Modeling and simulation of a 3-ф induction motor based on two types of WFA

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Article Info

ABSTRACT
This paper has been proposed to simulate the transient model of 3-Ф cage rotor induction motor based on winding function approach (WFA). According to this method the motor is assumed to be consist of an electrical circuits on both stator and rotor. The magneto motive forces (MMF) that have been generated by these circuits play a role for coupling them together. Then mutual and self-inductances will be easily computed using WFA. Two types of WFA have been used to build and simulate the model of the induction motor. In the one part type, it's assumed that the coupling MMF between stator and rotor have a non-sinusoidal shapes according to the actual windings distribution over the motor slots. While in second part type the generated MMF in are assumed to have sinusoidal waveform. The suggested models may be used to simulate the dynamic as well as steady state performance of a faulty and non-faulty motor. A simulation of the suggested models that consists of m-rotor bars and n-stator phases multiple coupled circuit-based has been performed using matlab m.file and the results of the motor current have been proved in its nonlinear way by using WFA.

Keywords:
Cage rotor
Induction motor
Matlab m.file
Steady state response
Winding function approach

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1. INTRODUCTION
Due to their robustness, reliability and low requirement for maintenance, three phase induction motors have a many application in industrial processes. Different methods had been used to compute the self and mutual inductances of electric machines. The winding function approach (WFA) had been extensively used to calculate these parameters. The principle of this approach is depending on solving the differential equations that explain the electromagnetic coupling between stator and rotor of the induction motor [1-3]. Calculation of coupled inductances are based on the basic laws of electrical circuit. Firstly, WFA had been applied to compute the inductances of healthy induction motors, then the formulas of this method had been modified to be suitable n-phase conventional induction motors. In the last decades of previous century it had been applied for analysis of linear induction motor [2, 4, 5]. Classical methods that had been used to simulate the electric machines have a limitation to add the effect of winding connection. Generally it assume that both stator and rotor are consists of three phases only and ignore the effect of winding distribution. This drawback had been removed by introducing the WFA where all types of winding can be introduced easily. Also the effects of materials types and saturation can be taken into consideration [6-9]. Then the WFA had been modified in order represents the actual distributions of magnetic flux which gives the correct simulation of electric machine parameters. However, obtaining lower harmonics and simple manufacturing process is a challenge in

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the design and optimization of analyst [10, 11]. Due to ability of WFA to simulate any type of winding distribution or a variable air-gap width around rotor, it has been effectively used for modeling and simulation of any type of abnormality of electrical machine operation. Fay model. The ability of WFA makes it more suitable to simulate the inter turn sort circuit between two windings or with other phases in addition to simulate the broken bars of electric machines. Finally it has been successfully for simulation the static, dynamic and mixed rotor eccentricity [10, 12]. Multiple coupled circuit is assumed that the stator of the induction motor consists of n-circuits while the rotor bars of m-circuits. These electrical circuits are mutually coupling and used to determine the inductances of the motor using simple mathematical formulas. WFA give an appropriate way to simulate all machine parameters, stator winding turns, rotor bar and air-gap eccentricities. According to principle of this theory, the mutual and self-inductances between stator winding and rotor bars (windings) stator-rotor circuits can be considered. The relation between these winding is time varying because it depends of the rotor position with respect each other and with stator winding. The leakage inductance of both stator winding and rotor bars are considered as constants and can be computed using special design formulas. In current work two procedure of WFA have been considered. The slot effect is not included in the first type where the coupling mmsfs have a sinusoidal shape. While in second approach a real winding distribution in the slots is assumed and mmsfs have a non-sinusoidal waveform. A complete model of 3-Ф cage induction motor which has an m-stator electrical circuits and n-rotor bars is implemented using matlab software [13]. An epic strategy to display electrical machines dependent on WFA is proposed. In traditional WFA, it is accepted that transition lines in air-gap are in outspread heading. The primary downside of this supposition that is incorrectness of the reproduction results. In this technique WFA is improved by limited component strategy in which the definite permeance work is determined by finite element method. This new methodology has a decent exactness adjacent to of low estimation preparing time. In this approach the simulation results are presented and show the performance of the proposed method and these results are compared to experimental results [14-16].

2. CONSTRUCTION OF THREE PHASE INDUCTION MOTOR MODEL USING WFA

As stated previously the model of 3-Ф Induction motor using a winding function theory has been constructed. It’s considers that the model of stator and rotor to be formulated of several electric circuits which are coupled together. Also it’s assumed the currents flow through these circuits as an independent variable. The analysis is based on the following assumptions:
- Non-sinusoidal distributed of MMF on the windings of stator where the actual distribution of windings over the stator slots have been taken into consideration.
- Infinity iron permeability (µ iron=∞) to introduce the correct behavior of actual motor.
- Uniform air gap and neglected inter-bar currents have been assumed, where each adjacent bar form a separate circuit.
- The rotor bars and the motor windings have been suggested to have a symmetric distribution.
- The effect of saturation at starting and heavy loading are neglected.
- Non-sinusoidal distributed of inductance due to slots effect also has been introduced.

2.1. Modeling of voltage equations

The stator of the induction motor model used in current work have a 3-Ф windings, therefore a three voltage equations are needed to simulate the stator part. The rotor part contains an equally spaced and symmetrical number of circuits formed by two bars that are connected together using the end rings segments as shown in Figure 1 [17].

![Figure 1. Rotor loops current distribution](image)
where Figure 1 display the currents distribution of the rotor loops. Both loop shares two rotor bars with the two national rings and two end ring sections with the two end-ring loops. Not the higher shape codes are explained later and according to previous assumptions the complete model of the cage rotor induction motor can be represented by:

\[
[V_s] = [R_s][I_s] + d[\phi_s]/dt \quad (1)
\]

\[
[0] = [Rr][I_r] + d[\phi_r]/dt \quad (2)
\]

\[
[\phi_s] = [L_s][I_s] + [Lsr][I_r] \quad (3)
\]

\[
[\phi_r] = [Lr][I_r] + [Lrs][I_s] \quad (4)
\]

where, \(V_s\) is the vector of the stator voltages, \(I_s\) is the vector of the stator currents, \(I_r\) is the vector of the rotor loops currents, \(R_s\) is the matrix of the stator windings resistances, \(R_r\) is the matrix of the rotor resistances, \(L_s\) is the matrix of the stator windings inductances, \(L_r\) is the matrix of the rotor inductances, \(Lsr\) and \(Lrs\) are the matrix of the mutual inductances between stator and rotor respectively \(\phi_s\) and \(\phi_r\) are the stator and rotor windings linkage flux respectively [12, 18, 19].

From (1) and (3), the voltage equations of the stator per phase may be written as:

\[
V_a = R_s I_a + (L_a + L_{1s}) \frac{dI_a}{dt} + L_{ab} \frac{dI_b}{dt} + L_{ac} \frac{dI_c}{dt} \quad (5)
\]

\[
V_b = R_s I_b + (L_b + L_{1s}) \frac{dI_b}{dt} + L_{ba} \frac{dI_a}{dt} + L_{bc} \frac{dI_c}{dt} \quad (6)
\]

\[
V_c = R_s I_c + (L_c + L_{1s}) \frac{dI_c}{dt} + L_{ca} \frac{dI_a}{dt} + L_{cb} \frac{dI_b}{dt} \quad (7)
\]

where \(V_a, V_b\) and \(V_c\) are the per phase stator voltages \(i_a, i_b\) and \(i_c\) are the per phase stator currents \(L_a, L_b\) and \(L_c\) are the per phase stator self-inductances, \(L_{ab}, L_{bc}\) and \(L_{ca}\) are the per phase stator mutual inductances \(L_{ik}\) is the leakage inductance of the stator per phase. These equations are rewritten in suitable way to suit the basic m.file refer to phase “a” the solve one phase current \(i_a(t)\) rewritten of equation in the following from:

\[
\frac{dI_a}{dt} = \frac{1}{L_a} \left( (V_a - R_s I_a + i_a) - \frac{d}{dt}(L_{ab}i_b + L_{ac}i_c + L_{ar}i_r) \right) \quad (8)
\]

where, \(L_{ab}, L_{ac}, L_{ar}\) are inductances and their value can be computed based on the position of rotor. Similarly the above equations for the other two phases \((\frac{dI_b}{dt} \text{ and } \frac{dI_c}{dt})\) currents and two damper windings currents. The complete m. file for three phase dynamic model of induction machine. The rotor loops currents \((ir_1, ir_2..., ir_n)\), \((Lar_1, Lar_2..., Larn)\) are mutual inductances of stator phase A to rotor loops, \((Lbr_1, Lbr_2..., Lbrn)\) are mutual inductances of stator phase B to rotor loops, and \((Lcr_1, Lcr_2..., Lcrn)\) are mutual inductances of stator phase C to rotor loops. All rotor circuits are mutually coupled with each other and with stator phase’s windings. Based on 2 and 4 and with depend on Figure 1, the rotor loop voltage equation k may be given as:

\[
0 = 2(R_b)I_{lk} - R_b(i_{r(k-1)} + i_{r(k+1)}) + 2(L_b + L_c) \frac{dI_{rb}}{dt} - L_b(\frac{dI_{r(k-1)}}{dt} + \frac{dI_{r(k+1)}}{dt}) + \frac{dI_{lk}}{dt} + \sum L_{rk} \frac{dI_{rk}}{dt} + \sum L_{rk} \frac{dI_{rk}}{dt} = d/dt(I_{abk}a + I_{bkb}b + I_{ckc}c \quad (9)
\]

where, \(Re, rb, Lb, Le\) are the resistance and inductance of each rotor bar. \(L_{rk}\) is the self inductance of each rotor loop \((L_{k1}, L_{k2}, ..., L_{k(k-1)}, L_{k(k+1)}..., L_kn)\) are the mutual inductances loop k to the other rotor loops \((Lk, Lkb, \text{and} Lck)\) are the stator mutual inductances with rotor loop k.

2.2. Equation of electromagnetic torque

The equation of the electromagnetic torque can be determined based on the energy conversion theory, the electromagnetic torque equation produced by a 3-phase induction motor can be computed using the following equation [12]:

\[
T_e = [i_s]^T \frac{dL_{sr}}{d\theta_r} [i_r] \quad (10)
\]

where, \(\theta_r\) is the angular position of rotor, \(T_e\) is the developed torque produced by the motor.
2.3. Electromechanical equation

The mechanical equation of rotor may be written as [12, 18]:

\[ T_r = J \frac{d\omega_r}{dt} + B \omega_r + TL \]  \hspace{1cm} (11)

where,
\( J \) is moment of inertia of rotor,
\( \omega_r \) is rotor angular speed,
\( B \) is the friction coefficient.
\( TL \) is the load torque,
\( \Theta_r \) is the angular position of the rotor.

3. NON-SINUSOIDAL SHAPES SELF AND MUTUAL INDUCTANCE FORMULA USING WFA

The important key of WFA of the induction motor is to compute the values of inductances equations accurately, in [2, 3] the winding function method had been applied in order calculate the inductances of the motor in terms angular position of rotor \( \Theta \). WFA is an analytical method used to calculate the self and mutual inductances based on the distribution of MMF in the air-gap region due to a follow a current of "1". A value pass through the winding under consideration. From the principles of WFA, it’s evident that it’s required to calculate all the inductances of the machine using the inductance matrices. These inductances are normally determined using WFA. The general formula that will be used to calculate the motor inductances (self and mutual) is defined by (11). Using winding function approach, the mutual inductance between two coupled electric circuits (stator phases or rotor bars) for any electrical machine may be determined using the following equation [20-24]:

\[ L_{ij}(\theta) = \mu_0 r / g \int_0^{2\pi} \lambda_i(\theta, \vartheta) \lambda_j(\theta, \vartheta) d \vartheta \]  \hspace{1cm} (12)

where: \( \mu_0 \) is the permeability of air and \( -4\pi \times 10^{-7} \) H. \( g \) is the width of the air gap, \( \theta \) is the angular position of the stator, \( r \) is the average radius of the air gap, \( l \) is the effect length of the motor stack, \( \vartheta \) is a certain point along the air gap, \( \lambda(\theta, \vartheta) \) is called the winding function and represents the distribution of the magnet motive force along the air gap when a current of '1' A flow in the winding. In current paper a three phase induction motor, 36 stator slots and symmetrical concentrate winding. While the cage rotor of the motor compromise a 28-equally spaced bars. The geometric construction of the motor has been taken into consideration, where the slots effect on the stator winding is considered. The self any two stator phases \( L_{aa}, L_{bb}, L_{cc} \) have been calculated based on WFM as given in (14). Where the motor inductances formulas can be written as [25-27]:

\[ L_a = L_b = L_c = (N_s / 2p)^2 (\pi \mu_0 r / g) = L_{ms} \]  \hspace{1cm} (13)

\[ L_{ab} = L_{bc} = (N_s / 2p)^2 (\pi \mu_0 r / 2g) = -L_{ms} / 2 \]  \hspace{1cm} (14)

\[ L_{kk} = \left( \frac{\alpha_k r}{g} \right) \left( 1 - \frac{\alpha_k}{2\pi} \right) \alpha_r \]  \hspace{1cm} (15)

\[ L_{k1} = L_{k2} = \cdots = L_{kn} = L_{kl} \left( \frac{\alpha_k r}{g} \right) \left( \alpha_r \right)^2 / 2 \pi \]  \hspace{1cm} (16)

\[ L_{ak} = L_m \cos(p(\theta_r + (k-1)\alpha_r + \delta)) \]  \hspace{1cm} (17)

\[ L_{bk} = L_m \cos(p(\theta_r + (k-1)\alpha_r + \delta) - 2\pi / 3) \]  \hspace{1cm} (18)

\[ L_{ck} = L_m \cos(p(\theta_r + (k-1)\alpha_r + \delta) + 2\pi / 3) \]  \hspace{1cm} (19)

where: \( N_s \) is the number of stator turns per phase, \( p \) is the number of pole pairs, \( \alpha_r \) is the angle between two neighboring bars \( L_m = (4 / \pi N_s) L_{ms} \sin(\delta) \), where \( \delta=\alpha_r / 2. \)
4. SIMULATION RESULTS

The flux equations have been used to simulate the proposed model. The simulated steps that have been used to solve the differential equations of the induction motor are explained in the algorithm shown in Figure 2. The integration of flux linkage derivatives with respect to time are calculated to obtain flux linkages, as follows:

\[
\frac{d\phi}{dt} = v_{\text{source}} - RI \tag{20}
\]

\[
\phi = \int (v_{\text{source}} - RI) = L_{\text{self}}i_{\text{self}} + \Sigma L_{\text{mutual}}i_i \tag{21}
\]

Matlab m-file has been used to simulate the dynamic equations of a 3-Φ cage rotor induction motor. The proposed mathematical model has been used to simulate the time response performance of a 4 kW induction motor with the parameters given in Table 1.

![Figure 2. Algorithm of simulation](image)

Table 1. Parameters of induction motor model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Supply Phase Voltage</td>
<td>220 Volt</td>
</tr>
<tr>
<td>Input Supply Frequency</td>
<td>50 Hz</td>
</tr>
<tr>
<td>STATOR PARAMETERS</td>
<td></td>
</tr>
<tr>
<td>Effective Number of Stator winding turns: (N_s)</td>
<td>156 turn, 36 stator slots</td>
</tr>
<tr>
<td>Stator windings Ohmic resistance: (R_s)</td>
<td>1.5 ohm</td>
</tr>
<tr>
<td>Stator windings Leakage inductance: (L_{1S})</td>
<td>7 Mh</td>
</tr>
<tr>
<td>Number of pole pairs: (p)</td>
<td>4</td>
</tr>
<tr>
<td>Rotor PARAMETERS</td>
<td></td>
</tr>
<tr>
<td>Number of rotor bars: (n)</td>
<td>28</td>
</tr>
<tr>
<td>Rotor bar resistance: (R_b)</td>
<td>96.940036 (\mu)Ω</td>
</tr>
<tr>
<td>End ring segment resistance: (R_a)</td>
<td>5 (\mu)Ω</td>
</tr>
<tr>
<td>Rotor bar self inductance: (L_b)</td>
<td>0.28 (\mu)H</td>
</tr>
<tr>
<td>End ring segment self inductance: (L_a)</td>
<td>0.036 (\mu)H</td>
</tr>
<tr>
<td>AIR GAP PARAMETERS</td>
<td></td>
</tr>
<tr>
<td>Air gap average radius: (r)</td>
<td>70 mm</td>
</tr>
<tr>
<td>Air gap length: (g)</td>
<td>0.28 mm</td>
</tr>
<tr>
<td>Rotor effective length: (L)</td>
<td>120 mm</td>
</tr>
<tr>
<td>MECHANICAL PARAMETERS</td>
<td></td>
</tr>
<tr>
<td>Inertia: (J)</td>
<td>0.007 N.sec²</td>
</tr>
<tr>
<td>Friction coefficient: (F)</td>
<td>0.001 N.sec</td>
</tr>
</tbody>
</table>

Modeling and simulation of a 3-Φ induction motor based on two types of WFA (Ahmed J. Ali)
According to the two suggested approaches, the model of motor has been simulated under operation conditions by applying a load torque of $T_L=3.5$ Nm on the shaft, starting time from zero, applying of 220 volt and 50 Hz. The time response of the stator current are shown in Figure 3 at Non-sinusoidal shapes self and mutual inductance. Also the dynamic characteristics (Mutual inductance, rotor speed and developed torque) are shown in Figures 4-6. The WFA principle is based on the coupling between stator and rotor as shown Figure 4. Then for the second approach, the model has been simulated under the same operating conditions mentioned for first approach. It’s clear from the simulation results of stator and rotor currents are shown in Figure 7 and Figure 8 that waveform have a smooth shape compare with Figure 3. This uniformly and regular current is due to sinusoidal assumption of mutual inductance that couple between the stator and rotor.

![Figure 3. The time response of the stator current, (a and b) stator current at non-sinusoidal shapes self and mutual inductance using WFA](image)

![Figure 4. Mutual inductance between stator and rotor](image)

![Figure 5. Rotor speed (rpm)](image)  

![Figure 6. Developed torque (N.m)](image)
Modeling and simulation of a 3-ф induction motor based on two types of WFA (Ahmed J. Ali)

5. CONCLUSION

Two types of winding function approaches have been modeled and simulated in this paper. It's obvious from the results that the first model has results different slightly from the second approach. For the stator, rotor currents and torque they have a non-smooth and ripple shapes compared with the results obtained from the second model. This non-linearity is due to the actual winding distribution on the motor slots. The stator MMF have a non-sinusoidal waveform due to slots effect which make this model more suitable to simulate the faulty motor compared with first type.

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