BER analysis of amplify-and-forward relaying FSO systems using APD receiver over strong atmospheric turbulence channels

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ABSTRACT
In this paper, we theoretically analyze the performance of amplify-and-forward (AF) serial relaying free-space optical (FSO) systems using avalanche photodiodes (APD) and subcarrier quadrature amplitude modulation (SC-QAM) over strong atmospheric turbulence channels modelled by gamma-gamma distribution. More specifically, an average bit error rate (BER) of system is theoretically derived taking into account APD shot noise, thermal noise as well as the impact of the number of AF relaying stations and turbulence. The numerical results show that using AF relay stations can extend the transmission distance and help to improve performance of FSO system significantly when compared with the direct transmission. Moreover, the selection of APD gain value is indispensable to the system performance. The proposed system could be achieved the best performance by selecting an optimal APD gain value. In addition, the optimal value of APD gain also significantly depends on various conditions, such as the number of relay stations, APD receiver noise and link distance.

Keywords:
Amplify-and-Forward
APD
Atmospheric turbulence
FSO
QAM

1. INTRODUCTION
Free-space optics (FSO) is known as green communication technology, in recent years for a lot of applications because of their ability to unlimited bandwidth, licensing-free requirements, high security, cost-effectiveness and simplicity of communication system design and deployment [1-2]. Although FSO communication system are many advantages like that, but the performance of FSO systems strongly depends on atmospheric turbulence conditions, which is cause by the temperature and pressure inhomogeneity of the atmosphere [3], when an optical beam traversing the atmosphere. It will negatively effects of scattering, absorption and turbulence result in power loss and the link distance limited over several kilometers [4].

In addition, to improve the low link distance transmission over the turbulence channel, relaying FSO systems have been proposed to extend the longer transmission and mitigate the turbulence included fading [5]. Outage performance of multihop FSO systems over strong turbulence channels modelled by the gamma-gamma distribution, has been studied in [6-7]. Most recently performance of relaying FSO systems over atmospheric turbulence channels has been studied focus on Amplify-and-Forward (AF) and Decode-and-Forward (DF) relaying FSO systems. The aim of this study, we theoretically analyze the performance FSO scheme AF serial system. In the AF relaying method, the receiver signal from source at each node will be
amplified before retransmitted to next relay node and repeated through intermediate nodes to the destination node, and AF relaying method is a potential solution to increase link distance transmission.

Previous works, FSO systems have mainly been implemented by employing OOK or PPM modulation techniques for commerce because of the simplicity and low cost for OOK and superior power efficiency for PPM modulation. However OOK modulation needs an adaptive threshold that is difficult adjustment to accomplish in the presence of atmospheric turbulence, also PPM modulation technical has a poor bandwidth efficient and require high transceiver synchronization [8-9]. To overcome the limitations of OOK and PPM, subcarrier intensity modulation quadrature amplitude modulation (SC-QAM) has been recently proposed. The performances of FSO system using SC-QAM over lognormal and gamma-gamma turbulence channels have been found in some recent studies [10-12]. Most recently, ASER performance analysis of MIMO/FSO systems using SC-QAM signaling over atmospheric turbulence channels have been reported [13].

Others technique can significantly improve the performance of FSO systems in receiver path is use avalanche photodiodes (APDs). APD are widely used because they provide higher values of responsivity compared with PIN photodiodes. The characteristics of APD devices and the performance of APD-based receivers have been studied extensively [14-16]. The use of APD for the cases of FSO systems using QAM modulations was reported in [17], Furthermore APD-based Amplify-And-Forward Serial Relaying FSO Systems has been studied in [18]. There has been, however, no study on BER analysis of AF relaying FSO system using APD receiver.

In this paper, we therefore analyze BER of AF relaying FSO/SC-QAM system using APD receiver over strong atmospheric turbulence channel. Moreover we obtain important performance for the practical system design, such as the effects of APD gain, transmitted power and link distance on the BER of FSO/SC-QAM systems over strong atmospheric turbulence channel.

2. SYSTEM DESCRIPTIONS

2.1. AF relaying FSO system using QAM signal

We start by investigating a typical serial an APD-based AF relaying FSO system, in which signal from the source node is transmitted to the destination through $N$ relaying nodes serially as shown in Figure 1. The source node, relaying node, and destination node schemes are illustrated in Figure 2.

In Figure 2a, which system was employed SC-QAM, electrical data from the signal source is firstly up-converted to an intermediate frequency $f_c$. This electrical subcarrier QAM signal is then used to modulate the intensity of a Laser.

Figure 1. An example of a serial relaying FSO communication system

![Figure 1. An example of a serial relaying FSO communication system](image)

![Figure 2. The source node, relaying node and destination node of FSO/QAM systems](image)
The electrical SC-QAM signal at the output of QAM modulator can be written as:

\[ e(t) = s_i(t) \cos(2pf_it) - s_Q(t) \sin(2pf_it) \]  \hspace{1cm} (1)

where \( s_i(t) = \sqrt{\frac{a}{j - \Delta}} \) and \( s_Q(t) = \sqrt{\frac{b}{j - \Delta}} \) are the in-phase and the quadrature signals, respectively, \( a(t) \), \( b(t) \) are respectively the in-phase and the quadrature component information amplitudes of the \( j-th \) transmitted data symbol, \( g(t) \) is the shaping pulse and \( T_s \) denotes the symbol interval.

The QAM signal is then used to modulate the intensity of an electrical-to-optical (E/O) laser before pointing laser beam through a telescope of the transmitter to the relaying node, the transmitted signal can be expressed as:

\[ s(t) = P_s \{ 1 + k \{ s_i(t) \cos(2pf_it) - s_Q(t) \sin(2pf_it) \} \} \]  \hspace{1cm} (2)

where \( P_s \) denotes the average transmitted optical power per symbol at each hop and \( k \) is the modulation index, \( 0 < k < 1 \). Due to the effects of link loss \( a_i \) and \( X_i(t) \) - the random process for the signal scintillation caused by atmospheric turbulence of the first hop, the received optical signal at the first relay node can be expressed as

\[ r_i(t) = a_i X_i(t) P_s \{ 1 + k \{ s_i(t) \cos(2pf_it) - s_Q(t) \sin(2pf_it) \} \} \]  \hspace{1cm} (3)

At each relay node, an APD receiver is used for amplifying signal as shown in Figure 2b. The received optical signal is firstly converted into electrical signal and then amplified by the APD, after has been amplified by the APD, this signal is used to re-modulate intensity of laser source and relayed to the next relay node. Due to slow turbulence changes, the DC component \( \{ a_i X_i(t) P_s \} \) can be filtered out by a bandpass filter. Therefore the electrical signal output of AF module at the first relay node will be

\[ e_1(t) = \sqrt{\frac{\lambda}{\lambda}} a_i X_i(t) P_s k e(t) + n_i(t) \]  \hspace{1cm} (4)

where \( \sqrt{\lambda} \), \( \lambda \), \( n_i(t) \) are the average APD gain, responsivity of photodiode and receiver noise at the first node, respectively.

The total receiver noise \( n_i(t) \) at the first node consisting of APD shot noise, thermal noise and can be modelled as a stationary random Gaussian process whose variance is given by \[19\]

\[ s_i^2 = 2q \sqrt{\frac{\lambda}{\lambda}} F_p k D_f a_i X_i + \frac{4k_B T}{R_i} F_n D_f \]  \hspace{1cm} (5)

where \( k_B \), \( T \), \( R_i \), \( F_p \), \( q \), \( \Delta f \), \( F_n \) represent the Boltzmann constant, the absolute temperature of receiver, the APD’s load resistance, the amplifier noise figure, the electron charge the symbol’s effective noise bandwidth and the excess noise factor. Where \( \Delta f = R_i / 2 \log_2(M) \) with \( R_i \) is the bit rate of the system and \( M = M_f \times M_Q \) and \( F_A = k_A \log(1 + k_A) (2 - 1/\gamma) \) with \( k_A \) denoting the ionization factor.

With AF relaying scheme, relay node first normalizes the received optical signal in (4) to unity and then is optically modulated with power \( P_s \) in order to transmitted power per symbol at the \( i-th \) node equal with transmitted power per symbol at the source and re-transmitted to the next node. The transmitted optical signal at the first node therefore can be expressed as \[18\]

\[ s_i(t) = P_s \{ 1 + X_i(t) k e(t) + \frac{n_i(t)}{A \sqrt{\lambda} a_i P_s} \} \]  \hspace{1cm} (6)

Repeating such steps above through the number of relay stations, \( N \), the electrical signal in the output of APD at the destination node can be derived as following
\[ e(t) = \tilde{g} \tilde{A} P a_{N+i} \bigg( \sum_{i=1}^{N+i} X_i(t) k e(t) + \tilde{a} \bigg) \frac{n_i(t) a_{N+i} \bigg( \sum_{j=i+1}^{N+i} X_j(t) \bigg)}{a_i} \] (7)

where \( a_i, X_i(t) \) are the path loss and atmospheric turbulence of the connection between the \((i-1)\)-th and \(i\)-th node respectively. Whereas \( n_i(t) \) is the receiver noise at the \(i\)-th node. Also the variance of total accumulated receiver noise at the destination node is given by

\[ s_{N+i}^2 = \tilde{a} \sum_{i=1}^{N+i} \frac{\tilde{A} P a_{N+i} X_i^2}{a_i} \bigg( 2qg^2 \tilde{A} P a_{N+i} X_i^2 + \frac{4kT}{R_L} F_n D_f \bigg) \] (8)

Similar to (7) and (8), the instantaneous electrical SNR per symbol, \( \gamma \), is the random variable at the output of the APD, as a function of instantaneous fading value \( X \), with system and channel parameters can be expressed as follows

\[ g = \frac{1}{s_{N+i}^2} \left( \sum_{i=1}^{N+i} X_i(t) \right)^2 = \frac{1}{s_{N+i}^2} \left( \sum_{i=1}^{N+i} X_i(t) \right)^2 \] (9)

2.2. Channel model

When an optical wave beam go through the free space atmosphere, it experiences the deformation caused various atmospheric condition, such as scattering, refraction, and absorption between the terminals of FSO link. In this study, we consider the factors characterizing an FSO channel including link loss, and atmospheric turbulence. The link loss is caused by both molecular absorption and aerosol scattering in the air. The total link loss of system can be expressed as

\[ a_s = \frac{A_s}{p L \frac{\phi^2}{2}} e^{-h_i L} \] (10)

where \( A_s, \phi, L, B_i \) denote the area of the receiver aperture, the optical beam’s divergence angle in radian, the link length between the source and destination and the atmospheric extinction coefficient, respectively [20].

The atmospheric turbulence caused the irradiance fluctuation represents signal scintillation. There are many distribution models to describe the atmosphere-induced turbulence. For strong turbulence condition, it is generally accepted that \( X(t) \) is a random process with gamma-gamma distribution. The probability density function (pdf) of the irradiance intensity in the strong turbulent is given by [13].

\[ f_X(X) = \frac{2(a \beta) \Gamma \left( \frac{\alpha+\beta}{2} \right)}{\Gamma(\alpha) \Gamma(\beta)} X^{\frac{\alpha+\beta}{2}-1} \Gamma_{\alpha-\beta} \left( 2a \beta X \right) \] (11)

where \( \Gamma(\cdot) \) is the Gamma function and \( \Gamma_{\alpha-\beta}(\cdot) \) denotes the modified Bessel function of the second kind of order \((\alpha-\beta)\). The positive parameter \( \alpha \) represents the effective number of large-scale cells of the scattering process, and the positive parameter \( \beta \) represents the effective number of small-scale cells of the scattering process in the atmosphere. It can be shown that the parameters \( \alpha \) and \( \beta \) are directly related to atmospheric conditions through the following expressions where \( \sigma_S \) is the scintillation index, and is defined at [13] as \( \sigma_S = \exp(\omega_1 + \omega_2) + 1 \); where

\[ \alpha = \left[ \exp \left( \frac{0.49\sigma_S^2}{1 + 1.87\sigma_S^2 + 0.56e^{2\sigma_S^2}\sigma_S^2} \right) - 1 \right]^{-1} \] (12)
\[
\beta = \left( \exp\left( \frac{0.51\sigma_1^2 + (1 + 0.69\sigma_2^{10/3})^{4/9}}{1 + 0.9d^2 + 0.62\sigma_2^{10/3}} \right) - 1 \right)^{1/4}
\] (13)

In these equations, \( d = \sqrt{kd^2/L} \) where \( k = 2\pi/\lambda \) is the optical wave number, \( \lambda \) is the wavelength, \( L \) is the link distance, and \( D \) is the radius of a circular receiving aperture, and \( \sigma_2 \) is the Rytov variance, and assuming spherical wave propagation, defined as:

\[
s_2^2 = 0.492C_n^2k^{7/6}L^{11/6}
\] (14)

In (14), \( C_n^2 \) is the refractive-index structure parameter, which is weather depended.

At the AF system, the probability distribution function of atmospheric turbulence through \( N \) nodes for FSO system is given in (15) [21].

\[
f_X(x) = \frac{2(\alpha\beta)^{\alpha+\beta/2}}{(N+1)\Gamma(\alpha)\Gamma(\beta)} X^{\alpha+\beta-N-1} K_{\alpha-\beta} \left( 2\alpha\beta \sqrt{x} \right),
\] (15)

3. BER ANALYSIS

In this section, we will analyze the BER performance of the AF relaying FSO/QAM system using APD receiver over strong atmospheric turbulence channel. The mathematical formula to perform bit error rate of QAM modulation can be found at:

\[
BER = \int_0^\infty x \cdot BER_{inst} f_X(x) dx
\] (16)

In which \( BER_{inst} \) instantaneous bit error rate of system in destination node, for the \( M_I \times M_Q \) rectangular QAM modulation, the instantaneous BER is given as

\[
BER_{inst} = \frac{1}{\log_2(M_I M_Q)} \left( \hat{\alpha} \log_2(M_I) P_{M_I}(k) + \hat{\gamma} \log_2(M_Q) P_{M_Q}(l) \right)
\] (17)

where \( P_{M_I}(k) \) and \( P_{M_Q}(l) \) are respectively denoted the BER occurring on the \( k-th \) bit of \( M_I-ary \) PAM and the \( l-th \) bit of \( M_Q-ary \) PAM. Using the derivation of the probability that the \( k-th \) bit in error for the \( I-ary \) PAM, \( P_{M_I} \) and \( P_{M_Q} \) can be respectively given by

\[
P_{M_I}(k) = \frac{1}{M_I} \sum_{i=0}^{M_I-1} \left( \frac{3g}{M_I} \right) \left( \frac{3g}{M_I} \right)^k \left( \frac{1}{M_I} \right) \left( \frac{3g}{M_I} \right)^{M_I-k-1} \left( \frac{2^{k-1} - \frac{3g}{M_I} 2^{k-1} + \frac{1}{M_I}}{2^{k-1}} \right)
\] (18)

\[
P_{M_Q}(l) = \frac{1}{M_Q} \sum_{j=0}^{M_Q-1} \left( \frac{3g}{M_Q} \right) \left( \frac{3g}{M_Q} \right)^j \left( \frac{1}{M_Q} \right) \left( \frac{3g}{M_Q} \right)^{M_Q-j-1} \left( \frac{2^{j-1} - \frac{3g}{M_Q} 2^{j-1} + \frac{1}{M_Q}}{2^{j-1}} \right)
\] (19)
In the (18), (19) \( \gamma \) and \( \zeta \) are the instantaneous electrical SNR per symbol and the quadrature to in-phase decision distance ratio with \( \zeta = d/\beta_d \). Replace (18), (19) with the parameter SNR \( \gamma \) calculated in (9) into (17), we have the instantaneous BER for rectangular QAM/FSO system for AF relaying using APD receiver. And then BER can be calculated by (16) with \( f_x(X) \) for the gamma-gamma channel given in (15). Define \( P_t \) and \( P_\circ \) as

\[
P_t = \sum_{k} \mathcal{A}_k P_{M_t}(k)f_x(x)dx
\]

(20)

\[
P_\circ = \sum_{l} \mathcal{A}_l P_{M_\circ}(l)f_x(x)dx
\]

(21)

The BER of FSO system can be written as

\[
\text{BER} = \frac{1}{\log_2(M_tM_\circ)} (P_t + P_\circ)
\]

(20)

4. NUMERICAL RESULTS

In this section, using previous derived expression, we present numerical results for BER analysis of AF relaying FSO/QAM system using APD receiver over strong atmospheric turbulence modeled by the gamma-gamma distribution. In our analysis, for fair comparison, BER is calculated as the function of the transmitted power per bit \( P_t= P_b/\log_2(M) \) with \( P_b \) is total transmitted power per symbol at source and \( N \) nodes. Also the denotes the average transmitted optical power per symbol at each node \( P_b=P_b/(N+1) \) so the relationship between \( P_b \) and \( P_t \) can be written as \( P_b = P_b \log_2(M)/(N+1) \). The performance analysis is carried out under the influence of different operating condition, which is include the average APD’s gain and thermal, the number of relay node, link distance. Relevant parameter consider in our analysis are provided in Table 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operational wavelength</td>
<td>( \lambda )</td>
<td>1550 nm</td>
</tr>
<tr>
<td>Photodiode responsivity</td>
<td>( \mathcal{A} )</td>
<td>1</td>
</tr>
<tr>
<td>APD’s resistance</td>
<td>( R_A )</td>
<td>1000Ω</td>
</tr>
<tr>
<td>Ionization factor</td>
<td>( k_\lambda )</td>
<td>0.7</td>
</tr>
<tr>
<td>Amplifier noise figure</td>
<td>( F_N )</td>
<td>2</td>
</tr>
<tr>
<td>Boltzmann constant</td>
<td>( k_b )</td>
<td>1.38x10^\text{-23}</td>
</tr>
<tr>
<td>Modulation index</td>
<td>( \kappa )</td>
<td>1</td>
</tr>
<tr>
<td>Electron charge</td>
<td>( e )</td>
<td>1.60x10^\text{-24}C</td>
</tr>
<tr>
<td>Index of refraction structure</td>
<td>( q )</td>
<td>9.10^\text{14} m^\text{-2}/s</td>
</tr>
<tr>
<td>Link distance</td>
<td>( L )</td>
<td>1-8 km</td>
</tr>
<tr>
<td>Aperture diameter</td>
<td>( D )</td>
<td>0.08 m</td>
</tr>
<tr>
<td>Atmospheric extinction coefficient</td>
<td>( \beta_l )</td>
<td>0.1 dB/km</td>
</tr>
<tr>
<td>Beam’s divergence angle</td>
<td>( \theta )</td>
<td>5 rad</td>
</tr>
<tr>
<td>Decision distance ratio</td>
<td>( \zeta )</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 3 show the system BER versus average APD gain \( \bar{g} \) with direct transmission and different relay stations, the number relay stations \( N \)=1,2,3 and 4 are used in analysis, the transmitted power per bit \( P_b=12 \text{ dBm} \) link distance \( L=6000 \text{ m} \), bit rate \( R_b=2 \text{ Gbps} \), \( T=300 \text{ K} \) and using 8x4QAM scheme. As the figure clearly show, BER parameter improves significantly with increasing the number of relay station, and better than direct transmission. For more importantly, it is also seen that the selection of APD’s gain \( \bar{g} \) has a significant impact on the system performance. More clearly, to reach the lowest BER value, the optimal gain value \( \bar{g} \) from 6 to 12 with different relay stations. Also the BER value will decrease when the average APD gain pass over the optimal gain value.

Figure 4 illustrates the BER performance of systems as a function of the average APD gain, for different values of link distance \( L=4000 \text{ m}, 6000 \text{ m}, 8000 \text{ m} \). The number relay stations \( N=3, P_b=8 \text{ dBm}, T=300 \text{ K}, R_b=2 \text{ Gbps} \) are used in this analysis. As you can see, increasing transmission link distance impact to system performance. More clearly, an increase in \( L \) from 4000 m to 6000 m and 8000 m results in a
significant bit error performance degradation. For example, the best BER value reached approximately $10^{-9}$ at $L = 4000$ m with optimal gain $\bar{g} = 6$ and reached from $8 \times 10^{-1}$ to $10^{-2}$ at $6000$ m and $8000$ m with $\bar{g} = 6$ and $\bar{g} = 10$ respectively. The optimal gain varies between 6 and 10.

In Figure 5, we further analysis the effect of temperatures noise on the system performance with transmitted power per bit $P_b = 3$ dBm, $R_b = 2$ Gbps, $L = 2000$ m, $N = 1$. As the figure clearly illustrates, when the degree of receiver increases lead to increase the thermal noise, therefore BER of system will be increased too. BER value reach to $10^{-11}$ at $T = 300$ K and decrease to $2 \times 10^{-8}$ at $T = 1100$ K. In addition, it is shown the impact of temperatures noise on the selection of optimal gain. It is seen that the optimal gain changes more significantly when the receiver temperature varies. The optimal gain also varies from 6 to 10 when the receiver temperature increases between 300K and 1100K.

Figure 6 shown the system BER versus transmitted power per bit $P_b$ at the link distance $L = 6000$ m and optimal gain $\bar{g} = 6$ is selected. The figure illustrates, when the directly transmission from source to destination node, the transmitted power per bit require to reach $BER = 10^{-8}$ about 19 dBm. With the $N$ relay stations, we can obtain performance improvements of 4dB, 6dB, 7dB for $N = 1, 2$ and 3 in comparison with direct transmission. Furthermore, we can see that, with the same transmitted power per bit requirement, while increasing the relay station ($N = 4, 5$), the system performance did not increasing significantly. So in this case, at the link distance $L = 6000$ m we should use maximum $N = 3$ relay nodes to reach the best BER value and reduce system and deployment cost.
5. CONCLUSION

In this paper, we have analyzed the bit error rate of amplify-and-forward relaying for FSO/SC-QAM scheme using APD receiver over gamma-gamma atmospheric turbulence channel. We have derived theoretical expressions for AF FSO/SC-QAM system considering with different relay stations, link distance, the average APD’s gain and thermal. We clearly seen a significantly improvement when deploying relay technique in comparison with direct transmission. Furthermore, the numerical results showed that using APD with proper selection of optimal gain could improve the system performance greatly, and in this work, the optimal gain to reach the best BER of AF FSO/QAM system could vary from 6 to 10 for typical APD receiver.

REFERENCES

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