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Comparative Study of Meta-heuristics Optimization Algorithm using Benchmark Function

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ABSTRACT

Meta-heuristics optimization is becoming a popular tool for solving numerous problems in real-world application due to the ability to overcome many shortcomings in traditional optimization. Despite of the good performance, there is limitation in some algorithms that deteriorates by certain degree of problem type. Therefore it is necessary to compare the performance of these algorithms with certain problem type. This paper compares 7 meta-heuristics optimization with 11 benchmark functions that exhibits certain difficulties and can be assumed as a simulation relevant to the real-world problems. The tested benchmark function has different type of problem such as modality, separability, discontinuity and surface effects with steep-drop global optimum, bowl- and plateau-typed function. Some of the proposed function has the combination of these problems, which might increase the difficulty level of search towards global optimum. The performance comparison includes computation time and convergence of global optimum.

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1. INTRODUCTION

The Meta-heuristic optimization is becoming more powerful method of solving optimization problem. Such optimization techniques are classified as stochastic optimization method. Their robustness and ability of finding global solution in various kind of field is proven in many literatures. The main characteristic of these algorithms is the dynamic balance of diversification and intensification in a gradient-free search space [1], [2].

Numerous meta-heuristic algorithms inspired by nature were introduced such as Simulated Annealing (SA), Particle Swarm Optimization (PSO), Firefly Algorithm (FFA), Harmony Search (HS) and many more. Each method has its own background philosophy of mimicking the nature and blended with the search strategy to explore and exploit a defined problem towards a global optimum. Such type of algorithm is also denoted as nature-inspired algorithm. It is also proven that these algorithms are capable to overcome many shortcomings of traditional algorithm application [4].

Despite of good performance, there are limitation that made these algorithms deteriorates by certain degree of problem types [5] especially in real-world problem which exhibit a large-scale property that grows exponentially by increasing number of variables and dimensions [2], [8]. This problem will continue to grow parallel with advance of science and technology. As a result of the increasing dimensionality, other factors such as interaction of variables (also referred as non-separability) and search space properties might result in difficulties of finding global optimum. Therefore any development, improvement or analysis of algorithm need to be verified with benchmark test functions [7], [8]. The so-called benchmark function is a mathematical functions that has a defined search spaces and exhibit certain difficulty classes such as separability, landscape which include multimodal functions, steep-drop, basin or valley-typed and function with null-space effects or plateau shaped. The properties of such difficulties are intended to simulate the characteristic of real-world problems. This paper presents a comparison of nature-inspired algorithms with defined class of benchmark problem. A short overview of 7 nature-inspired algorithms starting from long-established algorithm until recent optimization algorithm is presented in Section 2. Section 3 discusses the performance comparison of meta-heuristic method with benchmark function and the last section presents a conclusion of this paper.

2. RESEARCH METHOD

The overview of 7 meta-heuristic algorithms are summarised in Table 1. The performance of each algorithm is compared with benchmark function as provided in Table 2. These functions have different characteristic based on the difficulty class that can be simulated as a real-world problem.

Table 1. The overview of 7 meta-heuristic algorithm

Num	Algorithm (year)	Main features
1	Genetic Algorithm, GA (1960s)	Inspired from evolution's theory. Governed by 3 operators: 1) selection, 2) mutation, 3) crossover [2]
2	Differential Evolution, DE (1996)	Improvement of GA with same operators. Advantage: no coding needed. Decision factor by differential weight, $\delta = F(x_q - x_r)$ and crossover probability [2].
3	Simulated Annealing, SA (1983)	Trajectory-based algorithm inspired from metal cooling process. [2]
4	Particle Swarm Optimization, PSO (1995)	Swarm-based algorithm inspired from swarming of creatures. Solution is attracted to local and global best in each iteration [2], [3].
5	Firefly Algorithm, FFA (2008)	Inspired from flashing behaviour of fireflies. Each solution is attracted to potential solution based on fitness [2].
6	Cuckoo Search, CS (2009)	Inspired from Cuckoo bird parasitism method. Solution moved randomly with Lèvy flights. Some solution will be removed by probability, $pa = 0.25$ [2],[13]
7	Tree Physiology Optimization, TPO (2013)	Inspired from plant growth system with shoots and roots variables. Potential solution (shoots) search for optimum driven by amplification of root: root-shoot correlation search strategy [6].

Each defined meta-heuristic algorithm is compared with 11 test functions as summarized in Table 2. The characteristic of each test function includes modality, separability and continuity. With higher modality, the algorithm might trap in local minima, which results a negative impact on the search process away from true solution [7]. The separability is a measure of function difficulty, non-separable function is hard to solve due to compounded effect between each variables. Discontinuous function has step properties, which has certain flat and steep surface due to the *floor* effect of the function. This might lead to a slow convergence and local trapped optimum. Other properties of test function include bowl-shaped, valley-shaped or steep drops and flat surface. Flat surface problem will lead a poor algorithm to be trapped in local optimum as flatness of the function did not give any information towards global optimum.

Table 2. Benchmark function characteristic

Benchmark function		M	U	S	NS	D	B	SD	P
F1	Ackley	✓			✓				
F2	Damavandi	✓			✓		✓	✓	
F3	Easom		✓	✓				✓	✓
F4	Griewank	✓			✓		✓		
F5	Matyas		✓		✓				✓
F6	Michalewicz	✓			✓			✓	
F7	Rosenbrock		✓		✓				✓
F8	Shekel.F.	✓			✓			✓	✓
F9	Step		✓	✓		✓			✓
F10	WWavy	✓		✓					
F11	X.S.Yang 4	✓			✓			✓	

Benchmark function is selected from [7] with different characteristics; M=Multimodal, U= Unimodal, S= Separability, NS= Non-separable, D= Discontinuous, B= Bowl-type, SD= Steep-drop, and P= Plateau-shaped.

The parameters of each algorithm is designed with the best setting that it can converge towards global optimum. Different algorithm may have different setting depending on their nature of coding and search. The parameter settings are tabulated in Table 3.

Table 3.Simulation parameter for each algorithm

Algo.	Parameters	Algo.	Parameters
GA	Iteration = 30 Population = 50 Mutation = Gauss. Crossover = Scattered Selection = stoch.un.	SA	Init. temp. = 10 Final = 1e-10 alpha=0.95
DE	Iteration = 30 Diff. weight = 0.7 Crossover p. = 0.9	PSO	Iteration = 30 Pop. = 100 $\alpha = 0.6^{i}$;β =0.6
FFA	Fireflies = 100 Gen.= 100	CS	Nests = 25 Gen. = 100 pa rate= 0.25
TPO	Iteration = 30 Pop.= 30 leaves = 30 α = 0.3 β = 50 $\theta = 0.9^{i}$		

3. RESULTS AND ANALYSIS

The evaluation is based on computer processor of 2.6GHz. Each algorithm is simulated hundred times for every test function and the optimized parameters are compared. The parameters for comparison include computation time and convergence towards global optimum.

3.1. Computation time

The computation time of each algorithm is carried out only with a simple unimodal function, which make it suitable for benchmarking the convergence speed of meta-heuristic algorithm [1]. In this paper the computation time is compared with unimodal –typed function F3 and F5, as shown in Figure 8. Unimodal test function can be used as a benchmark for not only convergence speed, but also exploitation of the algorithm [1]. The computation time is dependent mainly on number of iteration and population size. In this study, each algorithm is executed until exceeding the number of iteration or until no improvement is achieved (global optimum solution).

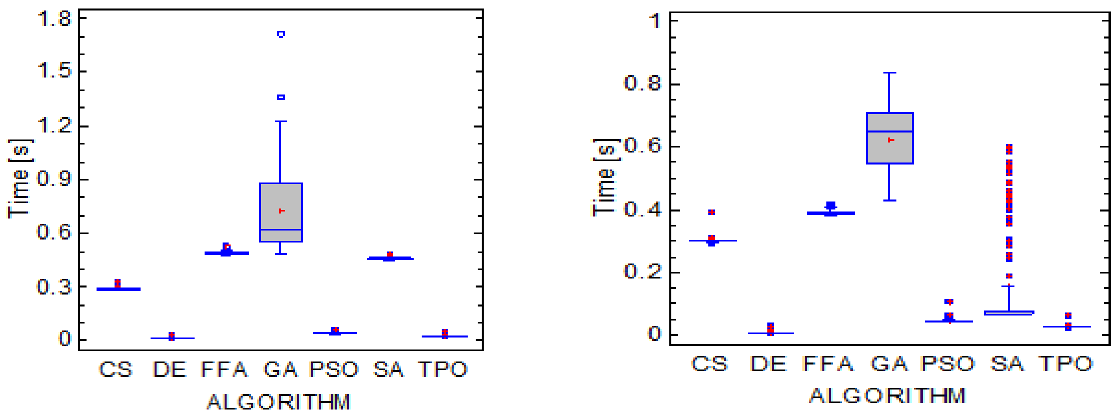

Figure 8. Computation time for F3 (right) and F5 (left)

Based on the comparison, TPO and DE outperformed other algorithm in the speed of convergence and followed by PSO. Among the slower converged are CS, FFA, and GA. DE is proven to converge better than GA which is supported in [2].

3.2. Covergence towards global optimum

The 11 benchmark functions have different difficulty level of problem-type as described in previous section. This will show the capability of each algorithm whether each of them can search efficiently in various problem types.

Based on Figure 9 and Table 4, the lowest variance of solution is found in TPO in all cases. CS show the second good performer as this algorithm converges consistently with lower variation except for F8 and F11. GA also shows consistently good convergence except for F4, F8, F10 and F11, these test functions have many local optimums with higher difficulty of surface. The performance of GA can be improved further by adding different strategy such as multi-parent crossover [16], dynamic adaptation of crossover and mutation [17], fine-tuning crossover [18] and many more. DE has some difficulty to track global optimum consistently in multimodal with steep-drop, and plateau-shaped. Furthermore DE trapped mostly in local optimum for F3. SA is trapped mostly in local optimum for F4, F8 and F 11. These functions have feature of multimodal and non-separable with steep-drop. FFA has a good convergence in plateau-shaped function except with discontinuous problem. This might be the reason of broader search ability of fireflies since it has unique function of comparison with different firefly companion [9], [10]. However FFA also trapped in local optimum as in F8, F9 and F10. PSO also show good convergence except in F2, F5 and F8. This might due to the fast behaviour of particles resulted in immature convergence for flat surface problem. Based on all tested benchmark function, F8 has the biggest variation of solution for all algorithms. The reason of such difficulty is combination of multimodal, a steep global optimum surrounded with bowl-shaped with several local optimum and also a plateau shape that covers 50% of the search space.

Table 4. Mean of 100 runs of each algorithm

Algo	Count	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11
GA	100	0.05	2.2	-0.75	0.01	0	-1.8	0.38	-5.03	0.19	0.03	-0.48
DE	100	0	2.18	-23	0.02	0	-1.83	0.24	-4.65	0	0.08	-0.09
SA	100	0.01	2.11	-0.24	0.02	0	-1.3	0	-3.35	0	0.01	-0.01
PSO	100	0.12	101.48	-0.98	0	0	-1.8	0.01	-4.81	0	0.01	-0.97
FFA	100	0	0	0	0	0	-1.76	0	-6.09	11.64	0.1	-0.25
CS	100	0.01	1.66	-0.86	0	0	-1.8	0.01	-6.36	0	0.01	-0.7
TPO	100	0	0	-1	0	0	-1.8	0	-6.45	0	0	-1

The convergence comparison with anova test from Figure 9 is tabulated in Table 5 that shows significant difference of algorithm by each test function (with p-value < 0.05) at 95% confidence level. The method being used to discriminate among the means is Fisher's least significant difference (LSD) procedure [15]. With this method, there is a 5.0% risk of calling each pair of means significantly different when the actual difference equals 0.

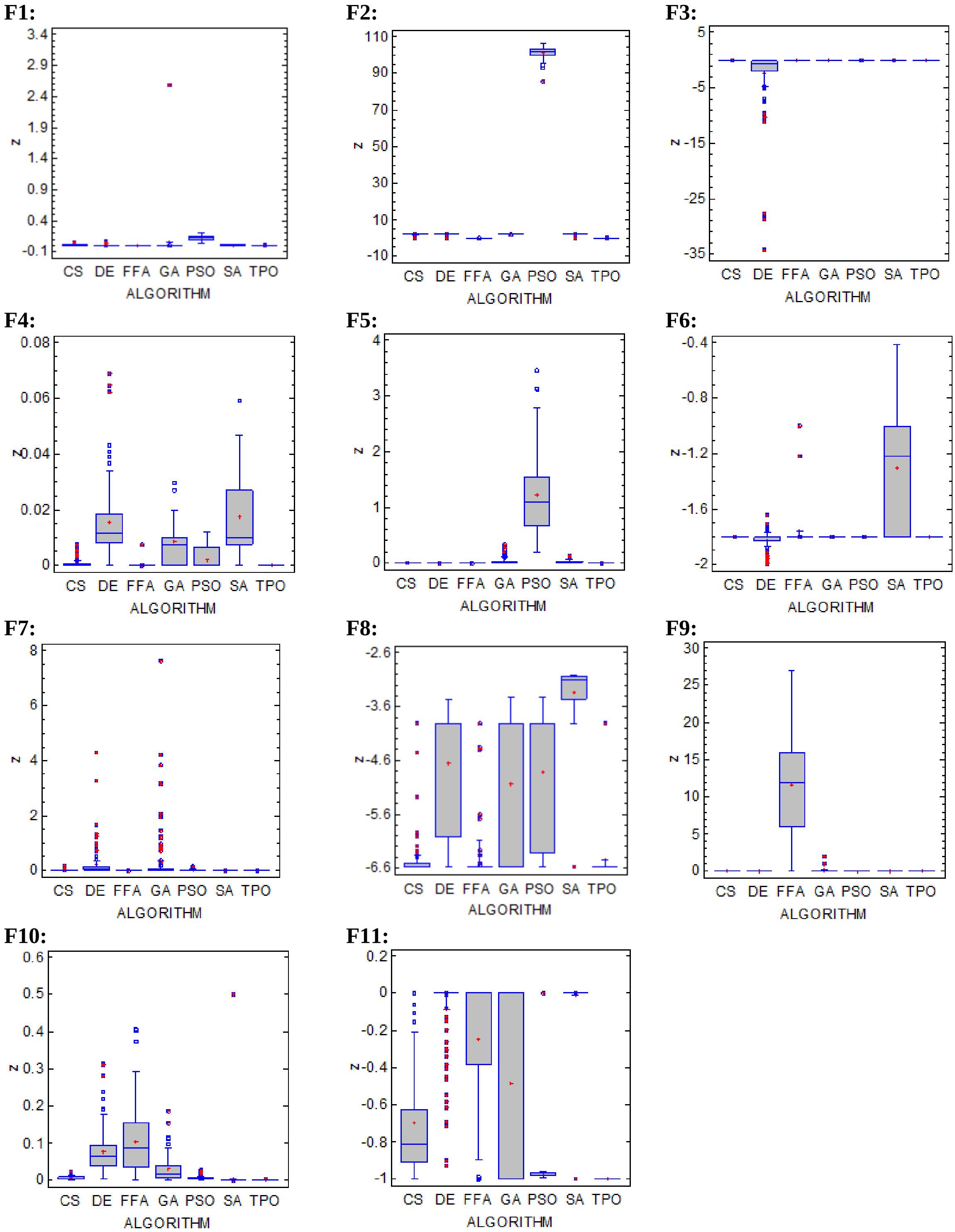

Figure 9. Distribution of convergence of each algorithm by benchmark function

Table 5. Significantly different algorithm according to Anova test

	CS	DE	FFA	GA	PSO	SA	TPO
F1	GA, PSO	GA, PSO	GA, PSO	SA, PSO, TPO	SA, TPO		SA
F2	DE, FFA, GA, PSO, SA, TPO	FFA, PSO, TPO	GA, PSO, SA	PSO, TPO	SA, TPO	TPO	CS, DE, GA, SO, SA
F3	DE	ALL	DE	DE	DE	DE	DE
F4	DE, GA, SA	FFA, GA, PSO, TPO	GA, SA	PSO, SA, TPO	SA	CS, FFA, GA, PSO,TPO	DE, TPO
F5	PSO	PSO	PSO	PSO	ALL	PSO	PSO
F6	SA	FFA, SA	DE, SA	SA	SA	ALL	SA
F7	DE, GA	FFA, GA, PSO, SA, TPO		PSO, SA, TPO			
F8	DE, FFA, GA, PSO, SA	FFA, GA, SA, TPO	GA, PSO, SA, TPO	SA, TPO	SA, TPO	TPO	DE, FFA, GA, PSO, SA
F9	FFA	FFA	GA, PSO, SA, TPO				
F10	DE, FFA, GA	ALL	GA, PSO, SA, TPO	SA, TPO	DE, FFA	DE, FFA, GA	DE, FFA, GA
F11	ALL	FFA, GA, PSO, TPO	GA, PSO, SA, TPO	PSO, SA, TPO	SA	TPO	CS, DE, FFA,GA, SA

The convergence dynamic in a single run is compared with two benchmark functions: F6 and F7 as depicted in Figure 10. It can be observed that GA, FFA, PSO and TPO converge towards global optimum faster compared to others.

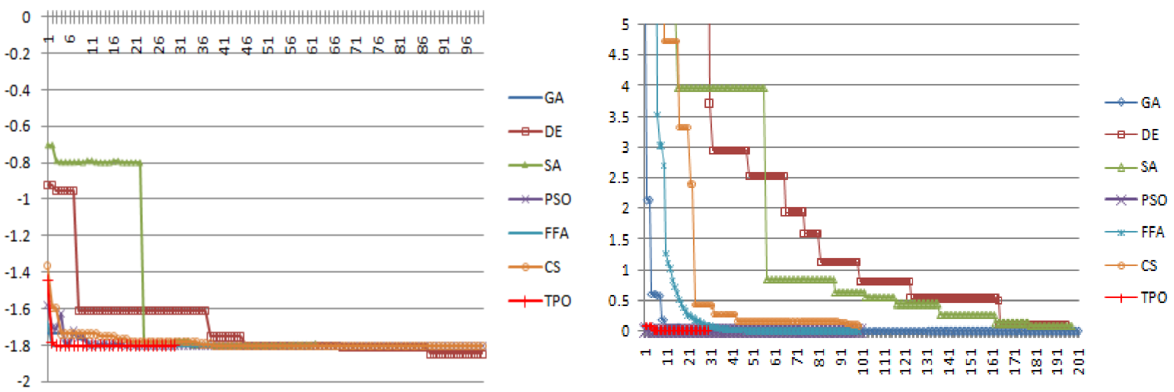

Figure 10. Convergence of best selected algorithm in one run for F6 (right) and F7 (left)

4. CONCLUSION

Meta-heuristic optimization algorithm is able to solve wide range of nonlinear optimization problems optimally [19]. The reason for these advantages is from its unique characteristic of diversification and exploitation capability. However each algorithm has different background philosophy that blended with dynamic search strategy. This leads to the difference in convergence and computation time, which might reveal the ability of reaching global optimum by different type of difficulties. In this paper, 7 meta-heuristics are compared with 11 benchmark functions. Based on the statistical comparison, TPO performs significantly better compared to other algorithms in most benchmark function. This is due to the parallel search of local optimum (individual leaf) and global optimum (branches). With the amplification search from root system, the search process become broader, thus the probability of finding a true solution will be greater [11]. CS is also able to search global optimum for various type of problem except for benchmark function F11 (function with many local optima and single steep global optimum). This may due to the random search via Lèvy flights [2].

The search using CS algorithm can be improved further if the number of nests are more than number of local optima [12]. This idea is also supported with TPO feature of wider search. PSO has successful solution in some benchmark functions, but its well-known stability problem restricts the success rate of this algorithm. FFA shows successful search in most plateau-type problem. However it has limitation with many local optima and discontinuous problem. Discontinuous function, F9 has several plateaus that might result in poor convergence. GA has better performance compared to DE as most of the solution in 100 runs converged towards global optimum. SA shows slightly poor convergence in specific multimodal typed with steep global optimum function (F4, F6, F8 and F11 are multimodal with more local optimum).

Computation time is compared using unimodal-typed function. DE and TPO computes the problem faster compared to other algorithm. Among the slower computation time is FFA and GA. The finding from this paper suggests a necessity of more comparison studies in different field especially in real world problem as it will grow parallel with advance of science and technology. Some proposed example of areas that might be considered for real world are manufacturing improvement, control engineering and routing system

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