Modified sub-gradient based combined objective technique and evolutionary programming approach for economic dispatch involving valve-point loading, enhanced prohibited zones and ramp rate constraints

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ABSTRACT
A security constrained non-convex power dispatch problem with prohibited operation zones and ramp rates is formulated and solved using an iterative solution method based on the feasible modified sub-gradient algorithm (FMSG). Since the cost function, all equality and inequality constraints in the nonlinear optimization model are written in terms of the bus voltage magnitudes, phase angles, off-nominal tap settings, and the Susceptance values of static VAR (SVAR) systems, they can be taken as independent variables. The actual power system loss is included in the current approach and the load flow equations are inserted into the model as the equality constraints. The proposed modified sub gradient based combined objective technique and evolutionary programming approach (MSGBCAEP) with λ as decision variable and cost function as fitness function is tested on the IEEE 30-bus 6 generator test case system. The absence of crossover operation and adoption of fast judicious modifications in initialization of parent population, offspring gene generation and normal distribution curve selection in EP enables the proposed MSGBCAEP approach to ascertain global optimal solution for cost of generation and emission level.

Keywords:
CCPP, CCPSO, CTPSO, FMSG, MSGBCAEP, SVAR

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1. INTRODUCTION
The basic theme of economic dispatch is to determine the optimal combination of power outputs of the generating units in electric power system so as to optimize the total fuel cost for a certain load demand satisfying operational constraints. The economic load dispatch (ELD) [1] problem is analyzed basically through the input output characteristic or through the heat rate input output characteristic by taking real power output of \( i^{th} \) generating unit (PG \( i \)) in the X axis and fuel input in rupees per hour in the Y axis. Input-output characteristic is approximated as a single quadratic variation curve which gives sub-optimal solutions. Usually the nature of Input-output characteristics of modern generating units is non-linear because of multi-fuel effects (MFE) using combined cycle power plants (CCPP) [2] and valve loading effects, which may lead to multiple local minimum points of cost functions. Hence it is more realistic to represent the input-output characteristic as a piece wise quadratic cost function to avoid huge revenue loss over time problems. In this paper a sub-gradient based modified dispatch approach [3] combined with EP technique (MSGBCAEP) has been attempted for solving economic dispatch problem involving cost objective function.

and emission objective function as well. Using sub gradient based FMSG technique inequality constraints are transformed to single equality constraint described in (18).

Economic load dispatch problem using FMSG Technique. The F-MSG is a deterministic solution method. It can solve security constrained non-convex power dispatch problems with prohibited operation zones and ramp rates [4, 5]. It is especially suitable to solve non-convex [6-10] dispatch problems where exact model of the test case system (optimal power flow problem) [11] is used. Since power flow calculation is not used in the computation process (except initial step), the solution time becomes lower than that of other algorithms mentioned in recent literature. Detailed explanation about the F-MSG method can be found in reference [12]. In this paper, application of the FMSG method is extended to non-convex [13, 14] dispatch problems with prohibited operation zones and ramp rates. Out performance of the FMSG algorithm with respect to some other economic dispatch algorithms based on heuristic and deterministic methods mentioned in recent literature is demonstrated on some well-known test systems. In those test systems, prohibited operation zones [15] and ramp rates of the generator are considered and exact or approximate model of power systems [16] are used.

2. PROBLEM FORMULATION

A nonlinear optimization model for an economic power dispatch problem can be described as follows:

\[ \text{Min } F_T = \sum_{i \in N_G} F_i (P_i) \]  

(1)

Subject to

\[ P_i - P_{load,i} - \sum_{j \in N_G} P_{ij} = 0 \]

(2)

\[ Q_i - Q_{load,i} - \sum_{j \in N_G} q_{ij} = 0, \quad i = 1, 2, ..., N \]

\[ p_i \in \{(p_i^{\min} \leq p_i < p_i^{z_1^+}) \cup (p_i^{z_1^+} < p_i < p_i^{z_2^-}) \cup \ldots \cup (p_i^{z_{p_1}^+} < p_i \leq p_i^{\max})\}, i \in N_G \]

\[ \min_{p_i} \text{min} \quad \min \quad P_i = \max(P_i, p_i^0 - DR_i) \]

\[ \max_{p_i} \text{max} \quad P_i = \min(P_i, p_i^0 + UR_i) \]

(4)

\[ Q_i^{\min} \leq Q_i \leq Q_i^{\max}, i \in N_Q \]

(5)

\[ p_i \leq p_i^{\max}, l \in L \]

(6)

\[ U_i^{\min} \leq U_i \leq U_i^{\max}, \]

\[ i = 1, 2, ..., N, i \neq \text{ref, vc} \]

(7)

\[ m_i^{\min} \leq m_i \leq m_i^{\max}, i \in N_{\text{tap}} \]

(8)
\[ m_2^{\min} \leq m_2 \leq m_2^{\max}, i \in N \]  
\[ s \text{ var} \]  
(9)

Note that the active power generation of the \(i^{th}\) unit \(p_i\) should satisfy one of the inequalities shown in (3). In other words, \(p_i\) should not be contained by any of the closed prohibited zone sets \(p_i \not\in [p_{z_{in}}^-, p_{z_{in}}^+], m = 1, 2, ..., n_{pci}\).

### 2.1. Determination of line flows and power generation

To express the total cost rate function in terms of independent variables of the proposed optimization model, the line flows need to be written in terms of the bus voltage, the off-nominal tap settings and the Susceptance values of SVAR systems (see (1) and (2)). The necessary equations, giving the active and reactive power flows \((p_{i,j}, q_{i,j})\) over the line that is connected between buses \(i\) and \(j\) in terms of the independent variables, can be found in reference. Using those equations and (2), the active and reactive power generations of other \(i^{th}\) unit connected to bus \(i\) can be calculated as under:

\[ p_i = p_{loadi} + \sum_{j \in N_G} p_{i,j} \]  
(10)

\[ q_i = q_{loadi} + \sum_{j \in N_G} q_{i,j} \]  
(11)

Also the total loss of the network can be calculated as:

\[ P_{loss}^{ij} = P_{ij}^{+} + P_{ji}^{+} \]  
(12)

\[ And \ P_{loss} = \sum_{i \in N} \sum_{j \in N, j \neq i} p_{ij}^{+} \]  
(13)

The non-convex cost rate function and the emission level function for the \(i^{th}\) generating unit are taken as:

\[ F_i(p_i) = m_1 + m_2 p_i + m_3 p_i^2 + \varepsilon_i (\sin(m_4(p_{i}^{\min} - p_i))), i \in N_G \]

\[ E(p_i) = [m_6 \varepsilon_i + m_7 \varepsilon_i \times (p_i - p_i^{\max}) + m_7 \varepsilon_i \times (p_i - p_i^{\max})^2] \]  
(14)

Where, \(m_1, m_2, m_3, m_4, m_5, m_6\) and \(m_7\) are constant coefficients for cost and emission objective functions. The sine term in (14) is added to the cost rate curve to reflect the valve point loading affect. The non-convex total cost rate is then determined as:

\[ F_T = \sum_{i \in N_G} F_i(p_i) \text{ in (Rs/h)} \]  
(15)

### 2.2. Converting inequality constraints into equality constraints

Since the FMSG algorithm requires that all constraints should be expressed in equality constraints form, the inequality constraints in the optimization model should be converted into corresponding equality constraints. The method described below is used for this purpose since it does not add any extra independent variable (like one in the slack variable approach) into the optimization model. Therefore, the solution time of the considered dispatch problem is reduced further. A double sided inequality \(x_i^- \leq x_i \leq x_i^+\) [17] can be written in form of the following two inequalities:
\[ h_i^+(x_i) = (x_i - x_i^+) \leq 0 \]
\[ \text{For } h_i^-(x_i) = (x_i^- - x_i) \leq 0 \]  

Then we can rewrite the above inequalities in a single equality constraint form as under:

\[ h_i^{eq}(x_i) = \max \{0, \max \{0, (x_i^- - x_i)\} \} = 0 \]

if \( x_i^- \leq x \leq x_i^+ \), it is obvious that \( (x_i^- - x_i^+) \leq 0 \), \( (x_i^- - x_i) \leq 0 \) and

\[ \max \{0, (x_i^- - x_i^-)\} = 0, \]
\[ \max \{0, (x_i^+ - x_i)\} = 0. \]

So, the inequality constraints in (16) can be represented by the corresponding single equality constraint in (17). In this paper, the double sided [17-20] inequality constraints given in (5) until (9) are converted into the corresponding single equality constraints in this manner. For the same reason, the union of two sided inequalities shown in (3) can be converted into the corresponding single equality constraint described by (18).

\[ h_i^{eq}(p_i) = \min \{\max \{0, (p_{i \min}^+ - p_i)\} + \max \{0, (p_i - p_{i \min})\} + \max \{0, (p_i - p_{i \min})\} \} = 0 \]

It should be noted that when \( p_i \) takes an in-feasible value, all quantities inside the square brackets in (18) become positive and therefore, the equality constraint is not satisfied. In the opposite case, once \( p_i \) takes a feasible value, one of the quantities contained by the square brackets becomes zero, so the equality constraint is satisfied in this case. Considering the non-convex nature of cost problem, we form the dual problem using the following sharp augmented La Grange function:

\[ L(x,u,c) = F_T(x) + c[h(x)] - u_h(x) \]
\[ = F_T(x) + c([h_1(x)]^2 + [h_2(x)]^2) \]
\[ + \ldots + [h_{eq}^n(x)]^{1/2} - (u_1h_1(x) + u_2h_2(x)) \]
\[ + \ldots + \cup_{eq} h_{eq}^n(x) \]

\[ H(u,c) = \min_{x \in K} L(x,u,c) \]

Then the dual problem is given by

\[ \text{Modified sub-gradient based combined objective technique and evolutionary ... (Susanta Kumar Gachhayat)} \]
For the given dual problem, the conditions of guaranteeing zero duality gaps are proven. The FMSG algorithm which was recently developed by Kasimbeyli is used to solve the dual problem given in this paper. It is a generalized version of modified sub-gradient algorithm.

3. THE MSGBCAEP ALGORITHM

The independent (decision) variables of the method are made up of voltage magnitudes and phase angles of the buses (except reference bus), the tap setting of the off nominal tap ratio transformers and the Susceptance values of the SVAR systems in the network. The method uses an augmented La Grange function that is called sharp LaGrange function. The FMSG Algorithm proposed to solve the dispatch problem is described in Section 2 involving the modified sub gradient method based on feasible values given in [3, 12]. The fitness function obtained from modified sub gradient method is used to create population for EP method which finally undergo mutation using Gaussian distribution function and selection process to obtain best feasible optimal solution. Using final value of selected cost function real power generation for various units are found out. MSGBCAEP process is repeated till cost function is obtained within desired accuracy.

3.1. Numeric example

In this section the proposed technique is going to be tested on a non-convex dispatch problem with ramp rate and prohibited zone constraints which were solved by heuristic method earlier. The test system includes IEEE 30 Bus 6 generators system shown in Figure 1. The simulation program is coded in MATLAB 2010. A PC with Intel core to duo, 2.20 GHZ CPU and 4 GB RAM is used for the proposed method. The cost curve coefficients and the emission curve coefficients corresponding to IEEE 30 bus test case system were tabulated in Table 1 for a 6 generator system. The detailed information on bus data and line data of IEEE 30 bus test case system shown in Figure 1 has been illustrated in Tables 2 and 3 respectively.

![Figure 1. IEEE 30 bus test case system](image)

<table>
<thead>
<tr>
<th>GENERATOR NUMBER</th>
<th>m1</th>
<th>m2</th>
<th>m3</th>
<th>m4</th>
<th>m5</th>
<th>P Max</th>
<th>P Min</th>
<th>Prohibited zones</th>
</tr>
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<tbody>
<tr>
<td>G1</td>
<td>0.00369</td>
<td>1.90</td>
<td>0</td>
<td>17</td>
<td>0.0310</td>
<td>300</td>
<td>10</td>
<td>363</td>
</tr>
<tr>
<td>G2</td>
<td>0.0169</td>
<td>1.69</td>
<td>0</td>
<td>15</td>
<td>0.032</td>
<td>96</td>
<td>4</td>
<td>116.16</td>
</tr>
<tr>
<td>G5</td>
<td>0.0619</td>
<td>0.90</td>
<td>0</td>
<td>13</td>
<td>0.03</td>
<td>60</td>
<td>3</td>
<td>72.6</td>
</tr>
<tr>
<td>G8</td>
<td>0.0077</td>
<td>3.19</td>
<td>0</td>
<td>11</td>
<td>0.039</td>
<td>42</td>
<td>2</td>
<td>50.82</td>
</tr>
<tr>
<td>G11</td>
<td>0.019</td>
<td>2</td>
<td>0</td>
<td>12</td>
<td>0.036</td>
<td>36</td>
<td>2</td>
<td>43.56</td>
</tr>
<tr>
<td>G13</td>
<td>0.019</td>
<td>2</td>
<td>0</td>
<td>12.5</td>
<td>0.037</td>
<td>48</td>
<td>2.4</td>
<td>58.08</td>
</tr>
</tbody>
</table>

Table 1. Cost and emission curve coefficients with minimum, maximum limits and prohibited operating zones
Modified sub-gradient based combined objective technique and evolutionary ... (Susanta Kumar Gachhayat)
The bus Number 1 was considered as slack bus with a voltage of 1.06<0 p u. The lower and upper limits of voltage magnitudes were chosen in between 1.0 p u and 1.082 p u respectively. The parameters for the FMSG algorithm are chosen as $\lambda = 1$, $\varepsilon = 0.001$, $u = [1.0, 1.1, 1.2, 1.3, 1.4, 1.5]$ and $c = 0.5$. The calculated initial total cost rate values for the given data are shown in Table 4. The non-convex generation dispatch problem for IEEE-30 bus test case system involving ramp rate constraints and valve point loading is solved involving evolutionary programming technique where in the decision variable is $\lambda$ that is incremental fuel cost instead of real power generation. This EP technique [9] involving $\lambda$ as decision variable and $F(P_j)$ as fitness function yields better optimal heuristic results over the traditional $\lambda$ iteration method and particle swarm optimization method.

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>$P_i$ (MW)</th>
<th>$F(P_i)$ (Rs./hr.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.08</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.289</td>
<td>1050</td>
</tr>
<tr>
<td>3</td>
<td>0.392</td>
<td>1100</td>
</tr>
<tr>
<td>4</td>
<td>0.482</td>
<td>1190</td>
</tr>
<tr>
<td>5</td>
<td>0.767</td>
<td>1310</td>
</tr>
<tr>
<td>6</td>
<td>0.92</td>
<td>1530</td>
</tr>
</tbody>
</table>

The optimal total cost rate and solution time values produced by FMSG and other heuristic methods like combined cycle particle swarm optimization (CCPSO), Group search optimizer (GSO) [13], Constraint treatment strategy particle swarm optimizer (CTPSO), chaotic sequences [21, 22] and cross over operation algorithm based particle swarm optimization (CCPSO), Continuous quick group search optimizer (CQGSO) and evolutionary programming (EP) techniques were compared and tabulated in Table 5. It is found that FMSG yields better optimal solution for prohibited operating zone accompanied by ramp rate and valve point constraints for a multi objective generation dispatch problem. It is seen through Figure 2 that FMSG yields a total cost rate of 1657Rs/hr. Over a solution time of 28.06 seconds while other heuristic methods like combined cycle particle swarm optimization (CCPSO), Group search optimizer (GSO), Constraint treatment strategy particle swarm optimizer (CTPSO), chaotic sequences and cross over operation algorithm particle swarm optimization (CCPSO), continuous quick group search optimizer (CQGSO) [23, 24] and evolutionary programming (EP) method etc. yield 1658 Rs./hr, 1728Rs./hr, 1639Rs./hr, 1658 Rs./hr and 1667 Rs./hr for solution times of 150 seconds, 53.80 seconds, 100 seconds, 150 seconds and 31.67 seconds respectively. Therefore, it is quite conspicuous that MSGBCAEP method outperforms the other heuristic methods involving non smooth cost functions, multi-objectives and valve point loading [25-27] in terms of total cost rate and solution time as well.

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>$P_i$ (MW)</th>
<th>$E(P_i)$ (MMT/hr.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.08</td>
<td>15.33</td>
</tr>
<tr>
<td>2</td>
<td>0.289</td>
<td>12.5</td>
</tr>
<tr>
<td>3</td>
<td>0.392</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>0.482</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>0.767</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>0.92</td>
<td>2</td>
</tr>
</tbody>
</table>

4. RESULT ANALYSIS

Results for Generation Cost rate vs. active power generation and emission level vs. active power generation for 6 generator IEEE 30 bus test case system through MSGBCAEP approach were tabulated in Table 5. The performance of MSGBCAEP method for cost rate and emission level rate was compared with other heuristic methods and the results were tabulated in Table 6.Classical methods like lambda iteration method and other heuristic methods could not meet out multiple constraints to near global so proposed approach because of fast judicious modification of initialization of parent population independent of crossover mechanism, resulted global optimal solution with less computational time for cost and emission objective functions. The proposed method can be applied into the areas subjected to multi valve, multi fuel, prohibited zones present in interconnected hybrid power systems subjected to multi capacity and multifaceted generating units having multi constraints for obtaining global optimal solution.
5. PERFORMANCE CHARACTERISTICS

Figure 2 and Figure 3 show the simulation result for generation cost and emission level versus real power for the proposed MSGBCAEP model wherein for increased real power generation, the cost of generation and emission level are found less while compared with the results obtained by other heuristic methods such as CCPSO, CTPSO, GSO, CQGSO and EP etc. and are tabulated in Table 6.

<table>
<thead>
<tr>
<th>Method</th>
<th>MSGBCAEP</th>
<th>CCPSO</th>
<th>CTPSO</th>
<th>GSO</th>
<th>CQGSO</th>
<th>EP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal total cost rate (R/h)</td>
<td>1557961.345</td>
<td>1558868.730</td>
<td>1559962.730</td>
<td>1628151.168</td>
<td>1567962.727</td>
<td>1558962.717</td>
</tr>
<tr>
<td>Emission level in MT / hr.</td>
<td>1.33</td>
<td>1.45</td>
<td>1.51</td>
<td>1.58</td>
<td>1.62</td>
<td>1.35</td>
</tr>
<tr>
<td>Simulation Time (sec)</td>
<td>28.06</td>
<td>150</td>
<td>100</td>
<td>53.80</td>
<td>31.67</td>
<td>30.05</td>
</tr>
</tbody>
</table>

Figure 2. Cost of generation versus real power
Figure 3. Emission versus real power

6. CONCLUSION

The optimal fuel costs of all the three methods are closer to each other with insignificant difference. The optimal generation schedules of the proposed MSGBCAEP method and that of \( \lambda \) iteration method exactly match while the schedule obtained with the other method significantly differs. The number of iterations of the proposed algorithm is reduced greatly compared to the other methods. A comparative study of the computation time is given in the last row of Table 6. It may be observed that on an average computation time is reduced by 90% which shows the computational effort of the proposed method. The above facts validate the statement that the proposed method is more efficient and fast converging as well. The MSGBCAEP approach developed in the present study can be utilized to solve a single objective as well as conflicting multi-objective power dispatch problems. The hill climbing algorithm and iterative gradient methods get trapped in local minima but have faster convergence. MSGBCAEP approach has global search characteristics and robustness that prevent them from getting trapped in local optima unlike hill climbing algorithm. Hence evolutionary algorithms can be applied in the initial stages of the solution for global search and the hill climbing algorithms can be applied in the later stages for fine local search. This MSGBCAEP approach may be able to combine the advantages of both FMSG and EP approaches and find optimal solutions more rapidly compared to using a single search technique throughout. The proposed work can be extended to solve different kinds of economic dispatch problems. The MSGBCAEP approach developed in this work will be extremely useful for electric power utilities with valve point loading, in enhancing the economy and security of operation in their systems. The comparison of the results obtained using the proposed method with those of outlined heuristic methods in Table 6 shows that evolutionary programming approach always leads to global or near global optimum solutions.

REFERENCES


**BIOGRAPHIES OF AUTHORS**

**Prof. Susanta Kumar Gachhayat** procured his UG degree from IE (India). Completed Master’s Program from SRM university, Chennai and working as an Asst. Professor in the Dept. of EE, KIST, Bhubaneswar. At present he is continuing PhD in BPUT under the guidance of Professor Dr. S. K. Dash.

**Professor Dr. S. K. Dash** received the UG degree in Electrical Engineering from I.E, India in 1991 and accomplished Master’s Program in electrical engineering from UCE, Burla (Sambalpur University), India, in 1998 and the PhD degree from Utkal University, Odisha, India in the year 2006. He has been with the Electrical Engineering Department, Gandhi Institute for Technological Advancement as a Professor and Head of the Department since 2005. Prior to it he worked in industry for 5 years and in OSME, Keonjhar, for 2 years and in Krupajal Engineering College for 4 years. His research areas include power system planning, operation, and optimization techniques relating to power system dispatch. Dr. Dash received Pundit Madan Mohan Malaviya award, Union Ministry of Power Prize and gold medals thereof for his research papers entitled “Economic load dispatching of generating units with multi-fuel options” and “Short term generation scheduling with take or pay fuel contract using evolutionary programming technique” on Multi Objective Generation Dispatch. He too authored two books entitled ‘Fundamentals of Electromagnetic Field Theory’ and ‘Basic Electrical Engineering’ under the umbrella of PHI Publication and YESDEE publication in the year 2010 and 2016 respectively. Dr. Dash is engaged as a reviewer of EPCS, and EPSR journals of IEEE. He is the recipient of best session chair award in the World Academy of Science, Engineering and Technology (WASET) Conference held in SANFRANCISCO, USA during 7th June 2017 and also received thereof best paper presentation award for two of his research papers on PSO based economic dispatch and EP based economic dispatch. He is too awarded with energy conservation award in the year 2015 and 2016 as well for bringing solar power plants in GITA, Bhubaneswar.