Inactive power detection in AC network

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ABSTRACT

Using the examples of wave and vector diagrams, we study the conditions for the appearance of components of inactive power in an AC network, which are known as reactive power and distortion power. It is shown that the components of the active, reactive power and distortion power are mutually orthogonal and form a power balance, which can be violated mainly due to methodological errors in calculating these components under conditions of non-stationary mode parameters. It is established that the interaction of reactive power and distortion power occurs at the instantaneous power level, and changing their phase shifts allows you to adjust the shape of the resulting power without involving additional active power in the AC network. The results obtained will allow not only to correctly determine the proportion and nature of the components of inactive capacities, which is valuable for solving the problems of optimizing modes in AC networks, but also to create effective technical means of compensating for the identified inactive capacities in the future.

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In one of the first theories of inactive powers, the power \( Q \) was determined [10] for all harmonics in the AC network. It was there that the \( D \) value was first described, characterized as the power of distortion, thereby indicating that it occurs when the current shape is distorted relative to the voltage shape. However, comments on this theory [11] remain, which relate primarily to unresolved issues of determining the power of \( D \) and the interpretation of the physical meaning of its components.

Scientific works are known in which the analysis of the properties of the capacities \( Q \) and \( D \) is based on spectral methods [12, 13], or integrated methods for estimating capacities using the summation of instantaneous values of \( i(t) \) and \( u(t) \) [14, 15]. Methods for separating currents into separate components for expressing inactive powers in special coordinates [16-18] or in individual energy flows in the form of orthogonal components [19, 20] have gained some development. Unfortunately, these works are contradictory with respect to the physical meaning of the components of inactive capacities, which in practice causes problems in their analysis and calculation.

Therefore, in this paper, the task is to show how the interaction of combinations of harmonics \( i(t) \) and \( u(t) \) leads to the appearance of power components \( Q_i \) and \( D_j \), whose indices \( i \) and \( j \) are determined by the serial numbers of these harmonics. This will allow a better understanding of the process of the occurrence of inactive capacities in AC networks, their analysis and calculation. Such results are necessary to create rational methods and construct algorithms for compensating inactive capacities by technical means [21, 22] that do not consume additional power from the network. In this case, the correction of the forms of curves of instantaneous consumer power in the AC network will be possible due to the redistribution of individual inactive components \( Q_i \) and \( D_j \).

2. WAVE AND VECTOR DIAGRAMS FOR THE ANALYSIS OF THE COMPONENTS OF INACTIVE POWER

In the theory of electrical engineering, the reasons for the appearance of power in an alternating current network with loads possessing linear CVC, but having a reactive nature are described quite well [5-8]. If the load current vector \( \vec{I}_i \) lags or is ahead of the supply voltage vector \( \vec{U}_i \) by the angle \( \varphi \), then, characterizing these quantities as \( u(t) = U_{im} \sin(\omega t) \) and \( i(t) = I_{im} \sin(\omega t - \varphi) \), the instantaneous load power \( p(t) = u(t) \cdot i(t) \) is conditionally divided into a constant component \( p_\sigma \) and an alternating component \( p_\pi \).

\[
p(t) = U_{im} \sin(\omega t) \cdot I_{im} \sin(\omega t - \varphi) = \frac{U_{im}I_{im}}{2} \cos(\varphi) \cos(2\omega t - \varphi) = p_\sigma + p_\pi
\]  

(1)

The integral sum of the components \( p_\sigma \) and \( p_\pi \) for the period \( T \) corresponding to the complete revolution of the vectors \( \vec{I}_i \) and \( \vec{U}_i \) can form the active power \( P_1 \), determined by the argument \( \varphi \)

\[
P_1 = \frac{1}{T} \int_0^T u_i(t)i_i(t) dt = \frac{U_{im}I_{im}}{2} \cos(\varphi) \sin(\omega t) \sin(\omega t - \varphi)
\]  

(2)

with \( \frac{1}{T} \int_0^T p_\sigma dt = P_\sigma \) and \( \frac{1}{T} \int_0^T p_\pi dt = 0 \). The appearance of the alternating \( p(t) \) in the combination of the components \( p_\sigma \) and \( p_\pi \) for the period \( T \) characterizes the effect of the exchange or reactive power \( Q \) between the elements of the AC network, for example, the source and the load. The change of sign \( p(t) \), as follows from expression (1), will occur with a frequency of \( 2\omega t \) twice during period \( T \).

The value of \( Q_i \) is traditionally expressed through the imbalance of the total power \( S_i = U_iI_i \) and \( P_1 \), using the well-known property of orthogonality of the components \( Q \) and \( P \)

\[
Q_i^2 = S_i^2 - P_1^2
\]  

(3)
As the vector $\mathbf{I}_1$ lags with respect to the vector $\mathbf{U}_1$, the component $p_1$ decreases and a range of negative $p(t)$ values appears, indicating that component $P_1$ decreases to zero and the growing component $Q_1$ appears. An imbalance between the capacities $S_1$ and $P_1$ in an alternating current network with a sinusoidal voltage source can also be observed when the load is operated with a purely active character, but non-linear CVC [5-7].

3. REACTIVE POWER OF ACTIVE LOAD IN AC NETWORK

It is known that it is enough to allow an artificial delay in opening electronic keys in an alternating current circuit supplying a purely active load in order to form a spectrum of higher current harmonics [23-25]. From this spectrum, only the first harmonic of the current will create the component $Q_1$. As an example, Figure 1 shows the oscillograms and the spectral composition of the load current at $R = 1$ r.u. in a network with a sinusoidal voltage source $U = 100$ r.u. and frequency $f = 50\text{Hz}$, when the delay in opening electronic keys was about 70%, that is, 0.007s. Spectral composition of active load current shown in Table 1.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Oscillograms: 1-supply voltage; 2-active load current}
\end{figure}

In this network with a sinusoidal voltage of the power supply, a load with a purely active character causes a phase shift of the main harmonic of the current of 54.8 degrees, as if the electric circuit of the load contained an inductive element. Formally, using the expression (3), one can calculate the reactive power of such an element.

<table>
<thead>
<tr>
<th>No of harmonic</th>
<th>harmonic frequency, $f_h$ (Hz)</th>
<th>amplitude $I_{m1}$ (A)</th>
<th>phase shift, $\varphi_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>24.46</td>
<td>-54.8°</td>
</tr>
<tr>
<td>3</td>
<td>150</td>
<td>20.11</td>
<td>16.9°</td>
</tr>
<tr>
<td>5</td>
<td>250</td>
<td>13.28</td>
<td>94.2°</td>
</tr>
<tr>
<td>7</td>
<td>350</td>
<td>7.16</td>
<td>188.5°</td>
</tr>
<tr>
<td>9</td>
<td>450</td>
<td>5.03</td>
<td>-51.6°</td>
</tr>
<tr>
<td>11</td>
<td>550</td>
<td>5.02</td>
<td>53.8°</td>
</tr>
<tr>
<td>13</td>
<td>650</td>
<td>4.12</td>
<td>150.8°</td>
</tr>
<tr>
<td>15</td>
<td>750</td>
<td>3.12</td>
<td>260.0°</td>
</tr>
</tbody>
</table>

However, to consider that in this and similar circuits there are reactive elements that exchange power with the power source, of course, is erroneous. Moreover, in separate works, for example [26-28], it was noted that the concept of reactive power, which “characterizes the energy pumped from the source to the reactive elements of the receiver, and then returned by these elements back to the source during one oscillation period, referred to this period” does not fully reflect the physical meaning.
4. ANALYSIS OF THE COMPONENTS OF INACTIVE POWER

Consider the action that each harmonic of the load current individually produces. The action of the first harmonic of the current lagging relative to the voltage, in accordance with (1), is characterized by the active component \( p_1 \), capable of performing work, and the inactive component of power. Their interaction leads to the appearance of an alternating \( p(t) \) formed by the first harmonic of the current. If other current harmonics did not exist in this example, it could be assumed that a reactive element with a power of \( Q \) is present in the network, which exchanges energy with a sinusoidal voltage source [5-8]. However, the higher harmonics of the current form their instantaneous power \( p(t) \), which, adding together arithmetically, will create a more complex picture of the behavior of the instantaneous power in the network

\[
p(t) = \sum_{i=1}^{\infty} p_i(t).
\]

The vectors of the higher harmonics of the current rotating on the complex plane will constantly be ahead of the existing main harmonic of the voltage. So, in Figure 2(a) vector of the third harmonic of the current \( I_3 \), constructed according to the data in Table 1, in one period \( T \) is three times ahead of the voltage vector \( U_1 \).

The orthogonality property of the components [13, 18] of the higher harmonics of the current and the supply voltage, due to the multiplicity of frequencies \( U_i \) and \( j_i \), leads to the fact that during the working period \( T \) this current harmonic is not able to perform work, since the power

\[
P_i = \frac{1}{T} \int_0^T u(t) \cdot i(t) dt = 0.
\]

Therefore, this power does not make sense in the future to decompose into the components \( p_1 \) and \( p_\sim \).

Therefore, ripples of alternating instantaneous power \( p(t) \) are distributed in such a way that the total area bounded by the curve \( p(t) \) in Figure 2(b) with positive values is equal to the total area bounded by this curve with negative values, and the total power formed by the main voltage harmonic and the third current harmonic, will be \( S_3 = U_1 I_3 \), and will determine the so-called distortion power \( S_3 = D_3 \).

\[
S_3 = D_3 = U_1 I_3.
\]

Figure 2. Third harmonic load current, (a) vector diagrams, (b) wave diagrams, (c) power supply voltage.

Similarly, the interaction of the remaining \( i \)-th higher harmonics of the current with a voltage \( U \), which leads to the appearance of the corresponding power values

\[
S_i = D_i = U_i I_i.
\]
for which

$$P_i = \frac{1}{T} \int_{0}^{T} u_i(t) \cdot i_i(t) dt = 0. \quad (6)$$

Figure 3 shows wave diagrams explaining the formation of powers $p_i(t)$ subsequent harmonics the fifth and seventh and obtaining the total instantaneous power according to expression (4). Thus, the effect of the reactive component $Q_i$ in $p_i(t)$ from the first harmonic of the current at time intervals when the electronic switch is open is almost completely neutralized by instantaneous values of $p_i(t)$ higher harmonics. This example shows that the power $Q_i = U_i I_i \sin(\varphi_i)$ from the first harmonic of the current is not the prerogative of the manifestation of the properties of reactive elements in an alternating current network, but can be considered as an imbalance in the instantaneous values of $p_i(t)$, which caused a shift of the components $p_\omega$ and $p_{\omega\prime}$.

![Wave diagrams explaining the formation of powers](image)

Figure 3. Instantaneous power values from various harmonics, (a) fifth, (b) seventh, (c) synthesis of total instantaneous power

Thus, the power $Q_i$ interacts with the power components $D_i$, however, this happens only at the level of redistribution of the total instantaneous values of $p(t)$, since the values of the components $Q_i$ and $D_i$ remain unchanged and cannot be mutually compensated [23-25]. This is due to the fact that one harmonic, characterizing a certain power component, as follows from the properties of the Fourier series, cannot be completely obtained or compensated by a combination of other harmonics.

This explains the possibility of correcting the shape of the $p(t)$ curve of any AC consumer by adjusting the phase shifts of the $p_i(t)$ values created using a special technical device, which will not consume additional power from the network due to properties (6). Such a technical device may be an automatic inactive power compensator with zero active power [21, 22].

The set of effective $D_i$ values with orthogonal harmonics and taking into account expression (5) can be written as

$$D_\omega = \sqrt{\sum_{i=2}^{\infty} D_i^2} = \sqrt{\sum_{i=2}^{\infty} U_i^2 I_i^2}. \quad (7)$$

This expression is calculated as $D_\omega = U_1 \sqrt{(I_2^2 + I_5^2 + I_7^2 + \ldots)}$ according to Table 1

$$D_\omega = \frac{100}{\sqrt{2}} \sqrt{\left(\frac{20.11}{\sqrt{2}}\right)^2 + \left(\frac{13.28}{\sqrt{2}}\right)^2 + \left(\frac{7.16}{\sqrt{2}}\right)^2 + \left(\frac{5.03}{\sqrt{2}}\right)^2 \ldots} = 1347.4 \text{ VA}.$$
Separately calculated values of orthogonal components

\[ P_i = U_i I_i \cos(\phi) = \frac{100}{\sqrt{2}} \frac{24.46}{\sqrt{2}} \cos(54.7) \approx 712 \text{ W} \quad \text{and} \quad Q_i = U_i I_i \sin(\phi) = \frac{100}{\sqrt{2}} \frac{24.46}{\sqrt{2}} \sin(54.7) \approx 1005 \text{ VA} \]

provide power balance \([10]\).

\[ S = \sqrt{P^2 + Q^2 + D^2} . \quad (8) \]

Since

\[ S = U \cdot I = \frac{100}{\sqrt{2}} \left( \frac{24.46}{\sqrt{2}} \right)^2 + \left( \frac{20.11}{\sqrt{2}} \right)^2 + \left( \frac{13.28}{\sqrt{2}} \right)^2 + \left( \frac{7.16}{\sqrt{2}} \right)^2 \approx 1825.46 \text{ VA} \]

Thus, the computed \(1825.46 = \sqrt{712^2 + 1005^2 + 1347.4^2}\) components form a balance. This confirms the idea that the powers of \(P\), \(Q\), and \(D\) are mutually orthogonal [10, 29], and the value of \(\sqrt{S^2 - P^2}\) determines the set of inactive powers of \(Q^2 + D^2 = S^2 - P^2\).

5. VIOLATION OF THE POWER BALANCE IN THE AC NETWORK

The question remains - why in a number of studies, for example, in [1, 11-14], the authors find that in practice expression (8) is not true for all values of consumer operation modes measured in an alternating current network. For the conditions considered, when the network has only one voltage source \(U_1\), a power balance violation (8) will occur if the previously accepted theoretical provisions are not fulfilled, and, first of all, the power property (6) based on the orthogonality of the components. This is possible if the parameters of the higher harmonics of the current (amplitude, phase shift) are not stationary during the working period \(T\), which will lead to a violation of the equality of the areas limited by the \(p(t)\) curves for positive and negative values in Figure 3.

In such cases, higher harmonics of the current with non-stationary parameters, interacting with the voltage of the network \(U_1\), are able to do the work, because for them \(P \neq 0\). Taking, for example, the condition that \(i_1(t) = I_1 \sin(3\pi t + \varphi_1)\), if \(i_1(t) \cdot u_1(t) \geq 0\) and \(i_1(t) = 0\), if \(i_1(t) \cdot u_1(t) < 0\). we obtain a pulsating, but positive power \(p(t)\) in Figure 2(c). As a result of the accepted condition, in curve \(i_1(t)\) in Figure 2(c) appears, in addition, the harmonic component \(i_1(t)\), whose lagging character with respect to \(u_1(t)\) creates an additional component \(Q = 31.8 \text{ VA}\). As can be seen from Table 2, even a small non-stationarity of the parameters of a single current harmonic (in the considered example, this is the third harmonic) makes one of the known expressions for determining the power balance in an alternating current network unjust.

In practice, the lack of stationarity of the parameters of the harmonic composition during the \(T\) working period is typical for modern AC consumers. Switching high-speed electronic keys in the power circuits of such consumers is capable of repeatedly changing the amplitudes of the higher harmonics of the current in one \(T\) period.

In such cases, algorithms for calculating conventional means of measuring and analyzing the spectral composition of currents are not always able to correctly analyze, arranging these currents in Fourier series, since they do not have constant amplitudes and phase shifts [1, 2, 6-8]. The powers \(P\), \(Q\), and \(D\), determined from such data will not have a balance. Of course, this only testifies to the fact that in fact the power balance always exists, and the most reliable information on the operating modes of consumers is provided by the instantaneous \(p(t)\) capacities [18-20], by which it is necessary to methodologically correctly determine the \(P\), \(Q\), and \(D\) components that will give an idea of the nature of the course of energy processes.

In such cases, and in particular, if the AC network has non-sinusoidal voltage sources, it will be more efficient to analyze the spectrum and then calculate the operating modes of consumers using additional capabilities of the mathematical apparatus of spectral analysis, for example, the Wavelet transform [30], which may be the subject of a separate study.
Table 2. Power $P_i$, $Q_i$, $D_i$ and $S_i$ of active nonlinear load in a network with voltage $U_i$

<table>
<thead>
<tr>
<th>No of harmonic</th>
<th>$i_i(t)$</th>
<th>$P_i$, W</th>
<th>$Q_i$, VA</th>
<th>$D_i$, VA</th>
<th>$S_i$, VA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24,46sin($wt - 54,8$)</td>
<td>712</td>
<td>1005</td>
<td>-</td>
<td>1231,6</td>
</tr>
<tr>
<td>2</td>
<td>20,11sin(3$wt + 16,9$), $if\ sin(3wt + 16,9) - \sin(wt) \geq 0$</td>
<td>417</td>
<td>31,8</td>
<td>711,8</td>
<td>825</td>
</tr>
<tr>
<td>3</td>
<td>0, $if\ sin(3wt + 16,9) - \sin(wt) &lt; 0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>13,28sin(5$wt + 94,2$)</td>
<td>0</td>
<td>-</td>
<td>664</td>
<td>664</td>
</tr>
<tr>
<td>5</td>
<td>7,16sin(7$wt + 188,5$)</td>
<td>0</td>
<td>-</td>
<td>358</td>
<td>358</td>
</tr>
<tr>
<td>6</td>
<td>5,03sin(9$wt - 51,6$)</td>
<td>0</td>
<td>-</td>
<td>251,5</td>
<td>251,5</td>
</tr>
<tr>
<td>7</td>
<td>5,02sin(11$wt + 53,8$)</td>
<td>0</td>
<td>-</td>
<td>251</td>
<td>251</td>
</tr>
<tr>
<td>8</td>
<td>4,12sin(13$wt + 150,8$)</td>
<td>0</td>
<td>-</td>
<td>206</td>
<td>206</td>
</tr>
<tr>
<td>9</td>
<td>3,12sin(15$wt + 260,0$)</td>
<td>0</td>
<td>-</td>
<td>156</td>
<td>156</td>
</tr>
</tbody>
</table>

$P_i = \sum_{i=1}^{11} p_i = 1129$ $Q_{\omega} = 1036,8$ $D_{\omega} = \sum_{i=1}^{11} D_i = 1145,2$ $S_{\omega} = \sqrt{P_{\omega}^2 + Q_{\omega}^2 + D_{\omega}^2} = 1732,8$

Balance check
$1732,8 < 1913,4 = \sqrt{1129^2 + 1036,8^2 + 1145,2^2}$

6. CONCLUSION

Based on the analysis of wave and vector diagrams, it is shown that the power $Q$ should not be unambiguously characterized as a quantitative indicator of the rate of energy exchange between reactive elements of the electric network, as was previously shown in the scientific literature. A new sign of determining the power $Q$, which is expressed in the formation of an alternating instantaneous power $p(t)$, pulsating twice during the working period in the analyzed electric circuit, is proposed. Consequently, not only reactive elements, but also elements with nonlinear CVC, for example, semiconductors, can participate in the creation of $Q$ power, since they are also able to create a phase shift between the harmonic current and voltage with the same numbers. Thus, it is proposed to consider the twofold alternating ripple $p(t)$ caused by the interaction of the voltage harmonic and current harmonic with the same numbers as the only sign of the difference between the inactive power $Q$ and the active power $D$.

The analysis and calculations confirm the validity of the Budeanu’s theory regarding the orthogonality of $P$, $Q$, and $D$ components and achieving a power balance in a network with non-linear CVC load, despite the lack of a rigorous definition of $P$ power in the scientific literature. It is shown in the work that the power $D$ is also created by an alternating ripple $p(t)$, however, unlike the power $Q$, the multiplicity of the ripple frequency $p(t)$ in this case is always greater than two and is determined not only by the mutual phase shift of the harmonics of the voltage $U_i$ and current $I_j$, but also by their serial numbers, moreover, $i \neq j$, which is a hallmark of the power of $D$.

It is shown that a dynamic change in the amplitudes and phase shifts of individual harmonics of current and voltage in a time shorter than the working period $T$ in practice complicates the calculation of the actual values of the powers $Q$ and $D$, and can lead to calculation errors. Since the zero average values of the $Q$ and $D$ powers allow the compensating device not to consume additional active power from the network by correcting the $p(t)$ curve shape of the AC consumer, this power property is proposed to be used as a simple but effective criterion for the absence of calculation errors in the control system. Thus, it is proposed in the work to automatically check for errors on the basis of achieving zero calculated $Q$ and $D$ values when $p(t)$ is formed from higher harmonics and there is no violation of the power balance produced by the control system of the compensating device.
REFERENCES


BIOGRAPHIES OF AUTHORS

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