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Model reduction design for continuous systems with finite frequency specifications

Miloud Koumir[1], **Abderrahim El−Amrani**[2], **Ismail Boumhidi**[3]
[1]LIPI Laboratory, ENS, Sidi Mohamed Ben Abdellah University, Fes, Morocco
[2,3]LISAC Laboratory, FSDM, Sidi Mohamed Ben Abdellah University, Fes, Morocco

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ABSTRACT

This paper is concerned with the problem of model reduction design for continuous systems in Takagi-Sugeno fuzzy model. Through the defined FF H_∞ gain performance, sufficient conditions are derived to design model reduction and to assure the fuzzy error system to be asymptotically stable with a FF H_∞ gain performance index. The explicit conditions of fuzzy model reduction are developed by solving linear matrix inequalities. Finally, a numerical example is given to illustrate the effectiveness of the proposed method.

Corresponding Author:

Abderrahim El−Amrani
LISAC Laboratory
Faculty of Sciences Dhar EL Mehraz (F.S.D.M)
Sidi Mohamed Ben Abdellah University
Fes, Morocco
Email : abderrahim.elamrani@usmba.ac.ma

1. INTRODUCTION

In the last few decades, many researchers have investigated model reduction including continuous and discrete settings as these systems have great applications in engineering fields. The problem is to design a low-order model to approach a higher order model given according to some specified criteria. Indeed, many results-based model reduction approach were presented [1]-[10].

In the practical, systems are always more or less disturbed, therefore, model reduction issues for nonlinear systems have been extensively discussed through the T-S fuzzy model approach, see [11]-[16]. Among the most of the existed literature on model reduction problems, the disturbances are considered in the entire frequency (EF) domain, which will bring overdesign in the filtering design. While many practical engineering problems are more suitable to be considered in finite frequency (FF) ranges [17]-[26].

The main objective of this paper is to design a model reduction for continuous T-S fuzzy systems with disturbance in FF domain. Through the defined FF H_∞ gain performance, sufficient conditions are derived to design model reduction and to assure the fuzzy error system to be asymptotically stable with a FF H_∞ gain performance index. The explicit conditions of fuzzy FF are developed by solving linear matrix inequalities. A systematic model reduction design scheme is proposed, which could reduce the conservatism of the results compared to the one considered in EF domain. Finally, a simulation example demonstrates the usefulness of the proposed method.

Notations: The notation $A > 0$ ($A \leq 0$) means that A is positive definite (positive semi-definite). A^{-1}, A^T, A^* denote the inverse, the transpose and the complex conjugate transpose of matrix A, respectively. Symbol $'*'$ represents the term originated by conjugate symmetry in a matrix.

2. PROBLEM FORMULATION

The plant under consideration is a continuous T-S fuzzy system described by its i-th rule as follows:
Plant Rule *i*: IF $\sigma_1(t)$ is T_1^i,... and $\sigma_p(t)$ is T_p^i THEN,

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + B_i u(t) \\ y(t) &= C_i x(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $y(t) \in \mathbb{R}^p$ is the measured output vector; $u(t) \in \mathbb{R}^m$ is the external noise signal of the following frequency sets,

$$\triangle \triangleq \begin{cases} \omega \in \mathbb{R}| \ |\omega| \leq \bar{\omega}_l, & \bar{\omega}_l \geq 0, \quad (LF) \\ \omega \in \mathbb{R}|\bar{\omega}_1 \leq \omega \leq \bar{\omega}_2 & \bar{\omega}_1, \bar{\omega}_2 \in [0, +\infty], \quad (MF) \\ \omega \in \mathbb{R}| \ |\omega| \geq \bar{\omega}_h, & \bar{\omega}_h \geq 0, \quad (HF) \end{cases} \tag{2}$$

where LF, MF and HF stand for low-, middle-, and high- frequency ranges, respectively.

Via using inference product, singleton fuzzifer and center-average defuzzifer, nonlinear system (1) can be described by:

$$\begin{aligned} \dot{x}(t) &= A(\rho)x(t) + B(\rho)u(t) \\ y(t) &= C(\rho)x(t) \end{aligned} \tag{3}$$

where

$$\begin{aligned} A(h) &= \sum_{i=1}^{r} h_i A_i; \quad B(h) = \sum_{i=1}^{r} h_i B_i; \\ C(h) &= \sum_{i=1}^{r} h_i C_i \end{aligned} \tag{4}$$

and

$$h_i(\sigma(t)) = \frac{\prod_{j=1}^{p} \mu_{ij}(\sigma_j(t))}{\sum_{i=1}^{r} \prod_{j=1}^{p} \mu_{ij}(\sigma_j(t))} \tag{5}$$

$$\rho_i \geq 0, \quad i = 1, ..., r, \quad \sum_{i=1}^{r} \rho_i = 1. \tag{6}$$

with $h := (h_1, h_2, ..., h_r) \in \delta$.

In this paper, we are interested in approximating the T-S fuzy system (4) by a stable $\hat{n}$th-order $(\hat{n} < n)$ reduced-order T-S model.

Plant Rule *i*: IF $\sigma_1(t)$ is N_1^i,... and $\sigma_p(t)$ is N_p^i THEN

$$\begin{aligned} \dot{\hat{x}}(t) &= \hat{A}_i \hat{x}(t) + \hat{B}_i u(t) \\ \hat{z}(t) &= \hat{C}_i \hat{x}(t) \end{aligned} \tag{7}$$

where $\hat{x}(t) \in \mathbb{R}^{\hat{n}} (\hat{n} < n)$ is the state of the reduced-order model, $\hat{y}(t) \in \mathbb{R}^p$ is the output of the reduced-order model.

Then, the fuzzy reduced-order model as (8),

$$(\Sigma_r): \quad \begin{aligned} \dot{\hat{x}}(t) &= \hat{A}(\rho)\hat{x}(t) + \hat{B}(\rho)u(t) \\ \hat{y}(t) &= \hat{C}(\rho)\hat{x}(t) \end{aligned} \tag{8}$$

where

$$\hat{A}(\rho) = \sum_{i=1}^{r}\rho_i(t)\hat{A}_i, \quad \hat{B}(\rho) = \sum_{i=1}^{r}\rho_i(t)\hat{B}_i,$$
$$\hat{C}(\rho) = \sum_{i=1}^{r}\rho_i(t)\hat{C}_i \tag{9}$$

Then, we have the error model:

$$\begin{aligned}(\Sigma_e): \quad \dot{\mathcal{X}}(t) &= \bar{A}(\rho,\rho)\mathcal{X}(t) + \bar{B}(\rho,\rho)u(t)\\ \mathcal{E}(t) &= \bar{C}(\rho,\rho)\mathcal{X}(t) + \bar{D}(\rho,\rho)u(t)\end{aligned} \tag{10}$$

where

$$\bar{A}(\rho,\rho) = \begin{bmatrix} A(\rho) & 0 \\ 0 & \hat{A}(\rho)\end{bmatrix}; \ \bar{B}(\rho,\rho) = \begin{bmatrix} B(\rho) \\ \hat{B}(\rho)\end{bmatrix};$$
$$\bar{C}(\rho,\rho) = \begin{bmatrix} C(\rho) & -\hat{C}(\rho)\end{bmatrix}$$
$$\mathcal{X}(t) = \begin{bmatrix} x(t) \\ \hat{x}(t)\end{bmatrix}; \ \mathcal{E}(t) = y(t) - \hat{y}(t) \tag{11}$$

Next, given a scalar γ and a rectangular FF domain, the error system (Σ_e) is said to have a FF H_∞ performance if it satisfies the following inequality holds,

$$\int_{\omega\in\triangle} E^T(\omega)E(\omega)d\omega \leq \gamma^2 \int_{\omega\in\triangle} U^T(\omega)U(\omega)d\omega \tag{12}$$

where

$$\triangle \triangleq \{w \in \mathbb{R} : |\omega| \leq \omega_l; \ \ \omega_l \in [0, +\infty]\} \tag{13}$$

The following inequalities hold [27]:

$$\begin{aligned}&\Theta_{ii} < 0; \qquad 1 \leq i \leq r\\ &\frac{1}{r-1}\Theta_{ii} + \frac{1}{2}[\Theta_{ij} + \Theta_{ji}] < 0; \quad 1 \leq i \neq j \leq r\end{aligned} \tag{14}$$

where

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\rho_i(t)\rho_j(t)\Theta_{ij} < 0 \tag{15}$$

Let $\triangle \in \mathbb{R}^n$, $\mathcal{J} \in \mathbb{R}^{n\times n}$, $\mathcal{X} \in \mathbb{R}^{m\times n}$, $rank\ (\mathcal{X}) = r < n$ and $\mathcal{X}^\perp \in \mathbb{X}^{n\times(n-r)}$ such that $\mathcal{X}\mathcal{X}^\perp = 0$ [28], so that the following conditions are equivalent:

- $\triangle^*\mathcal{J}\triangle < 0, \forall\triangle \neq 0 : \mathcal{X}\triangle = 0$
- $\mathcal{X}^{\perp *}\mathcal{J}\mathcal{X}^\perp < 0$
- $\exists\mathcal{T} \in \mathbb{R}^{n\times m} : \mathcal{J} + \mathcal{T}\mathcal{X} + \mathcal{X}^*\mathcal{T}^* < 0$

Error system (10) is stable and the FF H_∞ performance (12) is satisfied if there exist $\mathcal{P} = \mathcal{P}^T$, $\mathcal{Q} = \mathcal{Q}^T > 0$ such that [29].

$$\begin{bmatrix} \bar{A}(\rho,\rho) & \bar{B}(\rho,\rho) \\ I & 0\end{bmatrix}^T \begin{bmatrix} -\mathcal{Q} & \mathcal{P} \\ \mathcal{P} & \omega_l^2\mathcal{Q}\end{bmatrix} \begin{bmatrix} \bar{A}(\rho,\rho) & \bar{B}(\rho,\rho) \\ I & 0\end{bmatrix} + \begin{bmatrix} \bar{C}^T(\rho,\rho)\bar{C}(\rho,\rho) & 0 \\ 0 & -\gamma^2 I\end{bmatrix} < 0 \tag{16}$$

3. FF H_∞ PERFORMANCE DESIGN

Error model (8) is stable, if $\mathcal{P} \in \mathbb{H}_n$, $0 < \mathcal{Q} \in \mathbb{H}_n$, $0 < \mathcal{W} \in \mathbb{H}_n$, $\mathcal{F}$, $\mathcal{G}$, $\mathcal{L}$ $\mathcal{H}$ such that

$$\Upsilon = \begin{bmatrix} -\mathcal{F} - \mathcal{F}^T & \mathcal{W} + \mathcal{F}\bar{A}(\rho,\rho) - \mathcal{G}^T \\ * & He\{\mathcal{G}\bar{A}(\rho,\rho)\} \end{bmatrix} < 0 \tag{17}$$

$$\Omega = \begin{bmatrix} -\mathcal{Q} - He[\mathcal{F})] & \mathcal{P} - \mathcal{G}^T + \mathcal{F}\bar{A}(\rho,\rho) & \mathcal{F}\bar{B}(\rho,\rho) - \mathcal{L}^T(\rho,\rho) & -\mathcal{H}^T \\ * & \omega_l^2\mathcal{Q} + He[\mathcal{G}\bar{A}(\rho,\rho)] & \mathcal{G}\bar{B}(\rho,\rho) + \bar{A}^T(\rho,\rho)\mathcal{L}^T(\rho,\rho) & \bar{C}^T(\rho,\rho) + \bar{A}^T\rho,\rho)\mathcal{H}^T \\ * & * & -\gamma^2 I + He[\mathcal{L}\bar{B}(\rho,\rho)] & \bar{B}^T\rho,\rho)\mathcal{H}^T \\ * & * & * & -I \end{bmatrix} < 0 \tag{18}$$

First. Define:

$$\begin{bmatrix} \bar{A}(\rho,\rho) \\ I \end{bmatrix}^T \begin{bmatrix} 0 & \mathcal{W} \\ \mathcal{W} & 0 \end{bmatrix} \begin{bmatrix} \bar{A}(\rho,\rho) \\ I \end{bmatrix} < 0 \tag{19}$$

$$\mathcal{J} = \begin{bmatrix} 0 & \mathcal{W} \\ \mathcal{W} & 0 \end{bmatrix};\ \mathcal{T} = \begin{bmatrix} \mathcal{F} \\ \mathcal{G} \end{bmatrix};\ \mathcal{X} = \begin{bmatrix} -I & \bar{A}(\rho,\rho) \end{bmatrix};\ \mathcal{X}^\perp = \begin{bmatrix} \bar{A}(\rho,\rho) \\ I \end{bmatrix} \tag{20}$$

then

$$\begin{bmatrix} 0 & \mathcal{W} \\ \mathcal{W} & 0 \end{bmatrix} + \begin{bmatrix} \mathcal{F} \\ \mathcal{G} \end{bmatrix} \begin{bmatrix} -I & \bar{A}(h,\hat{h}) \end{bmatrix} + \begin{bmatrix} -I & \bar{A}(\rho,\rho) \end{bmatrix}^T \begin{bmatrix} \mathcal{F} \\ \mathcal{G} \end{bmatrix}^T < 0 \tag{21}$$

which is nothing but (17).

On the other hand, Let:

$$\mathcal{J} = \begin{bmatrix} -\mathcal{Q} & \mathcal{P} & 0 & 0 \\ \mathcal{P} & \omega_l^2\mathcal{Q} & 0 & \bar{C}^T(\rho,\rho) \\ 0 & 0 & -\gamma^2 I & 0 \\ 0 & \bar{C}(\rho,\rho) & 0 & -I \end{bmatrix};\mathcal{T} = \begin{bmatrix} \mathcal{F}^T & \mathcal{G}^T & \mathcal{L}^T & \mathcal{H}^T \end{bmatrix}^T;$$

$$\mathcal{X} = \begin{bmatrix} -I & \bar{A}(\rho,\rho) & \bar{B}(\rho,\rho) & 0 \end{bmatrix}$$

then, we obtain (16). Error system in (8) is stable with an H_∞ performance bound γ, if there exist $\mathcal{P} = \begin{bmatrix} \mathcal{P}_1 & \mathcal{P}_2 \\ * & \mathcal{P}_3 \end{bmatrix}$, $Q = \begin{bmatrix} \mathcal{Q}_1 & \mathcal{Q}_2 \\ * & \mathcal{Q}_3 \end{bmatrix} > 0$, $\mathcal{W} = \begin{bmatrix} \mathcal{W}_1 & \mathcal{W}_2 \\ * & \mathcal{W}_3 \end{bmatrix} > 0$, $\breve{\mathcal{A}}_i$, $\breve{\mathcal{B}}_i$, $\breve{\mathcal{C}}_i$, $\mathcal{F}_1$, $\mathcal{F}_2$, $\mathcal{G}_1$, $\mathcal{G}_2$, $\mathcal{H}_1$, $E = \begin{bmatrix} I & 0 \end{bmatrix}^T$ and $\mathcal{V}$, such that

$$\tilde{\Omega}_{ii} < 0; \quad \tilde{\Upsilon}_{ii} < 0; \quad 1 \le i \le r \tag{22}$$

$$\frac{1}{r-1}\tilde{\Omega}_{ii} + \frac{1}{2}[\tilde{\Omega}_{ij} + \tilde{\Omega}_{ji}] < 0; \quad 1 \le i \ne j \le r \tag{23}$$

$$\frac{1}{r-1}\tilde{\Upsilon}_{ii} + \frac{1}{2}[\tilde{\Upsilon}_{ij} + \tilde{\Upsilon}_{ji}] < 0; \quad 1 \le i \ne j \le r \tag{24}$$

$$\tilde{\Omega} = \begin{bmatrix} \tilde{\Omega}_{11} & \tilde{\Omega}_{12} & \mathcal{P}_1 - \mathcal{G}_1^T + \mathcal{F}_1 A_j & \mathcal{P}_2 - \mathcal{G}_2^T + E\breve{\mathcal{A}}_i & \mathcal{F}_1 B_j + E\breve{\mathcal{B}}_i - \mathcal{L}_1^T & -\mathcal{H}_1^T \\ * & \tilde{\Omega}_{22} & \mathcal{P}_2 - \mathcal{V}^T E^T + \mathcal{F}_2 A_j & \mathcal{P}_3 - \mathcal{V}^T + \breve{\mathcal{A}}_i & \mathcal{G}_2 B_j + \breve{\mathcal{A}}_i & A_j^T \mathcal{H}_1^T \\ * & * & \omega_l^2 \mathcal{Q}_1 + \mathcal{G}_1 A_j + A_j^T \mathcal{G}_1^T & \omega_l^2 \mathcal{Q}_2 + A_j^T \mathcal{G}_2^T + E\breve{\mathcal{A}}_i & \mathcal{G}_1 B_j + E\breve{\mathcal{B}}_i + A_j^T \mathcal{L}_1^T & C_j^T \\ * & * & * & \omega_l^2 \mathcal{Q}_3 + \breve{\mathcal{A}}_i + E\breve{\mathcal{A}}_i^T & \mathcal{G}_2 B_j + \breve{\mathcal{B}}_i & -\breve{\mathcal{C}}_j^T \\ * & * & * & * & -\gamma^2 I + He[\mathcal{L}_1 B_j] & B_j^T \mathcal{H}_1^T \\ * & * & * & * & * & -I \end{bmatrix}$$

$$\tilde{\Upsilon} = \begin{bmatrix} -\mathcal{F}_1 - \mathcal{F}_1^T & -E\mathcal{V} - \mathcal{F}_2^T & \mathcal{P}_1 - \mathcal{G}_1^T + \mathcal{F}_1 A_j & \mathcal{P}_2 - \mathcal{G}_2^T + E\breve{\mathcal{A}}_i \\ * & -\mathcal{V} - \mathcal{V}^T & \mathcal{W}_2 + \mathcal{F}_2 A_j - \mathcal{V}^T E^T & \mathcal{W}_3 + \breve{\mathcal{A}}_i - \mathcal{V}^T \\ * & * & \mathcal{G}_1 A_j + A_j^T \mathcal{G}_1^T & E\breve{\mathcal{A}}_j + A_j^T \mathcal{G}_2^T \\ * & * & * & He[\breve{\mathcal{A}}_i] \end{bmatrix}$$

$$\bar{\Omega}_{11} = \mathcal{Q}_1 - \mathcal{F}_1 - \mathcal{F}_1^T;\ \bar{\Omega}_{12} = \mathcal{Q}_2 - E\mathcal{V} - \mathcal{F}_2^T; \bar{\Omega}_{22} = \mathcal{Q}_3 - \mathcal{V} - \mathcal{V}^T$$

Built on Th. 3., we pick of parameters $\mathcal{F}$, $\mathcal{G}$, $\mathcal{L}$, $\mathcal{H}$:

$$\begin{aligned} \mathcal{F} &= \begin{bmatrix} \mathcal{F}_1 & E\mathcal{V} \\ \mathcal{F}_2 & \mathcal{V} \end{bmatrix};\ \mathcal{G} = \begin{bmatrix} \mathcal{G}_1 & E\mathcal{V} \\ \mathcal{G}_2 & \mathcal{V} \end{bmatrix}; \\ \mathcal{L} &= \begin{bmatrix} \mathcal{L}_1 & 0 \end{bmatrix};\ \mathcal{H} = \begin{bmatrix} \mathcal{H}_1 & 0 \end{bmatrix} \end{aligned} \tag{25}$$

and

$$\breve{\mathcal{A}}(\rho) = \mathcal{V}\hat{A}(\rho);\ \breve{\mathcal{B}}(\rho) = \mathcal{V}\hat{B}(\rho) \tag{26}$$

Finally, by applying Lemma 2, we have Th. 3.

Moreover, under the above conditions, we can obtain a state-space realization of model reduction (8) with the following parameters as (27).

$$\breve{A}_i = \mathcal{V}^{-1}\hat{A}_i; \qquad \breve{B}_i = \mathcal{V}^{-1}\hat{B}_i;; \qquad \breve{C}_i = \hat{C}_i \tag{27}$$

4. NUMERICAL EXAMPLE

Consider tunnel diode circuit shown in Figure 1 with two rules [30]:
Plant Rule 1: IF $x_1(t)$ is $M_1(x_1(t))$ THEN

$$\begin{aligned} \dot{x}(t) &= A_1 x(t) + B_1 u(t) \\ y(t) &= C_1 x(t) \end{aligned} \tag{28}$$

Plant Rule 2: IF $x_1(t)$ is $M_2(x_1(t))$ THEN

$$\begin{aligned} \dot{x}(t) &= A_2 x(t) + B_2 u(t) \\ y(t) &= C_2 x(t) \end{aligned} \tag{29}$$

where

$$\begin{aligned} A_1 &= \begin{bmatrix} -0.2 & 100 & 0 & 0 \\ -10 & -66.6667 & 3.3333 & -66.6667 \\ 0 & -33.3333 & -1.6667 & -16.6667 \\ 0 & -33.3333 & -1.6667 & -33.3333 \end{bmatrix};\ B_1 = B_2 = \begin{bmatrix} 0 \\ 0 \\ -1.6667 \\ -3.3333 \end{bmatrix}; \\ A_2 &= \begin{bmatrix} -9.2 & 100 & 0 & 0 \\ -10 & -66.6667 & 3.3333 & -66.6667 \\ 0 & -33.3333 & -1.6667 & -16.6667 \\ 0 & -33.3333 & -1.6667 & -33.3333 \end{bmatrix};\ C_1 = C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}. \end{aligned} \tag{30}$$

The membership functions:

$$M_1(x_1(t)) = 1 - \frac{x_1^2(t)}{9} \; ; \; M_2(x_1(t)) = 1 - M_1(x_1(t)) \tag{31}$$

when $x_1(t)$ is almost ± 3 and 0.

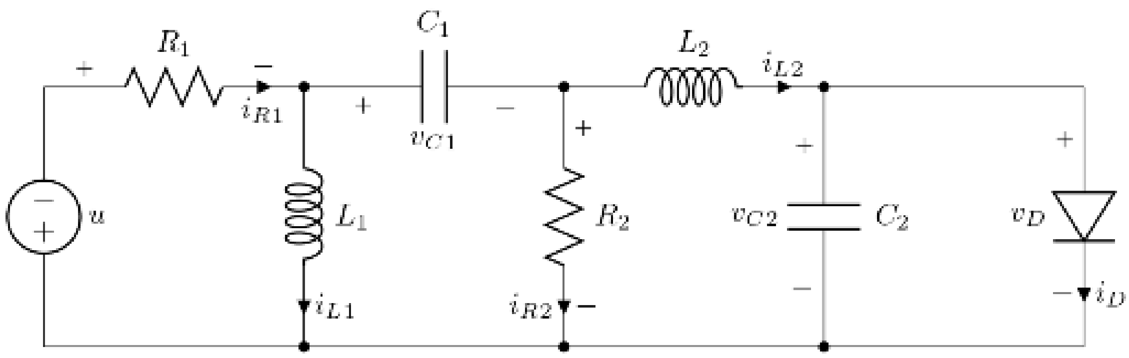


Figure 1. A tunnel diode circuit

As an example, let the disturbance input be the following form (32).

$$u(t) \quad = \quad 0.1\sin(2t), \quad i.e., .|\omega| \leq 2rad/s. \tag{32}$$

By Theorem 3, the reduced-order models are given by (33),

$$\left[\begin{array}{c|c} \hat{A}_1 & \hat{B}_1 \\ \hline \hat{C}_1 & - \end{array}\right] = \left[\begin{array}{cc|c} -11.1147 & 55.2147 & -8.7325 \\ -1.2574 & -4.1258 & -1.0245 \\ \hline -0.2487 & 0.4175 & - \end{array}\right];$$

$$\left[\begin{array}{c|c} \hat{A}_2 & \hat{B}_2 \\ \hline \hat{C}_2 & - \end{array}\right] = \left[\begin{array}{cc|c} -10.4785 & 56.0147 & -9.0147 \\ -1.2104 & -4.3301 & -0.9782 \\ \hline -0.2501 & 0.3305 & - \end{array}\right] \tag{33}$$

We propose in Table 1 shows the values of γ obtained in different frequency ranges. We can see from Table 1 shows the values of γ obtained with the approaches existing in [5], [22], [23] and Theorem 3. We can see that the proposed method provides better results than the existing ones.

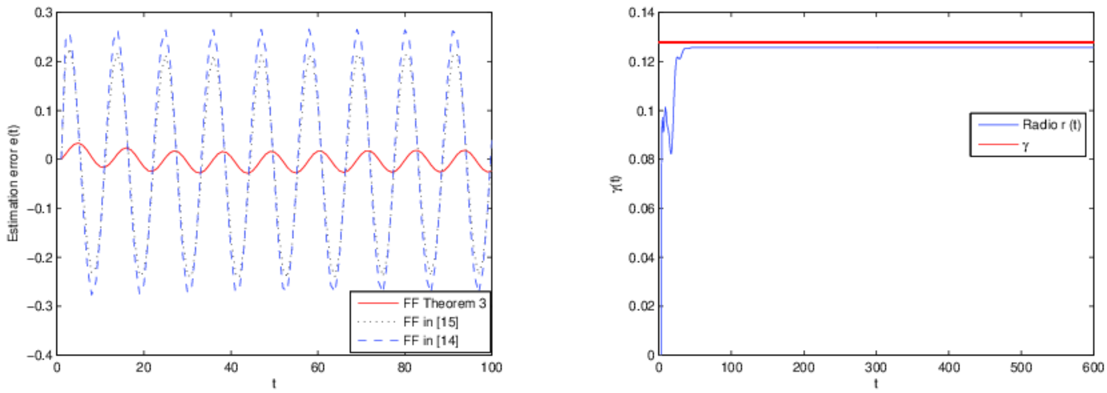
Table 1. H_∞ performance bounds γ by different domains

Frequency	*Methods*	γ_{min}	*Max error*
$0 \leq \omega \leq \infty$	[5]	1.1457	-
$\lvert\omega\rvert \leq 2$	[22]	0.3587	0.3245
$\lvert\omega\rvert \leq 2$	[23]	0.2324	0.2325
$\lvert\omega\rvert \leq 2$	Th 3.	0.1279	0.0332

Next, let:

$$\mu(t) \quad = \quad \sqrt{\sum_{t=0}^{\infty} \mathcal{E}^T(t)\mathcal{E}(t) / \sum_{t=0}^{\infty} u^T(t)u(t)} \tag{34}$$

Figures 2(a) and (b) present the estimation error $\mathcal{E}(t)$ for different methods and evolution of ratio $\mu(t)$ in (34). From Figure 2(a), the asymptotic stability of the error system can be clearly observed, while under the zero boundary conditions and the disturbance input (32). From Figure 2(b), the ratio tends to a constant value 0.1234 in $|\omega| \leq 2$ domains.



(a) (b)

Figure 2. Trajectories of $\mathcal{E}(t)$ and $\mu(t)$ for $|\omega| \leq 2$ range; (a) Estimation error $\mathcal{E}(t)$ from different methods, (b) Estimation $\mu(t)$

5. CONCLUSION

This paper has concerned with the problem of the model reduction design for continuous T-S fuzzy systems with FF disturbances. Assuming the disturbances is dominated in a known FF domain. Through applying a more general linearization procedure, systematic methods have been proposed for model reduction design, which guarantees the asymptotic stability and the FF H_∞ gain performance of the error system. A simulation example has been given to illustrate the effectiveness of the proposed method.

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