area scheme is presented in Figure 3(a), where numbers denote the area: 1 is core; 2 is inner bark; 3 is bark; 4 is top branches; 5 is reactive wood (the bottom of branches); 6 is the inner bark area part that has the same properties as area 4; 7 is the inner bark area part that has the same properties as area 5; 8 is the part of core that has the same properties as area 4; 9 is the part of core that has the same properties as area 5; 10, 11 are air. The boundaries of subdomains are designated in Figure 3(b).

![Figure 3](image)

(a) Decision area scheme (a) and boundaries of subdomains (b)

The heat transfer in the decision area is described by the system of unsteady heat conduction Equations:

\[
\rho c_{l_1} \frac{\partial T_1}{\partial t} = \frac{\lambda_1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_1}{\partial r} \right) + \frac{\lambda_2}{\partial z^2} T_2, \quad 0 \leq r \leq R_2, \quad 0 \leq r \leq R_{\text{react}}, \quad 0 \leq z \leq H_2, \quad 0 \leq z \leq H_3, \quad H_3 \leq z \leq H_4.
\]

(1)
\[ \rho_2 c_2 \frac{\partial T_2}{\partial t} = \frac{\lambda_2}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_2}{\partial r} \right) + \lambda_2 \frac{\partial^2 T_2}{\partial z^2} + JU, \quad z_0 \leq z \leq H_1, \quad H_3 \leq z \leq H_s, \quad r_2 \leq r \leq r_1, \]  

\[ \rho_3 c_3 \frac{\partial T_3}{\partial t} = \frac{\lambda_3}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_3}{\partial r} \right) + \lambda_3 \frac{\partial^2 T_3}{\partial z^2}, \quad z_0 \leq z \leq H_1, \quad H_3 \leq z \leq H_s, \quad r_1 \leq r \leq r_4, \]  

\[ \rho_4 c_4 \frac{\partial T_4}{\partial t} = \frac{\lambda_4}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_4}{\partial r} \right) + \lambda_4 \frac{\partial^2 T_4}{\partial z^2} + JU, \quad H_2 \leq z \leq H_3, \quad r_2 \leq r \leq R_{\text{react2}}, \]  

\[ \rho_5 c_5 \frac{\partial T_5}{\partial t} = \frac{\lambda_5}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_5}{\partial r} \right) + \lambda_5 \frac{\partial^2 T_5}{\partial z^2}, \quad H_1 \leq z \leq H_2, \quad R_{\text{react}} \leq r \leq R_2, \]  

\[ \rho_6 c_6 \frac{\partial T_6}{\partial t} = \frac{\lambda_6}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T_6}{\partial r} \right) + \lambda_6 \frac{\partial^2 T_6}{\partial z^2}, \quad z_0 \leq z \leq H_1, \quad H_3 \leq z \leq H_s, \quad r_2 \leq r \leq R_{\text{react2}}, \quad R_s \leq r \leq R_{\text{react2}}. \]  

The temperature field is constant at the initial time:

\[ T_i = T_{i0}, \]  

The boundary conditions for Equations (1) – (10):

\[ \Gamma_0: \quad \lambda_1 \frac{\partial T_1}{\partial r} = 0, \]  

\[ \Gamma_{el}: \quad \lambda_i \frac{\partial T_i}{\partial z} = 0, \quad i=1,2,3, \]  

\[ \Gamma_j: \quad -\lambda_j \frac{\partial T_j}{\partial z} = \alpha_j (T_s - T_j), \quad i=1,2,3, \]  

\[ \Gamma_{el}: \quad \lambda_3 \frac{\partial T_3}{\partial r} = \alpha_e (T_s - T_3), \]  

\[ \Gamma_{el}: \quad \lambda_3 \frac{\partial T_3}{\partial r} = \alpha_e (T_s - T_3), \]  

\[ \Gamma_{el}: \quad \lambda_4 \frac{\partial T_4}{\partial z} = \lambda_4 \frac{\partial T_4}{\partial z}, \quad T_4 = T_1, \]  

\[ \Gamma_{el}: \quad \lambda_4 \frac{\partial T_4}{\partial z} = \lambda_4 \frac{\partial T_4}{\partial z}, \quad T_4 = T_1, \]  

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\[ \Gamma_{1,2}: \quad \lambda_i \frac{\partial T_i}{\partial z} = \alpha_e (T_e - T_4), \tag{18} \]

\[ \Gamma_{2,1}: \quad \lambda_3 \frac{\partial T_3}{\partial z} = \lambda_4 \frac{\partial T_4}{\partial z}, \quad T_5 = T_4 \tag{19} \]

\[ \Gamma_{2,2}: \quad - \lambda_3 \frac{\partial T_3}{\partial z} = \alpha_s (T_e - T_5), \tag{20} \]

\[ \Gamma_{3}: \quad \lambda_4 \frac{\partial T_4}{\partial z} = \lambda_5 \frac{\partial T_5}{\partial z}, \quad T_4 = T_5 \tag{21} \]

\[ \Gamma_{4,1}: \quad \lambda_4 \frac{\partial T_4}{\partial r} = \lambda_5 \frac{\partial T_5}{\partial r}, \quad T_1 = T_4 \tag{22} \]

\[ \Gamma_{4,2}: \quad \lambda_4 \frac{\partial T_4}{\partial r} = \lambda_5 \frac{\partial T_5}{\partial r}, \quad T_1 = T_5 \tag{23} \]

\[ \Gamma_{5,1}: \quad \lambda_5 \frac{\partial T_5}{\partial r} = \alpha_e (T_e - T_4), \tag{24} \]

\[ \Gamma_{5,2}: \quad \lambda_5 \frac{\partial T_5}{\partial r} = \alpha_s (T_e - T_3), \tag{25} \]

\[ \Gamma_{6,1}: \quad T_6 = T_e, \tag{26} \]

\[ \Gamma_{6,2}: \quad T_6 = T_e, \tag{27} \]

\[ \Gamma_{6,3}: \quad T_6 = T_e, \tag{28} \]

\[ \Gamma_{6,4}: \quad - \lambda_6 \frac{\partial T_6}{\partial z} = \alpha_s (T_s - T_6), \tag{29} \]

\[ \alpha_e, \alpha_s, \lambda, c, \rho, J, T, U \] are the temperature, density, thermal conductivity, and heat capacity of subregions \((i = 1, ..., 6)\), \(J\) is current, \(U\) is voltage, \(\alpha_e, \alpha_s\) are heat transfer coefficients, \(r, z\) are space coordinates, \(t\) is time, \(R_s\) is the outer radius of the tree trunk, \(R_1\) is the boundary of bark and inner bark areas, \(R_2\) is the boundary of core section and inner bark, \(H_s\) is the height of the tree trunk, \(H_{1H2}\) is the thickness of the reactive wood zone (the lower branch zone), \(H_{2H3}\) is the thickness of the upper branch zone, \(\Gamma_i\) are boundaries.

Numerical investigations were carried out using the following input data (pine wood, core) [14]:
- \(\rho=500 \text{ kg/m}^3\); \(c=1670 \text{ J/(kg K)}\); \(\lambda=0.12 \text{ W/(m K)}\).
- The inner bark layer parameters: \(\rho=500 \text{ kg/m}^3\); \(c=2600 \text{ J/(kg K)}\); \(\lambda=0.35 \text{ W/(m K)}\).
- The bark thermal characteristics: \(\rho=500 \text{ kg/m}^3\); \(c=1670 \text{ J/(kg K)}\); \(\lambda=0.12 \text{ W/(m K)}\).
- The reactive wood thermal characteristics: \(\rho=550 \text{ kg/m}^3\); \(c=1670 \text{ J/(kg K)}\); \(\lambda=0.12 \text{ W/(m K)}\).
- The geometrical characteristics of the decision area: \(R_c=0.25 \text{ m}\); \(R_1=0.245 \text{ m}\); \(R_2=0.235 \text{ m}\); \(R_{react}=0.225 \text{ m}\); \(R_{react}=0.5 \text{ m}\); \(H_1=17 \text{ m}\); \(H_{1H2}=0.05 \text{ m}\); \(H_{2H3}=0.05 \text{ m}\). Environmental parameters: \(T_e=300 \text{ K}\); \(T_s=297 \text{ K}\); \(\alpha_e=80 \text{ W/(m}^2 \text{ K)}\); \(\alpha_s=20 \text{ W/(m}^2 \text{ K)}\).

### 4. RESULTS AND ANALYSIS

The system of Equations (1) - (10) with boundary and initial conditions (11) - (29) is solved by a finite difference method. The multi-dimensional heat conduction Equations are solved by a locally one-dimensional method. A marching method is used to solve the Equations of one-dimensional difference analogues [23, 24].

We consider the typical scenario of ignition: the cloud-to-ground lightning discharge of negative polarity (duration is 500 ms, peak electric current is approximately 23.5 kA, and voltage is approximately 100 kV [13]) strikes in a pine tree trunk. Figure 4 shows the temperature distribution along the radius and height of the coniferous tree trunk at different time points: a) \(t=0.01 \text{ sec}\); b) \(t=0.1 \text{ sec}\); c) \(t=0.3 \text{ sec}\); d) \(t=0.5 \text{ sec}\).
sec. Figure 5 shows the heat flux from the inner bark area of the tree trunk at the time of height varying from land surface. Figure 6 shows the dependence of the inner bark boundary temperature on time at the certain levels above land surface. In Figure 5 and Figure 6 numerals 1 and 2 denote the curves for normal \((z=8.2 \text{ m})\) and reactive \((z=8.47 \text{ m})\) wood correspondingly.

We propose using the experimentally determined ignition conditions [14] as an ignition criterion in this research. The conditions for the normal and reactive wood ignition are numerically determined in the framework of this research. The criteria [14] are used for the ignition conditions (Table 1). A similar approach to determine the ignition conditions is used in the investigation [25].

![Figure 4](image1.png)

Figure 4. Temperature distribution along radius and height of tree trunk at different time points: (a) \(- t = 0.01 \text{ sec} \); (b) \(- 0.1 \text{ sec} \); (c) \(- 0.3 \text{ sec} \); (d) \(- 0.5 \text{ sec} \);

<table>
<thead>
<tr>
<th>Ignition delay, sec</th>
<th>Heat flux, kW/m²</th>
<th>Surface temperature, K</th>
</tr>
</thead>
<tbody>
<tr>
<td>63.5</td>
<td>12.5</td>
<td>658</td>
</tr>
<tr>
<td>45.0</td>
<td>21</td>
<td>700</td>
</tr>
<tr>
<td>11.1</td>
<td>42</td>
<td>726</td>
</tr>
<tr>
<td>2.6</td>
<td>84</td>
<td>773</td>
</tr>
<tr>
<td>0.4</td>
<td>210</td>
<td>867</td>
</tr>
</tbody>
</table>

The influence of the current-voltage characteristics of the cloud-to-ground lightning discharge on tree trunk ignition has been studied. Table 2 shows the numerically determined ignition conditions depending
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on the voltage of the cloud-to-ground lightning discharge at an approximate current of J=23.5 kA for the representative cross-section of the tree trunk (z=8.2 m).

Table 2. Numerically Determined Conditions of Coniferous tree Ignition Depending on Discharge Voltage at Approximate Current of J=23.5 kA (z=8.2 m)

<table>
<thead>
<tr>
<th>Voltage, U, kV</th>
<th>Conditions [14] performance</th>
<th>Surface temperature, K</th>
<th>Heat flux to ignition surface, kW/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – 85</td>
<td>No</td>
<td>&lt;867</td>
<td>&lt;210</td>
</tr>
<tr>
<td>90</td>
<td>Yes</td>
<td>867</td>
<td>242</td>
</tr>
<tr>
<td>95</td>
<td>Yes</td>
<td>867</td>
<td>246</td>
</tr>
<tr>
<td>100</td>
<td>Yes</td>
<td>867</td>
<td>249</td>
</tr>
<tr>
<td>105</td>
<td>Yes</td>
<td>867</td>
<td>252</td>
</tr>
<tr>
<td>110</td>
<td>Yes</td>
<td>867</td>
<td>255</td>
</tr>
</tbody>
</table>

Figure 5. Heat flux dependence on time from inner bark to ignition surface: 1 is normal wood (z=8.2 m, r=0.244 m) and 2 is reactive wood (z=8.47 m, r=0.244 m)

Figure 6. Inner bark area boundary temperature dependence on time: 1 is normal wood (z=8.2 m, r=0.244 m) and 2 is reactive wood (z=8.47 m, r=0.244 m)

Table 3 shows the numerically determined ignition conditions depending on the current of the cloud-to-ground lightning discharge at an approximate voltage of U=100 kV for the representative cross-section of the tree trunk.

Table 3 Numerically determined conditions of tree trunk ignition depending on current at approximate voltage of U=100 kV (z=8.2 m)

<table>
<thead>
<tr>
<th>Current, J, kA</th>
<th>Conditions [14] performance</th>
<th>Surface temperature, K</th>
<th>Heat flux to ignition surface, kW/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – 20</td>
<td>No</td>
<td>&lt;867</td>
<td>&lt;210</td>
</tr>
<tr>
<td>23.5</td>
<td>Yes</td>
<td>867</td>
<td>249</td>
</tr>
<tr>
<td>30</td>
<td>Yes</td>
<td>867</td>
<td>264</td>
</tr>
<tr>
<td>35</td>
<td>Yes</td>
<td>867</td>
<td>274</td>
</tr>
</tbody>
</table>

The analysis of the results shown in Tables 2 and 3 shows that a typical cloud-to-ground lightning discharge with the parameters (U=100 - 110 kV and J=23.5 - 35 kA) causes the ignition of the normal wood of a pine tree. These conditions have been previously established in the approximation of the one-dimensional mathematical model of coniferous tree trunk ignition by the cloud-to-ground lightning discharge [26]. It can be concluded that the majority of cloud-to-ground lightning discharges cause the ignition of the normal wood of a pine tree.

Table 4 shows the theoretically determined conditions of the reactive wood ignition depending on the voltage of the cloud-to-ground lightning discharge at an approximate current of J=23.5 kA (z=8.47 m).

Table 5 presents the theoretically determined conditions of reactive wood ignition depending on the current of the cloud-to-ground lightning discharge at an approximate voltage of U= 00 kV (z=8.47 m).

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The analysis of the numerical simulation results leads to the conclusion that reactive wood does not always meet the ignition conditions [14] while they are observed for normal wood. It is determined in the parametric study of the effect of the cloud-to-ground lightning discharge voltage that the ignition conditions [14] are achieved only at an approximate voltage of 110 kV and higher. The similar variation of the cloud-to-ground lightning discharge current has shown that the ignition conditions are achieved only when the current is approximately 30 kA and higher [14].

Analyzing the results presented in Figure 4, we can conclude that the tree trunk in the inner bark area heated to temperatures of forest fuel burning (more than 1200 K) due to the considered cloud-to-ground lightning discharge. Therefore, it can be concluded that the tree trunk ignites under these conditions. The lower temperature field in reactive wood with other identical conditions is the logical consequence of the differences in the thermal properties of normal and reactive wood. The above mentioned cloud-to-ground lightning discharge with average current-voltage characteristics can not lead to the ignition of the coniferous tree reactive wood.

The analysis of the heat flux and inner bark boundary temperature dependence (Figure 5 and Figure 6) shows that the temperature (867 K) and heat flux (249 kW / m²) conditions for ignition are achieved for normal wood rather than for the typical parameters of the lightning discharge. At the same time a delay in the surface heating and lower heat fluxes is observed for the reactive wood. These results suggest that the presence of twigs and large branches changes the character of the coniferous tree trunk ignition. This effect can not be taken into account within the framework of one-dimensional and two-dimensional horizontal statements [26], [27].

5. CONCLUSION

The important scientific and practical problem (the development of the physical and mathematical models of coniferous tree ignition taking into account the localization of reactive wood) is solved. The ignition conditions characteristic for the typical range of the cloud-to-ground lightning discharge parameters variation are obtained. The reactive wood area is formed lower temperature field. As a consequence we can expect that the lightning discharge with the same current-voltage characteristics will more likely lead to the ignition of tall trees with few branches. Trees with the expanded system of branches are unlikely to ignite.

The current research results create the conditions to develop the flammable materials ignition models and deterministic and probabilistic approaches in future to assess forest fire danger [28-30].

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BIOGRAPHIES OF AUTHORS

**Nikolay V. Baranovskiy** obtained Specialist Degree in Mechanics from Tomsk State University in 2000. He obtained PhD Degree in Physics and Mathematics from Tomsk State University in 2007. He has 16 years experience in research activity and 9 years in total in academic activity. He works at Associate Professor position in Tomsk Polytechnic University (Department of Theoretical and Industrial Heat Systems Engineering, Institute of Power Engineering) at present time. Field of research: fire safety and ecology, heat and mass transfer, computer science. He has more than 100 publications in peer-reviewed journals in English and Russian.

**Geniy V. Kuznetsov** obtained Specialist Degree in Physics from Tomsk State University in 1973. He obtained PhD Degree in Physics and Mathematics from Tomsk State University in 1981. He has 43 years experience in research and academic activity. He works at the position of Head of Departement of Theoretical and Heat Systems Engineering (Institute of Power Engineering) in Tomsk Polytechnic University at present time. Field of research: fire safety and fire suppression, power engineering, heat and mass transfer. He has more than 600 publications in peer-reviewed journals in English and Russian.

**Tatiana N. Nemova** obtained Specialist Degree in Physics in 1973. She obtained PhD Degree in Physics and Mathematics from Tomsk State University in 1983. She has 43 years experience in research and academic activity. She works at the Full Professor position of Departement of Heat and Gas Supply in Tomsk State University of Architecture and Building. Field of research: heat and mass transfer, thermophysics, energy losses. She has more than 50 publications in peer-reviewed journals in English and Russian.