Efficient power allocation method for non orthogonal multiple access 5G systems

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ABSTRACT

One of the hot research topics for the upcoming 5G (fifth-generation) wireless communication networks is the non orthogonal multiple access (NOMA) systems, where it have attracted both industrial and academic fields to improve the existing spectral efficiency. In fact, the multiuser detection process for NOMA systems is largely affected by the power distribution of the received signals. In this paper, a new method has been proposed to control the transmit power among active users in one of the promising NOMA systems; the interleave division multiple access (IDMA) which has been adopted here for consideration. Unlike conventional methods, where tedious mathematical computations are required; a simple and direct method has been derived. The proposed method has been applied to IDMA system with different FEC codes. The obtained results show that the proposed method outperforms the conventional one as compared to optimal results.

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1. INTRODUCTION

Researchers have considered multiuser detection techniques for spread spectrum wireless communications systems and, recently, for non orthogonal multiple access (NOMA) systems. In fact, the multiuser detection process is largely affected by the power of the received signals [1-3]. Thus, managing these powers is crucial for interference suppression and the stability of the NOMA systems [4, 5]. In addition, one of the major design concerns is the power consumption reduction for mobile devices. Thus, minimization of total power received by uplink receiver is required while maintaining the SINR (signal to interference plus noise ratio) on certain desired level at final iteration of multiuser detector. That is so called the ‘power allocation problem’. Recently, researchers spare no effort to solve the power allocation problem in NOMA system [1,2,6-9], while early researches on power allocation problem were equally important that can be found elsewhere [10-15].

Although the methods that have been proposed in the literature have involved difficult computations; they did not lead to exact solutions as one may expect. This, in turn, encouraged me to derive a simple method to solve power allocation problem in IDMA as one of the promising NOMA systems for 5G. The contribution of this paper can be summerized with the following points:

(a) This paper presents a method which could be an efficient alternative to the existing power allocation methods for IDMA system.
(b) The proposed method draws its strength from being simple and direct. It depends only on the forward error correcting (FEC) rate to solve power allocation problem in IDMA system; whereas other methods usually use linear programing or complex analytical approaches.

This paper is organized as follows: after the introduction, the IDMA uplink system is briefly described in section 2 in order for a reader to focus only on the power allocation problem which is presented in section 3. Whereas a modelling of the proposed method is considered in section 4. Numerical results are then discussed in section 5. Finally, the conclusions are drawn in section 6.

2. IDMA UPLINK SYSTEM

Interleave Division Multiple Access (IDMA) is a promising non orthogonal multiple access system [16-19], so it has been adopted for consideration in this research. It can be considered as a special case of spread spectrum systems. In IDMA system the users are identified by their unique interleavers [20-27], while in spread spectrum systems such as CDMA the users are identified by their unique codes. The mechanism of the IDMA uplink system can be clarified according to the schematic diagram shown in Figure 1.

At the transmit side, the information bits of each user are encoded using certain FEC (forward-error-correction) codes that are assumed the same for all users. Each FEC code is achieved by serial concatenation of convolutional code of rate \( R_c \) along with a repetition code of rate \( R_p \) resulting a total FEC code rate of \( R_t = R_c R_p \). Consequently, the system can be considered to have a total spreading factor \( SF = 1/R_t \). Figure 2. illustrates some well-known FEC codes that have been used in this paper. After that, the encoded bits (also called chips) are organized into frames each has length of 1024; to be prepared for the interleaving process where each user is assigned a unique interleaver. In IDMA, the key principle is that the user specific interleavers should have minimum cross correlation and must be generated randomly and independently. In fact, these user specific interleavers make a dispersion in the encoded sequences which produces an approximately uncorrelated adjacent chips; this in turn facilitates the receiver detection process. The interleavers at receiver and transmitter sides should be stored somewhere at the base station by utilizing a memory prepared for this purpose. Throughout the duration of the initial link setting-up stage, the base station and the mobile stations must exchange messages to tell each other about the employed interleavers. In this paper, in order to focus on the power allocation problem, it is assumed that the employed interleavers are optimum.
The interleaver output is then scaled by coefficient for each user \( h_k \); which is assumed to constitute the combined effect of the channel loss and power control factor. In other words, the distribution of \( h_k^2 \) can be fitted via power control process. The signals of all active users are assumed to be superimposed into a wireless flat fading channel. At the receive side, the receiver is composed of elementary signal estimator (ESE) along with a bank of \( K \) decoders (DECs) corresponding to \( K \) users. The DECs are a posteriori probability (APP) decoders. A related point to consider, is that the operation of the receiver is mainly based on multiple access channel and the FEC code constraints. The outcomes of the ESE and DECs are combined, via an iterative process, to retrieve the transmitted bits. It is suitable here to mention that we have designed and achieved the IDMA uplink system simulator using MATLAB, but it has not been included in this paper to avoid redundancy.

3. PROBLEM FORMULATION

As mentioned before, the performance of IDMA system can be enhanced through suitable power allocation. The proper power distribution among users will highly increase the chance for correct signal detection at the receiver. Strong signals are firstly detected. Consequently, their interference effect on weak signals can be correctly removed which in turn increase the chance of weak signal detection.

The main research question here is: how to allocate the power levels among users?. In general, the power allocation problem can be solved by dividing the total number of multiple access users into groups, where for each a certain power level is assigned. As a consequence, the former research question triggers the following three sub-questions:

(a) What is the proper size for each group?
(b) What is the proper power level for each group to achieve feasible target BER?
(c) If 1 and 2 are satisfied, has the total transmitted power \( P_t \) from all active users been minimized as a result?

Early researches tried to solve this issue; for instance Schlegel et al. [28] proved that the minimum total power can be achieved when the groups have an equal number of users; so one can adopt this assumption as a foundation to meet other sub-questions. Therefore, the objective of this research becomes how to find the proper power level for each group so that the minimum total power condition is satisfied with feasible BER (typically \( 10^{-6} \)).

4. MODELLING OF PROPOSED METHOD

Unlike the existing researches, this paper have revealed a new feature for FEC codes which can be utilized to solve the power allocation problem. The feature tells us that all FEC codes can support limited number of users (users with same power level). However, exceeding this limit leads to degradation in system performance. Figure 3 shows the simulation results of the BER with number of equal power users for different FEC codes. One can observe that for each FEC code, a target BER of \( 10^{-6} \) can be achieved as long as the number of users do not exceed certain threshold say \( k_{max} \). Above this threshold, the BER jumps to infeasible values.

In this work, such a feature has been exploited to solve the power allocation problem. It is obvious, from the former figure, that \( k_{max} \) for the codes FEC1, FEC2, and FEC3, are 12, 13, and 18, respectively.

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Figure 2 demonstrates some FEC codes with their corresponding $k_{\text{max}}$s. To clarify our proposed method, it is suitable to discuss the following example. Assume an IDMA (NOMA) system with BPSK mapping of $SNR = 10dB$ and with the following specifications: the total number of users ($K$) is 32; a FEC code consisting of convolutional code of rate 1/2 serially concatenated with a repetition code of length 8 (i.e $R_p = 1/8$). Thus the total FEC rate is $R_t = 1/16$.

In order to distribute the power among active users; first, one should divide the total number of users $K$ into groups. We start with the case of 32 groups, each group consists of one user. Now, assume that the power set $\{p_1^*, p_2^* ... p_{32}^*\}$ represents a solution to the power allocation problem. The power set should satisfy two conditions: a minimum total power; and convergence to feasible BER. By assuming that the up-link users have the same transmission rate, one can prove that the power set should follow a constant ratio according to [29]; hence:

$$p_2^*/p_1^* = p_3^*/p_2^* = \ldots = p_K^*/p_{K-1}^* = \Psi$$

where $\Psi$ represents power ratio. Assume $p_1^*$ represents the minimum transmit power allowed for each user ($p_{\text{min}}$), hence the power set can be rewritten as:

$$\{p_{\text{min}}, \beta p_{\text{min}}, \beta^2 p_{\text{min}}, \ldots, \beta^{31} p_{\text{min}}\}$$

or

$$p_{\text{min}} \{1, \beta, \beta^2, \ldots, \beta^{31}\}$$

The power ratio ($\Psi$) in this case is equal to $\beta$. It is worth mentioning that $p_{\text{min}}$ can be determined according to the wireless system requirements. Thus, solution of the power allocation problem is mainly achieved by finding $\beta$. Since total power should be minimized, hence the power allocation problem can be solved by finding the minimum $\beta$ (i.e. $\beta_{\text{min}}$), or for the sake of generality $\Psi_{\text{min}}$.

![Figure 3. BER vs. number of equal power users](image)

**4.1. Finding minimum power ratio ($\beta_{\text{min}}$)**

Going forward to find $\beta_{\text{min}}$, assume that $g$ represents the number of users per group (or the group size). Thus, for the same total number of users $K = 32$, in order to increase the group size from 1 to 2 one should raise the power ratio $\Psi$ from $\beta$ to $\beta^2$ as shown in Figure 4; starting from rightmost column. This is logically acceptable if one thinks of it as to ensure that each group will have enough power for converging to the target BER. By the same way, in order to increase $g$ to 4, the power ratio $\Psi$ should be increased to $\beta^4$, and so on. Figure 4 shows the possible group sizes along with the corresponding power ratios. In general, one can observe that for certain group size ($g$); the corresponding power ratio should be $\beta^g$. Figure 5 illustrates the effect of group size on the power ratio. It is obvious that as the group size increases, the required power ratio also increases. While for certain FEC code, as the group size approaches $k_{\text{max}}$, the required power ratio fairly increases. Thus, in order to avoid unwanted increase in total power, we recommend that the maximum group size should not approach $k_{\text{max}}$ for given FEC code.
Total number of users = 32

<table>
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<tr>
<th>2 groups</th>
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<th>32 groups</th>
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<td>g = 16 users/group</td>
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<td>$\beta^{31}$</td>
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</tbody>
</table>

$P_{t1} = g \ P_{\text{min}} (\beta^0 + \beta^1 + \beta^2 + \cdots + \beta^{31})$

$P_{t2} = g \ P_{\text{min}} (\beta^0 + \beta^2 + \beta^4 + \cdots + \beta^{30})$

$P_{t3} = g \ P_{\text{min}} (\beta^0 + \beta^4 + \beta^8 + \cdots + \beta^{28})$

$P_{t4} = g \ P_{\text{min}} (\beta^0 + \beta^8 + \beta^{16} + \beta^{24})$

$P_{t5} = g \ P_{\text{min}} (\beta^0 + \beta^{16})$

Figure 4. Estimated power ratios for different group sizes

Figure 5. Power ratio vs. group size for different FEC codes
The important note here is that for certain FEC code (say FEC3), the sum of the users’ normalized powers should not exceed $k_{\text{max}}$ as seen by the receiver; otherwise the BER jumps to infeasible values as shown in Figure 3. So:

$$\frac{P_{\text{Total}}}{P_{\text{max}}} \leq k_{\text{max}}$$ (1)

Now, assume that the number of groups is decreased from 32 down to 2. Thus, each of the resulting two groups would have the maximum number of users $g_{\text{max}}$ (where $g_{\text{max}} = 16$ in our example). Hence, one can derive:

$$\frac{(g_{\text{max}}P_{\text{min}} + g_{\text{max}}P_{\text{max}})}{P_{\text{max}}} \leq k_{\text{max}}$$ (2)

where in this case $P_{\text{max}} = \beta g_{\text{max}} P_{\text{min}}$, thus;

$$\frac{g_{\text{max}}}{\beta g_{\text{max}}} + g_{\text{max}} \leq k_{\text{max}}$$ (3)

which leads to;

$$\beta \geq \left( \frac{g_{\text{max}}}{k_{\text{max}} - g_{\text{max}}} \right) g_{\text{max}}$$ (4)

Since total power should be minimized, which means minimum power ratio among groups, hence one can write;

$$\beta_{\text{min}} = \left( \frac{g_{\text{max}}}{k_{\text{max}} - g_{\text{max}}} \right) g_{\text{max}}$$ (5)

where $g_{\text{max}} < k_{\text{max}}$. Knowing that $g_{\text{max}}$ and $k_{\text{max}}$ are integers, and $g_{\text{max}}$ should have maximum value; thus $g_{\text{max}} = k_{\text{max}} - 1$. Substituting for $g_{\text{max}}$;

$$\beta_{\text{min}} = (k_{\text{max}} - 1) \left( \frac{g_{\text{max}}}{k_{\text{max}} - g_{\text{max}}} \right)$$ (6)

It is obvious that one can find a solution for power allocation problem for any FEC code by knowing its $k_{\text{max}}$, which is the objective of this research. However, by knowing that every FEC code has its own $k_{\text{max}}$, which may be somewhat difficult to find; hence it is preferable if one can make further simplification to equation (6).

4.2. Simplification of equation (6)

In order to understand (6), it is preferable to study its characteristics. Figure 6 shows the variation of the minimum power ratio $\beta_{\text{min}}$ versus range of $k_{\text{max}}$ integer values. One can observe that for $k_{\text{max}} > 4$ (which is the case in practical FEC codes), increasing $k_{\text{max}}$ always results in decreasing the power ratio $\beta_{\text{min}}$. This will in turn reduce the total power. Although larger values of $\beta_{\text{min}}$ ensure iterative receiver convergence to the target BER with fewer iterations, but it will be at the expense of unwanted increase in total power. Thus, a reasonable trade-off should be achieved.
In order to make a proper trade-off, Figure 3 should be studied carefully. By looking from the line \( N = 16 = SF \), one can note that strong FEC codes such as FEC1 and FEC2 have \( k_{max} < SF \). Whereas weak FEC codes (such as FEC3) have \( k_{max} > SF \). Thus, it may be suitable to consider \( SF \) as the average value for \( k_{max} \) for a range of FEC codes; from the strongest one to weakest.

According to the previous discussion, an acceptable trade-off can be achieved by replacing \( k_{max} \) in (6) by \( SF \). This replacement leads to simplify (6) to:

\[
\beta_{min}^* \approx (SF - 1)^{\frac{1}{SF - 1}}
\]  

Thus, in order to find the power ratio \( \beta_{min}^* \) we only need to know the total spreading factor \( SF = 1/R_{s}R_{g} \) rather than knowing \( k_{max} \) for each code. So, generally speaking, to find the minimum power ratio \( \Psi_{min} \) for certain group size \( g \), the rule can be rewritten in the following final general form;

\[
\Psi_{min}|_{g} = (\beta_{min}^*)^g \approx (SF - 1)^{\frac{1}{SF - 1}}
\]  

It is worth mentioning that the rule in equation (8) is derived under assumption that the users who undergo the power control process should utilize the same total FEC rate.

### 4.3. Proposed power allocation algorithm

Figure 7 shows the proposed power allocation algorithm. It starts by entering total number of users, total FEC rate, and the minimum transmit power allowed for each user. Then an initial value for group size \( g \) is assumed to be 1. After that, the algorithm runs to find the group size that satisfies the minimum total power; then allocates the power among users accordingly.

### 5. RESULTS AND DISCUSSION

To verify the ability of our proposed method to solve the power allocation problem, the method has been applied to different FEC codes. For the sake of simplicity, FEC3 is discussed. Figure 8 shows two methods of finding the power ratio; our proposed method (green color) and one of the conventional methods for NOMA (brown color) [1,2,4,6-10,28] compared to the corresponding optimum method (blue color). Compared with the optimal results, the proposed method seems in good agreement. However, saying "in good agreement" can not be considered as a measure. So, one should put a measure to describe the degree of agreement precisely. Thus, in this paper a term coincidence \( (C) \) has been used to represent this measure of agreement; more generally the percentage coincidence which can be expressed as \( C = (1 - \epsilon) \times 100\% \), where:

\[
\epsilon = \frac{\Psi_{estimated} - \Psi_{optimum}}{\Psi_{optimum}}
\]

\[
C = \left(1 - \left(\frac{\Psi_{estimated} - \Psi_{optimum}}{\Psi_{optimum}}\right)\right) \times 100\%
\]
Start

Enter
K: Total Number of Users.
Total FEC Rate: \( R_t = R_c \times R_p \).
\( P_{min} \): Minimum Transmit Power Allowed For User.

Calculate the Spreading Factor:
\[ SF = \frac{1}{R_t} \]

Set Counter
\( i = 1 \).

Distribute Users into Groups:
Group Size \( g = 2^{(i-1)} \).

Calculate the Minimum Power Ratio:
\[ \beta_{min} = (SF - 1)^{(\frac{1}{SF})} \]

Calculate the Power Ratio (\( \Psi \)) According to the Group Size:
\[ \Psi_{min}(g) = \beta_{min}^g \]

Calculate Total Power:
\[ P_t = g \times P_{min} \times \sum_{k=0}^{(i-1)} \beta_{min}^{(i-1)g} \]

Follow the Steps to Allocate the Power Among Users:
1- Distribute Users into \( g \) Groups.
2- Follow Table 2 for Suitable Power Allocation.

FIGURE 7. Proposed power allocation algorithm

FIGURE 8. Comparison among different methods of finding the power ratio for FEC3
Figure 9 illustrates the degree of coincidence of the proposed method as compared to the conventional one. It is clear that for the conventional method, the coincidence ($C$) is excellent for $g = 4$ or fewer; whereas it drops down for $g$ larger than 4. However, our proposed method shows better performance especially for $g$ larger than 4. Accordingly, one may decide to distribute users into small groups (low values of $g$) for the sake of maximum coincidence. However, in the power allocation problem some other parameters must be taken into account. One of these, is the total power consumed by active users ($P_t$). It is known that the purpose behind the power allocation is to distribute power among users to facilitate the multi-user detection process, with minimum total power consumption. Thus, one should choose minimum $P_t$. Figure 10 illustrates that our proposed method can achieve a minimum total power of $25dB$ at $g = 16$, whereas the conventional method achieves $27dB$ at $g = 1$.

![Figure 9. Percentage coincidence vs. $g$ for FEC3](image1)

![Figure 10. Relative total power vs. $g$ for FEC3](image2)

It is clear that our proposed method reduces the total power by $2dB$ at $g = 16$. According to Figure 8, $g = 16$ corresponds to significant increase in the power ratio (green color) as compared to the optimum one. Fortunately, this increase will facilitate the detection process by reducing the number of iterations needed to reach the target BER as shown in Figure 11.

![Figure 11. Receiver iterations vs. group size. A comparison between the proposed method and the simulation results for FEC3](image3)

It is important here to mention that the ‘optimum method’ denotes to the method that achieves the minimum total power regardless the number of receiver iterations required to reach target BER. Now, referring to Figure 11, our proposed method shows considerable reduction in the number of iterations over the possible values of $g$. This can be interpreted as follows: from the iterative receiver point of view, the increase in the power ratio $\Psi$ increases the receiver ability to discriminate among users groups. This in turn
enhances the receiver ability to converge to the target BER \(10^{-6}\) with fewer number of iterations. In other words, it means reducing the time required by the receiver to achieve its assigned task. Figure 12 illustrates the percentage reduction in the time required by the receiver to converge to the target BER which is called here “Task Time Reduction”. It is clear that choosing \(g = 16\) can reduce task time by 47%.

Moreover, to further verify the validity of our proposed method; we have applied it to different FEC codes. Figure 13 and Figure 14 show the relative total power graphs for FEC2 and FEC1, respectively. It is clear that our proposed method outperforms the conventional one by increasing the FEC code constraint length. In other words, the proposed method performs better as the FEC code becoming stronger.

![Figure 12. The percentage Task Time reduction vs. \(g\) for FEC3](image)

Figure 12. The percentage Task Time reduction vs. \(g\) for FEC3

![Figure 13. Relative total power vs. \(g\) for FEC2](image)

Figure 13. Relative total power vs. \(g\) for FEC2

![Figure 14. Relative total power vs. \(g\) for FEC1](image)

Figure 14. Relative total power vs. \(g\) for FEC1

6. **CONCLUSIONS**

In this paper, a new method has been derived to simplify the solution of the power allocation problem in one of the promising NOMA systems; that is the IDMA system. Compared with the conventional methods, the proposed method is proved to be efficient and easy to apply because it needs only total system FEC rate to find a solution to the power allocation problem. Finally, the obtained numerical results show that this simple method is more feasible than its counterparts and could be applied right away.

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